Electric Circuits and Networks	**18/09/2018**

Lecture 19:Two port networks

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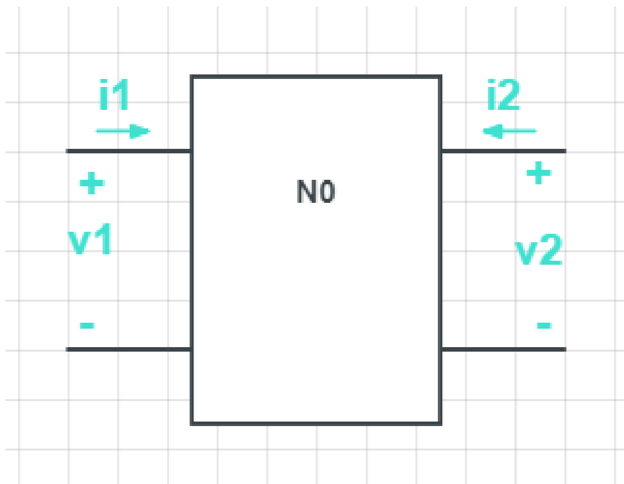
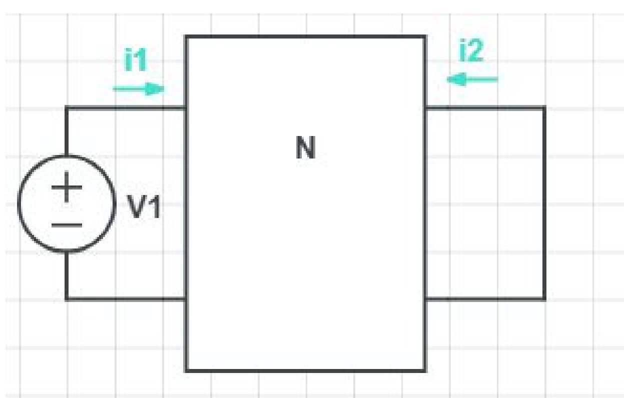
Single port two terminal network: Can connect a current source and measure voltage (find driving point impedance) or can connect a voltage source and find the current (driving point admittance). The admittance and impedances are reciprocals of each other.

Two port four terminal network - two sets of currents and voltages. We can choose any two quantities as inputs and two as outputs, so there are six possibilities.

V_1, V_2, I_1, I_2 are the voltages and currents. By convention, current going into the network is taken as positive.



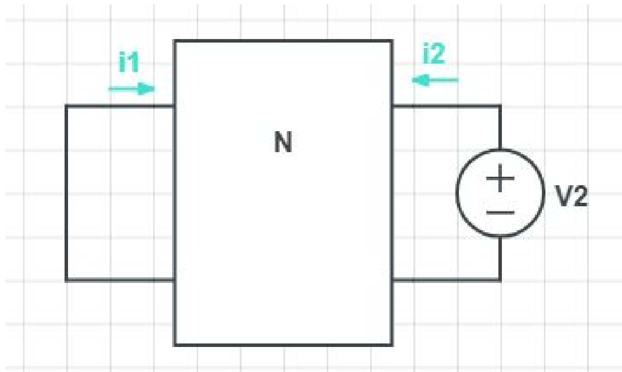
First we choose V_1, V_2 as inputs and I_1, I_2 as outputs and set $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
Measure the Y parameters as follows.



$$y_{11} = \frac{I_1}{V_2}|_{V_2=0}$$

$$y_{21} = \frac{I_2}{V_2}|_{V_2=0}$$

y_{11}, y_{21} are calculated by setting $V_2 = 0$ i.e shorting the output port, adding voltage source at the input port and finding I_1, I_2.



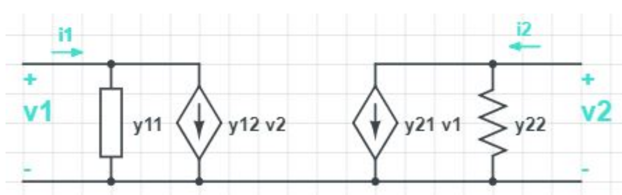
y_{12}, y_{22} are calculated by setting $V_1 = 0$ i.e shorting the input port, adding voltage source at the output port and finding I_1, I_2.

$$y_{12} = \frac{I_1}{V_2}|_{V_1=0}$$
$$y_{22} = \frac{I_2}{V_2}|_{V_1=0}$$

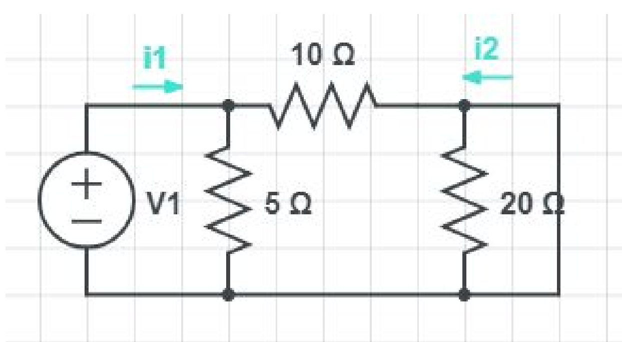
so, the equations are

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The equivalent circuit corresponding to these equations are

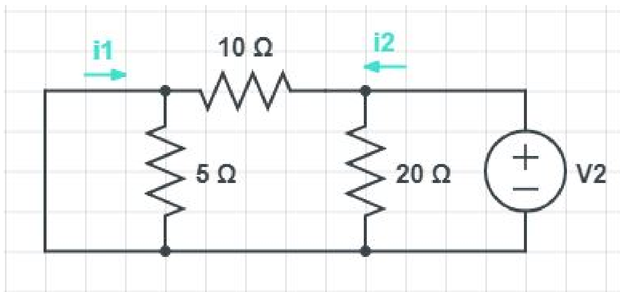


Example: Find Y parameter of the network given below



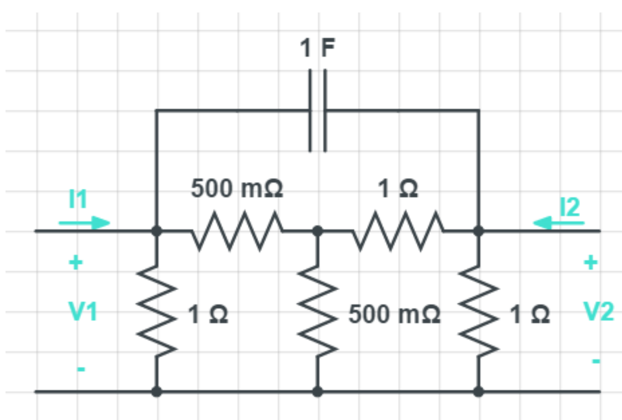
5Ω and 10Ω are in parallel

$$
\begin{aligned}
I_1 &= \frac{V_1}{5} + \frac{V_1}{10} \\
I_2 &= \frac{-V_1}{10} \\
y_{11} &= \frac{1}{5} + \frac{1}{10} = \frac{3}{10}S \\
y_{21} &= \frac{-1}{10}S
\end{aligned}
$$



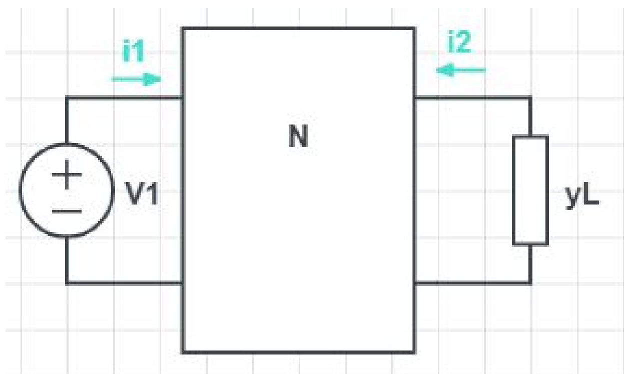
$$
\begin{aligned}
I_1 &= \frac{-V_2}{10} \\
\implies y_{12} &= \frac{-1}{10}S \\
I_2 &= \frac{V_2}{20} + \frac{V_2}{10} \\
\implies y_{22} &= \frac{3}{20}S
\end{aligned}
$$

Example 2: Find Y parameters of the network



$$y_{11} = s + \frac{11}{5}$$
$$y_{21} = -(s + 0.4) = y_{12}$$
$$y_{22} = s + \frac{9}{5}$$

Question:
If a load y_L(Admittance) is connected to output port of 2 port network as shown below, then determine the driving point admittance seen at the input port.



solution:

The admittance at the input port is $\frac{I_1}{V_1}$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Applying ohm's law at output port we get:

$$I_2 = -y_L V_2$$

using the above equations we can get the following matrix:

$$\begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} + y_L \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\implies 0 = y_{21} V_1 + (y_{22} + y_L) V_2$$
$$\implies V_2 = \frac{-y_{21}}{y_{22} + y_L} V_1$$

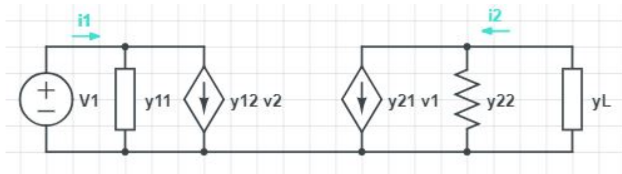
Since

$$I_1 = y_{11} V_1 + y_{12} V_2$$

Substitute V_2 value in above equation to get

$$I_1 = (y_{11} - \frac{y_{21} y_{12}}{y_L + y_{22}}) V_1$$
$$\implies y_{in} = \frac{I_1}{V_1} = y_{11} - \frac{y_{21} y_{12}}{y_L + y_{22}}$$

The network N can be represented as a circuit with admittances and controlled sources, as shown below.



$$gain = \frac{V_{out}}{V_{in}}$$

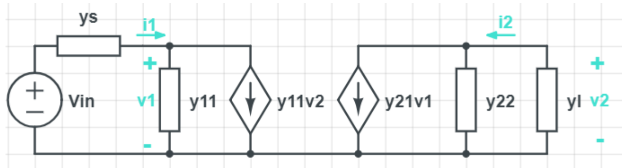
Gain is defined as the ratio between output voltage and input voltage. Here output voltage is voltage across load, i.e voltage at output port.

$$\begin{aligned} gain &= \frac{V_2}{V_1} \\ &= \frac{-y_{21}}{y_{22} + y_L} \end{aligned}$$

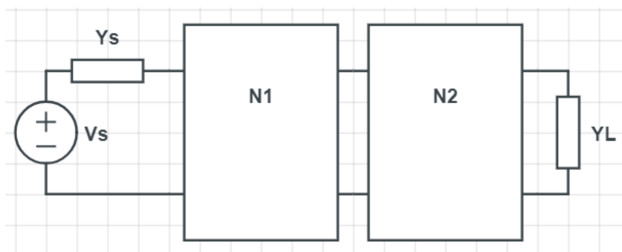
Electric Circuits and Networks	**19/09/2018**

Lecture 20:Two port network (Contd...)

Lecturer: Dr. Vinita Vasudevan — *Scribe: Pradeep*



$$
\begin{aligned}
gain &= \frac{V_2}{V_{in}} \\
&= \frac{V_2}{V_1}\frac{V_1}{V_{in}} \\
&= \frac{-y_{21}}{y_{22}+y_L}\frac{y_s}{y_s+y_{in}}
\end{aligned}
$$



If we have cascaded networks, y_{in_2} will be the load admittance for N_1.
HW: Find the overall gain of the cascaded networks.

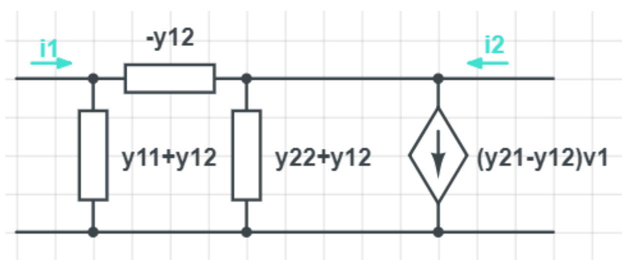
Another equivalent circuit:

$$
\begin{aligned}
I_1 &= y_{11}V_1 + y_{12}V_2 \\
I_2 &= y_{21}V_1 + y_{22}V_2
\end{aligned}
$$

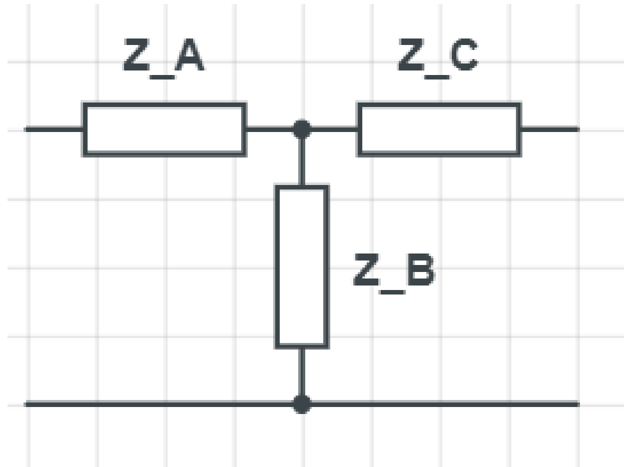
the above two equations can be rewritten as

$$
\begin{aligned}
I_1 &= (y_{11}+y_{12})V_1 + (-y_{12})(V_1 - V_2) \\
I_2 &= (y_{21}-y_{12})V_1 + (y_{22}+y_{12})V_2 + (-y_{12})(V_2 - V_1)
\end{aligned}
$$

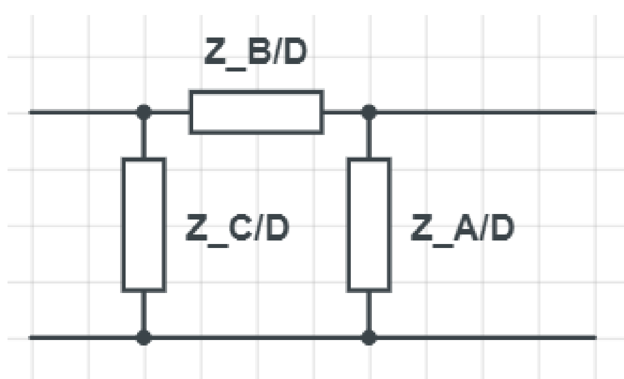
These equations can be represented in a circuit form as follows.



Find the y parameters and represent the following circuit in this equivalent form.

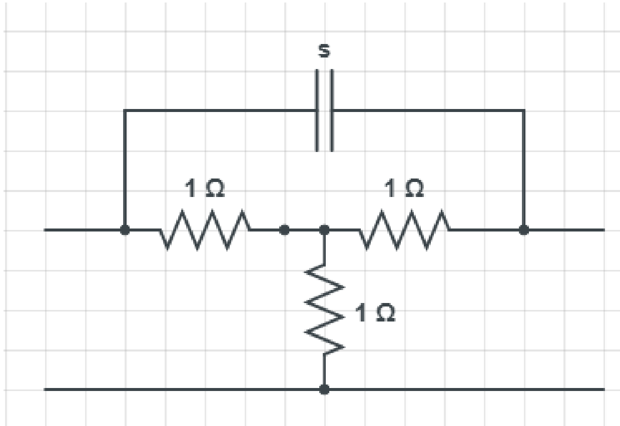


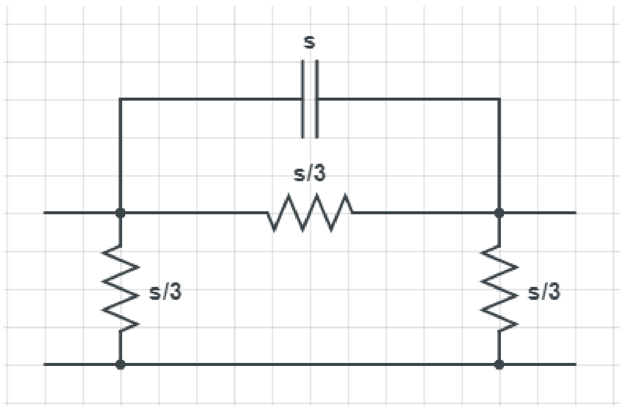
$$
\begin{aligned}
y_{11} &= \frac{(\frac{1}{Z_A})(\frac{1}{Z_B}+\frac{1}{Z_C})}{\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_C}} \\
&= \frac{Z_B Z_C}{Z_A Z_B + Z_B Z_C + Z_C Z_A} \\
\implies V_X &= \frac{V_1(\frac{Z_B Z_C}{Z_B+Z_C})}{Z_A + \frac{Z_B Z_C}{Z_B+Z_C}} \\
I_2 &= \frac{-V_X}{Z_C} \\
y_{21} &= \frac{-Z_B}{Z_A Z_B + Z_B Z_C + Z_C Z_A} = y_{12} \\
y_{22} &= \frac{Z_A + Z_B}{Z_A Z_B + Z_B Z_C + Z_C Z_A} \\
\text{Let } D &= Z_A Z_B + Z_B Z_C + Z_C Z_A
\end{aligned}
$$



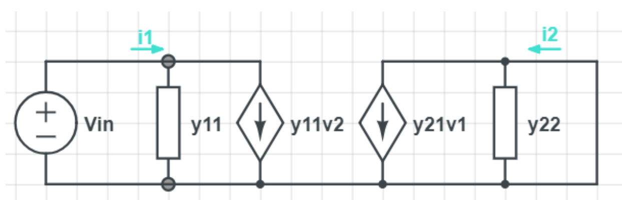
This is called Y-Δ transformation or π-T transformation. Networks for which $y_{12} = y_{21}$ are called reciprocal networks.

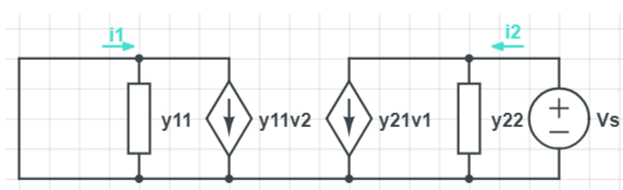
This transformation can be used to simplify analysis of circuits as shown in the following example.





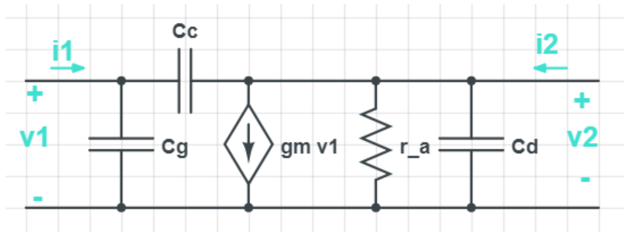
Reciprocal network:





If $y_{12} = y_{21}$; $i_1 = i_2$. Can interchange the position of the voltage source and the point at which current is measured.

Home work:

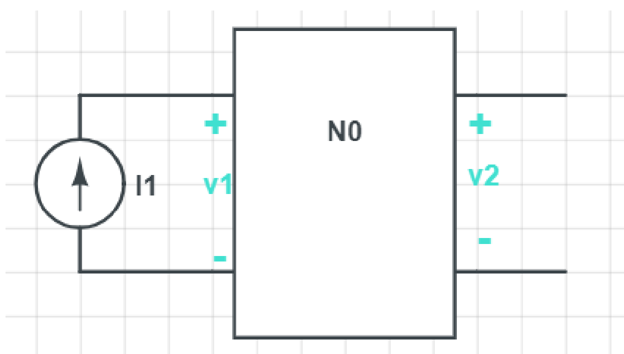


Find Y parameters of above circuit.

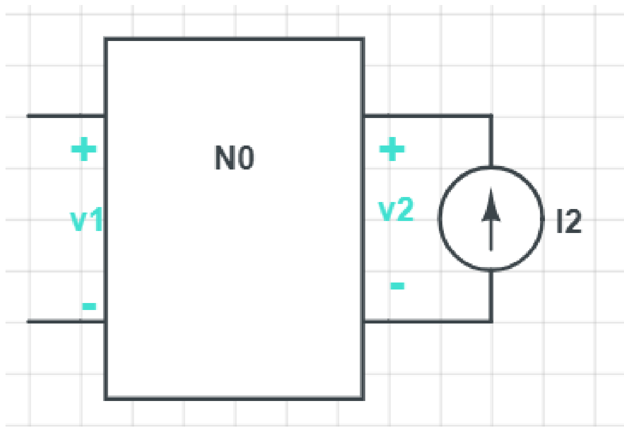
Impedance parameters
2 port equations are:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \frac{v_1}{I_1}|_{I_2=0}$$
$$z_{21} = \frac{V_2}{I_1}|_{I_2=0}$$



$$z_{12} = \frac{V_1}{I_2}|_{I_1=0}$$
$$z_{22} = \frac{V_2}{I_2}|_{I_1=0}$$



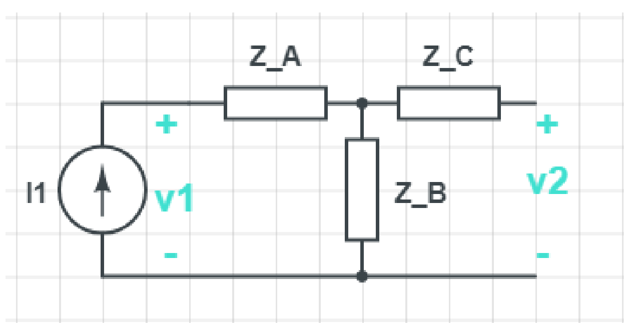
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V = ZI$$

Since

$$I = YV$$

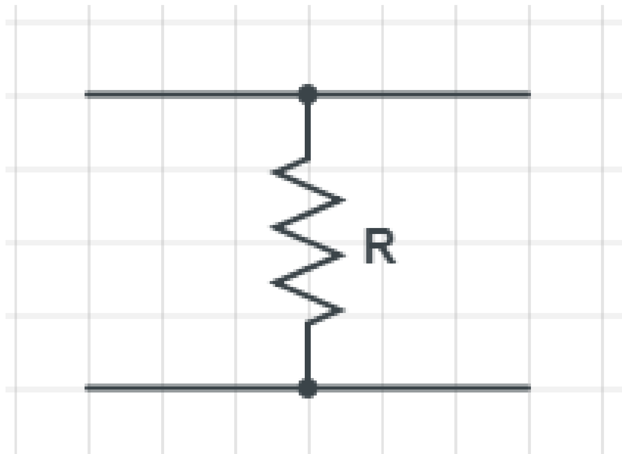
$$\begin{aligned} Z &= Y^{-1} \\ \implies z_{11} &\neq \frac{1}{y_{11}} \\ z_{11} &= \frac{y_{22}}{y_{11}y_{22} - y_{12}y_{21}} \end{aligned}$$



Easy to find Z parameters of a T-network.

$$\begin{aligned} z_{11} &= z_A + z_B \\ z_{21} &= z_B = z_{12} \\ z_{22} &= z_B + z_C \end{aligned}$$

Homework:
Find z parameters of a π network and transform to a T-network.



$$Z = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

The equivalent circuit for the Z parameters is as follows.