Electric Circuits and Networks **28-08-2018**

Lecture 12: Natural frequencies

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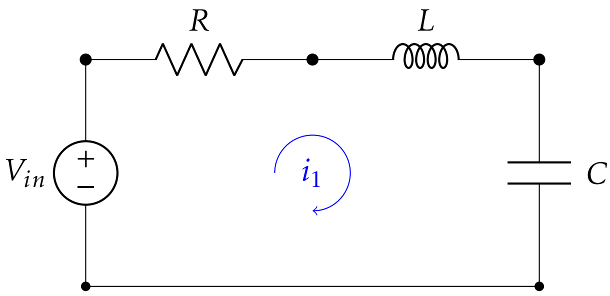
Natural frequencies



$$V_{in} = iR + L\frac{di}{dt} + \frac{1}{C}\int i dt$$
$$\frac{dV_{in}}{dt} = L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C}$$

characteristic equation: Assume solution is e^{st} and substitute in differential equation.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Solve for roots of the characteristic equation and get natural frequencies

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

solution :$k_1 e^{s_1 t} + k_2 e^{s_2 t}$, $s_1, s_2$are the natural frequencies

Equivalently,

$$V_{in}(s) = \left(R + sL + \frac{1}{sC}\right) I(s)$$
$$= \frac{s^2 LC + sRC + 1}{sC} I(s)$$

$$I(s) = \frac{sC}{s^2 LC + sRC + 1} V_{in}(s)$$

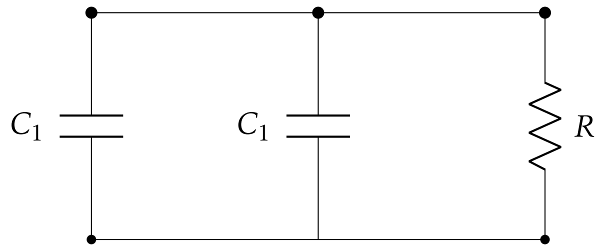
Roots of the denominator polynomial (poles) are the natural frequencies

* Independent of input - roots depends on $R, L, C \Rightarrow$ property of the system and nothing to do with input.

* unforced network; set all input = 0 (short voltage source, open circuit current sources); solve for all branch currents/ voltages using initial conditions.

$$k_1 e^{s_1 t} + k_2 e^{s_2 t} + \cdots + k_n e^{s_n t}$$

zero-input solution: solve $k_1, k_2, \ldots k_n$ using initial voltages across capacitors and currents through inductors. If there is a initial condition such that $k_i \neq 0$ then s_i is a natural frequency.
The number of independent initial conditions that can be specified in the network determines the number of natural frequencies.



initial condition for both capcitors can not be specified independently

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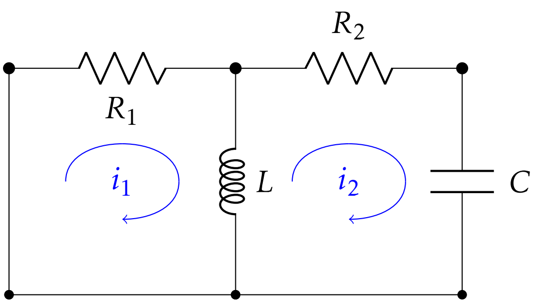


$$\underbrace{\begin{bmatrix} R_1 + sL & -sL \\ -sL & R_2 + sL + 1/sC \end{bmatrix}}_{\text{Network matrix (mesh basis)}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\text{initial conditions}}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} R_2 + sL + 1/sC & sL \\ sL & R_1 + sL \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$I_1 = \frac{(R_2 + sL + 1/sC)\, a + sL\,(b)}{D}$$

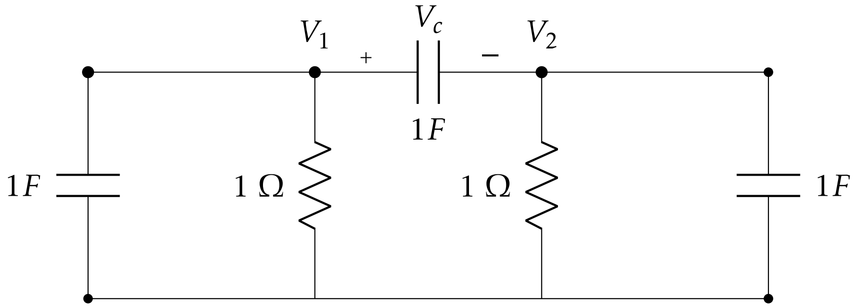
$$D = \frac{s^2 LC\,(R_1 + R_2) + s\,(R_1 R_2 C + L) + R_1}{sC}$$

$$I_2 = \frac{sLa + (R_1 + sL)\, b}{D}$$

Denominator polynomial determines the natural frequencies. Given by the determinant of the network matrix. To get natural frequencies solve for roots of $D = 0$. Note that Both I_1 and I_2 have same denominator polynomial.
The natural frequencies of any response in the circuit will belong to the set given by roots of $D = 0$. All network variables need not have all the natural frequencies.

$$k_1 e^{s_1 t} + k_2 e^{s_2 t} + \cdots + k_n e^{s_n t}$$

Note:
zero input solution: $k_1, k_2, \ldots k_n$ is determined from initial conditions.
Natural response: $k_1, k_2, \ldots k_n$ is determined using both inputs and initial conditions.
When initial conditionas are zero, the zero input reponse is zero, but the natural response is not zero.



$$\underbrace{\begin{bmatrix} 2s+1 & -s \\ -s & 2s+1 \end{bmatrix}}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Node basis Network matrix (admittance matrix)

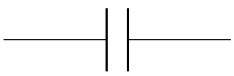
$$D = 3s^2 + 4s + 1, s = -1, -1/3$$

All capacitor voltages cannot be specified independently.

$$\begin{aligned} V_c &= V_1 - V_2 \\ &= \frac{s+1}{D}a - \frac{s+1}{D}b \\ D &= (s+1)(s+1/3) \end{aligned}$$

V_c has only one natural frequency; $s = -1/3$

i, v will have same natural frequency



$$i = C\frac{dV}{dt}$$

$$V = L\frac{di}{dt}$$

same natural frequency

$$\frac{d}{dt}e^{s_1 t} = s_1 e^{s_1 t}$$

$$\int e^{s_1 t} = \frac{1}{s_1}e^{s_1 t}$$

eigenfunction

For all three elements, i and v have the same natural frequencies (Eigenfunction property).