**Electric Circuits and Networks** **xx-10-2018**

# Lecture 31: Polyphase Circuits

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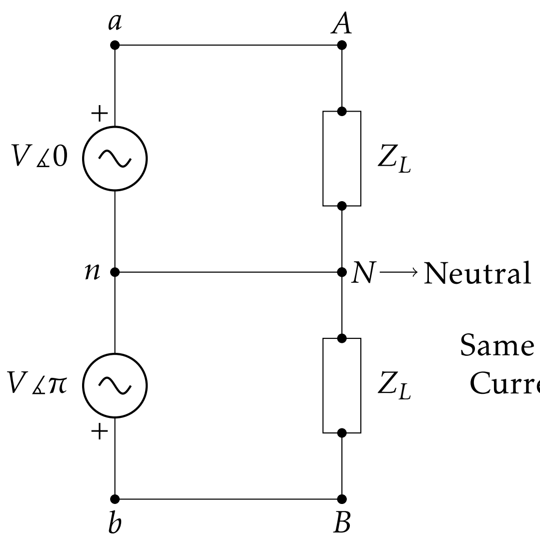
**Polyphase Circuits**

$$
\begin{aligned}
v(t) &= V_m \cos(\omega t + \theta_v) \\
i(t) &= I_m \cos(\omega t + \theta_v - \alpha) \\
& \alpha : \text{impedance angle} \\
p(t) &= v(t)\, i(t) \\
&= V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_v - \alpha) \\
&= \frac{V_m I_m}{2} \left[\cos(2(\omega t + \theta_v) - \alpha) + \cos\alpha\right] \\
&= \frac{V_m I_m}{2} \left[\cos(2(\omega t + \theta_v)) \cos\alpha + \sin(2(\omega t + \theta_v)) \sin\alpha + \cos\alpha\right] \\
&= P\left[1 + \cos(2(\omega t + \theta_v))\right] + Q \sin(2(\omega t + \theta_v))
\end{aligned}
$$

where $P$ is **active power** = $V_{eff} I_{eff} \cos\alpha$. Average value of p(t) $P$. and $Q$ is **reactive power** = $V_{eff} I_{eff} \sin\alpha$. Active power; average is non-zero,but there is a large variation around the average. Motor speed will vary with time. To make speed more uniform; reduce this variation - have multiple signals that have maximum power at different times.

**Bicycles :** 2 power strokes per cycle;
- similar principle also used in multi-cylinder engines in cars.

Consider two sources 180° out of phase

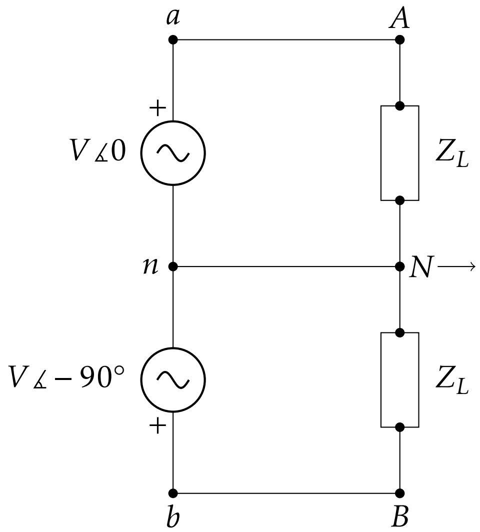



$$V_1(t) = V \cos\omega t$$

$$V_2(t) = -V \cos\omega t$$

$$\Rightarrow p(t) = 2p_1(t)$$

Same variation in power - Not useful
Current in the neutral line (nN) = 0




$$p_1(t) = P(1+\cos 2\omega t) + Q\sin 2\omega t$$

$$p_2(t) = P(1+\cos(2\omega t - \pi)) + Q\sin(2\omega t - \pi)$$

$$P = \frac{|V|^2}{|Z_L|}\cos\alpha$$

$$Q = \frac{|V|^2}{|Z_L|}\sin\alpha$$

$$p_1(t) + p_2(t) = 2P$$ → Instantaneous power is constant

$$I_{Nn} = \frac{V}{Z_L} + \frac{Ve^{-j\pi/2}}{Z_L} \neq 0$$

- 2 phase balanced system. If both phase have the same load, the instantaneous power = constant. However , current in the neutral is not zero.

**3 Phase Systems**

$$\begin{aligned} V_a &= V\measuredangle 0^\circ \\ V_b &= V\measuredangle -120^\circ \\ V_c &= V\measuredangle 120^\circ \end{aligned}$$

If $Z_L$ is the same for all 3 phases, the complex power in each phase can be calculated as follows.

$$S = P + jQ = \frac{|V|^2}{2|Z|}(\cos\alpha + j\sin\alpha)$$

where, $\alpha$ is the impedance angle

$$p_a(t) = P(1+\cos 2\omega t) + Q\sin 2\omega t$$

$$p_b(t) = P\left[1+\cos\left(2\omega t - \frac{4\pi}{3}\right)\right] + Q\sin\left(2\omega t - \frac{4\pi}{3}\right)$$

$$p_c(t) = P\left[1+\cos\left(2\omega t + \frac{4\pi}{3}\right)\right] + Q\sin\left(2\omega t + \frac{4\pi}{3}\right)$$

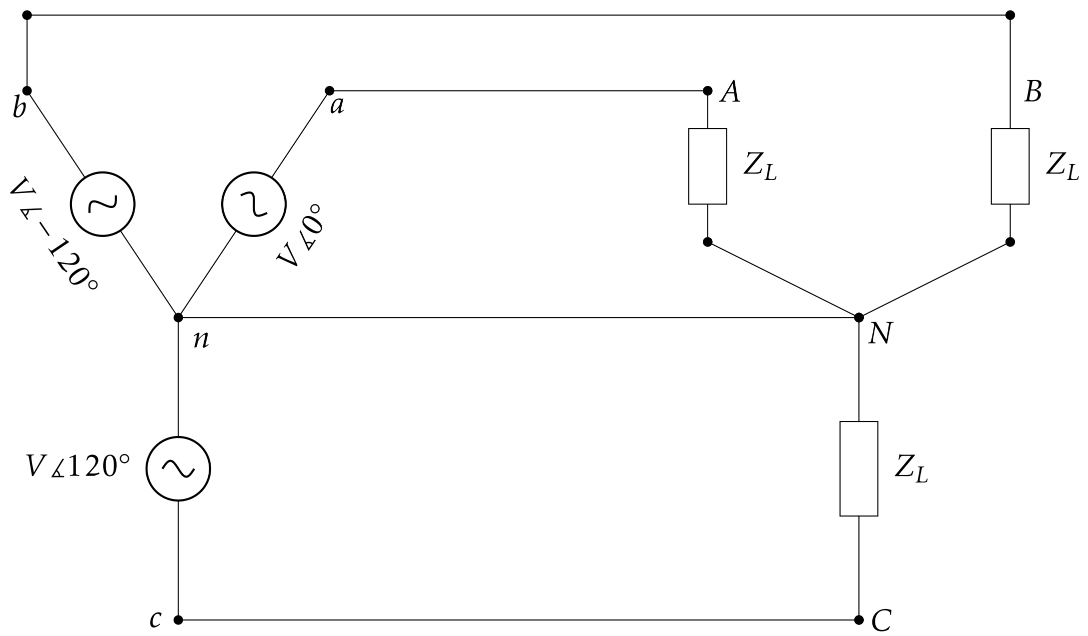
$$\begin{aligned} \cos\left(2\omega t - \frac{4\pi}{3}\right) &= \cos\left(2\omega t + \frac{2\pi}{3} - 2\pi\right) \\ &= \cos\left(2\omega t + \frac{2\pi}{3}\right) \\ \cos\left(2\omega t + \frac{4\pi}{3}\right) &= \cos\left(2\omega t - \frac{2\pi}{3} + 2\pi\right) \\ &= \cos\left(2\omega t - \frac{2\pi}{3}\right) \end{aligned}$$

$$\cos 2\omega t + \cos\left(2\omega t + \frac{2\pi}{3}\right) + \cos\left(2\omega t - \frac{2\pi}{3}\right) = 0$$

&

$$\sin 2\omega t + \sin\left(2\omega t + \frac{2\pi}{3}\right) + \sin\left(2\omega t - \frac{2\pi}{3}\right) = 0$$

$$\begin{aligned}\text{Total Power} &= p_a(t) + p_b(t) + p_c(t)\\ &= 3P \qquad (constant)\end{aligned}$$




$a\ b\ c \rightarrow +ve$ sequence
$a\ c\ b \rightarrow -ve$ sequence

$$\begin{aligned}I_{aA} &= \frac{V}{|Z|}e^{-j\alpha}\\ I_{bB} &= \frac{V}{|Z|}e^{-j(2\pi/3+\alpha)}\\ I_{cC} &= \frac{V}{|Z|}e^{j(2\pi/3-\alpha)}\\ I_{aA} + I_{bB} + I_{cC} &= \frac{V}{|Z|}e^{-j\alpha}\left(1 + e^{-j2\pi/3} + e^{j2\pi/3}\right)\\ &= 0\end{aligned}$$

$I_{Nn}$ : current in neutral = 0.
For 3 phase system, when we have balanced loads, instantaneous power = constant and current in neutral = 0.This is what is used in power distribution systems.