Electric Circuits and Networks **10-08-2018**

Lecture 6: Laplace domain analysis

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Laplace Transform

KCL: $\sum i_k(t) = 0$ at any node, at any instant of time.

$$i_1(t) + i_2(t) + ... + i_n(t) = 0$$

Taking Laplace transform (L.T) on both sides

$$\int_{0^-}^{\infty} (\sum_k i_k(t))e^{-st}\mathrm{d}t = 0$$
$$\implies \sum_k \Big(\int_{0^-}^{\infty} i_k(t)e^{-st}\mathrm{d}t \Big) = 0$$
$$\implies \sum_k I_k(s) = 0, \qquad I_k(s) : \text{Ampere-sec units}$$

Similarly KVL: $\sum_k v_k(t) = 0 \implies \sum_k V_k(s) = 0$, Units: Volt-sec. Consider all circuits to be LTI unless mentioned otherwise.

Resistor

$$v(t) = Ri(t)$$

Taking L.T on both sides

$$V(s) = RI(s)$$

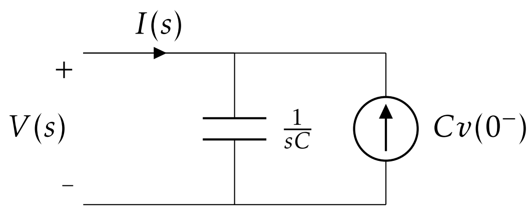
Capacitor

$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

Taking L.T on both sides

$$\begin{aligned} I(s) &= C\int_{0^-}^{\infty} \frac{\mathrm{d}v(t)}{\mathrm{d}t}e^{-st}\mathrm{d}t \\ &= C\Big(v(t)e^{-st}\Big|_{0^-}^{\infty} + s\int_{0^-}^{\infty} v(t)e^{-st}\mathrm{d}t \Big) \\ &= C\Big(-v(0^-) + sV(s) \Big) = -Cv(0^-) + sCV(s) \end{aligned}$$

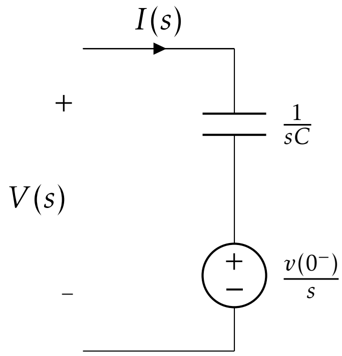
where $v(0^-)$ is the initial voltage across the capacitor. sC is called admittance of capacitor.



Circuit representation 1

$$V(s) = \frac{1}{sC} I(s) + \frac{1}{s} v(0^-)$$

$\frac{1}{sC}$ is called as impedance of capacitor = 1/admittance. Units of impedance is same as resistance.



Circuit representation 2

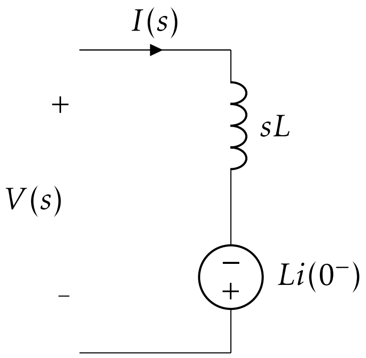
Both circuit representation 1 and 2 equivalent. Choose the one which is easy for analysis.

Inductor

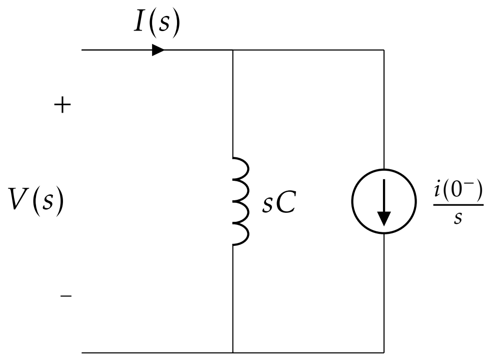
$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

taking L.T

$$V(s) = -Li(0^-) + sLI(s)$$
$$I(s) = \frac{1}{sL} V(s) + \frac{1}{s} i(0^-)$$



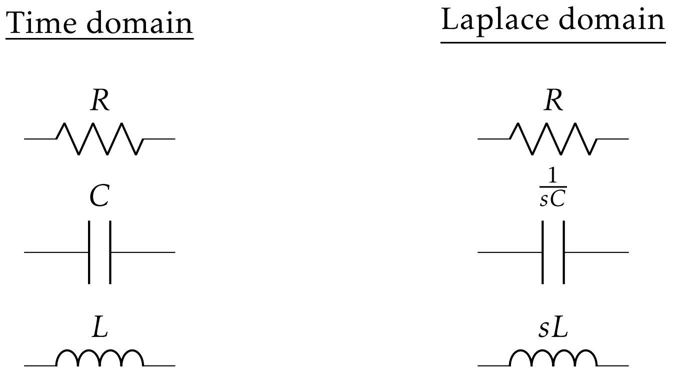
Circuit representation 1



Circuit representation 2

sL is called as impedance of inductor.

Time domain vs Laplace domain



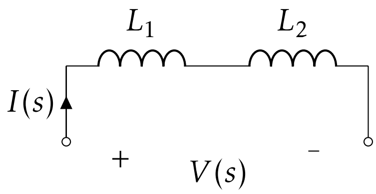
Zero initial conditions

Under zero initial conditions

- For a resistor $V(s) = RI(s)$
- For a capacitor $V(s) = (1/sC)I(s)$
- For an inductor $V(s) = sLI(s)$

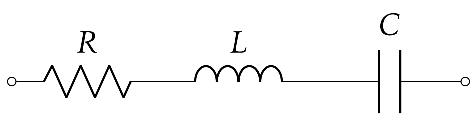
Impedance is represented with $Z(s)$ and admittance is represented with $Y(s) = \frac{1}{Z(s)}$. Impedance in Laplace domain behaves exactly like resistance in time domain.

Example 1. *Impedance for the following circuit*



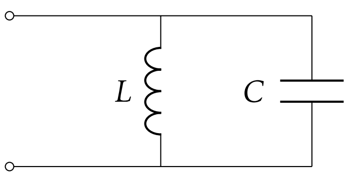
$$\begin{aligned} V(s) &= I(s)(sL_1) + I(s)(sL_2) \\ &= I(s)(sL_1 + sL_2) \\ Z(s) &= sL_1 + sL_2 \end{aligned}$$

Example 2. *Impedance for the following circuit*



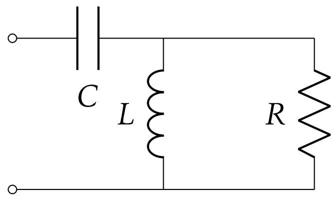
$$Z(s) = R + 1/sC + sL$$

Example 3. *Admittance for the following circuit*



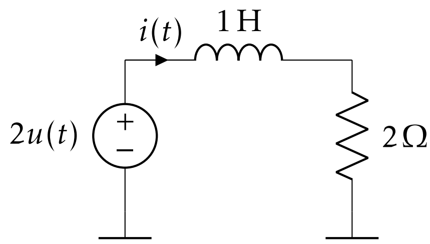
$$Y(s) = sC + 1/sL$$

Example 4. *Admittance for the following circuit*



$$Y(s) = \frac{sC(1/R + 1/sL)}{sC + 1/R + 1/sL}$$

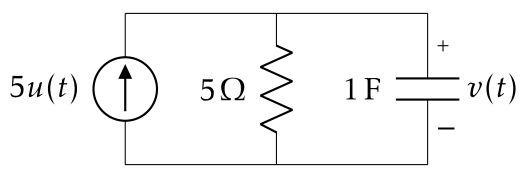
Example 5. *Calculate $i(t)$ in the following circuit. Zero initial conditions.*



$$\begin{aligned} V(s) &= L.T(2.u(t)) = 2/s \\ \implies 2/s &= I(s)(s+2) \\ \implies I(s) &= \frac{2}{s(s+2)} = \frac{1}{s} - \frac{1}{s+2} \end{aligned}$$

$i(t) = L.T^{-1}(I(s)) = (1 - e^{-2t})u(t)$. As $t \to \infty$, $i(t) = 1$. At $t = 0^+$, $i(t) = 0$. For a DC, if the circuit is ON for a long time then the inductor is a short circuit.

Example 6. *Calculate $v(t)$ in the following circuit*



$$\begin{aligned} Y(s) &= 1/5 + s \\ I(s) &= Y(s) \cdot V(s) \\ \implies V(s) &= \frac{5}{s}\left(\frac{5}{1+5s}\right) = 25\left(\frac{1}{s} - \frac{1}{s+1/5}\right) \end{aligned}$$

$v(t) = L.T^{-1}(V(s)) = 25(1 - e^{-t/5})u(t)$. As $t \to \infty$, $v(t) = 25\text{V} \implies$ Capacitor is an open circuit.

In DC, Inductor is a short circuit and capacitor is an open circuit.