Quantification of Uncertainty in Radar Backscatter Due to Variable Soil Moisture

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Objectives of this talk are:

1. Describe a new, efficient method of computing radar backscatter from random rough surfaces using FEM
2. Quantify the uncertainty in radar backscatter due to variability in soil moisture

Usage scenario

A fieldwork campaign in support of a SAR mission measures soil moisture to calibrate inversion models. How accurate must these measurements be?

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Computing backscatter by Monte Carlo

1. Surface is modelled as a stochastic process (gaussian/exponential correlation functions used). Parameters[1] rms roughness kh, correlation length kl
2. To simulate what the radar observes, multiple computations on multiple surface instances needed & ensemble average
3. How much is good enough? Specify: confidence level (CL) & confidence interval (CI) to estimate statistical significance
   e.g. CI = 1 dB at CL = 95%
   i.e. 19 out of 20 ensemble averages will bracket true mean within 1 dB

[1] $k = 2\pi/\lambda$, where λ is radar wavelength

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Random surface description

1. Traditionally: Filter a sequence of random points by the F.T. of the correlation function[2]
2. Instead: Kosambi-Karhunen-Loeve (KL) expansion[3] is widely used to represent random processes: $s(x,\theta) = s_0(x) + \sum_{k=1}^{\infty} \sqrt{\eta_k} f_k(x) z_k(\theta)$
   - $s_0(x)$ is the mean of the random process
   - η, f solve this eigenvalue problem: $\int C(i,j) f_k(j) dj = \eta_k f_k(i)$ where $C(i,j) = \text{cov}(z_i, z_j)$ is the correlation between two RVs, z_i, z_j
   - $z(\theta)$ represents mutually uncorrelated normal RVs ($\langle z_k \rangle = 0$)
   - Expansion truncated to d terms in practice

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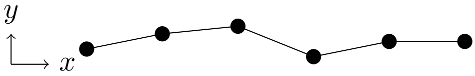
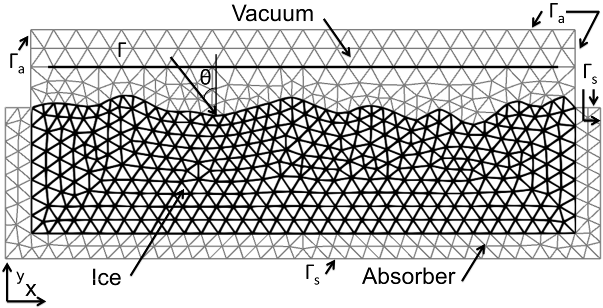
Random surface description : applied to FEM mesh

- KL expansion: $s(x, \theta) = s_0(x) + \sum_{k=1}^{d} \sqrt{\eta_k} f_k(x) z_k(\theta)$
- Discretize this to get a sequence of points:

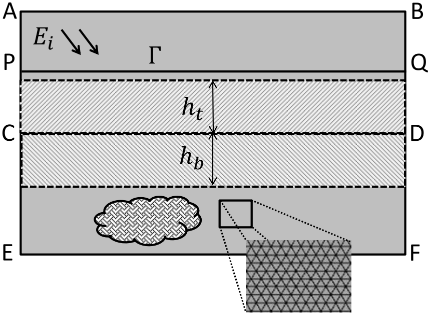


- Apply to whole mesh:



Handle the rough surface intelligently[4]

Partition the domain into parts that move & those that don't



- Move each node smoothly within 'sandwich' region: $y \to y + \Delta y$

$$\Delta y = \begin{cases} s(x)\left(\frac{h_t - y}{h_t}\right), & 0 < y < h_t \\ s(x)\left(\frac{y + h_b}{h_b}\right), & -h_b < y < 0 \end{cases}$$

- CD will deform to rough surface
- Zero deformation by the time $y = h_t$ or $y = -h_b$

[4] Khankhoje et al. , 'Stochastic solutions to rough surface scattering using the finite element method', IEEE TAP 65(8), 2017

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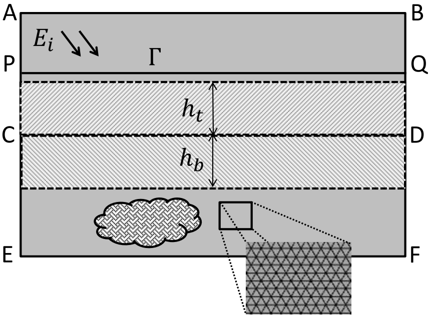

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Motivations of this study

1. In fieldwork campaign, soil moisture measured at few locations
2. This data is used to calibrate an inversion algorithm, but how sensitive is backscatter σ to errors in measuring mv?

Earlier we wrote $\sigma = \sum_{i=1}^{n} \frac{1}{n}\sigma_i(z_1^{(i)}, z_2^{(i)}, \ldots, z_d^{(i)}, mv_o)$
Now: make mv stochastic, e.g. normal distr. $mv = \mathcal{N}(mv_o, \Delta mv)$ and compute $\bar{\sigma} = \sum_{i=1}^{n'} \frac{1}{n'}\sigma_i(z_1^{(i)}, z_2^{(i)}, \ldots, z_d^{(i)}, mv^{(i)})$

- Ask: How are σ, $\bar{\sigma}$ related as a function of Δmv?
- How does it depend on the values of kh, kl, mv_o?

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Setup of numerical experiments

Strategy

Fix: kh, l/h, mv_o, and see $\{\sigma, \bar{\sigma}\}$ for different Δmv

Combinations of following parameters were simulated:

kh	l/h	mv_o	$\bullet$	Δmv
0.05	5	5		0
0.1	20	25		4
0.3	200	-		10

Note: *Entire* domain gets the same (random) value of soil moisture for one simulation

Covers a variety of roughness, correlation lengths, and soil moisture values. Fixed soil composition to {sand=0.51, clay=0.13, silt=0.36}, Hallikainen model[5] to convert soil moisture to permittivity.

[5] Hallikainen et al. , IEEE TGRS 23(1), 1985

Results of numerical experiments – 1

$kh = 0.3, l/h = 5$

$kh = 0.3, l/h = 20$

← HH →

$kh = 0.3, l/h = 5$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	4
5	-13.5	-14
25	-10.2	-10

$kh = 0.3, l/h = 20$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	4
5	-25.5	-25.3
25	-21.8	-21.8

← VV →

$kh = 0.3, l/h = 5$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	4
5	-10	-10.2
25	-3.87	-3.87

$kh = 0.3, l/h = 20$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	4
5	-24.8	-24.8
25	-19.9	-19.9

Recall that all results are within a CI of 1 dB at 95% CL

$\Longrightarrow$ no statistical significance of backscatter variation for rough soils!

Results of numerical experiments – 2

$kh = 0.1, l/h = 20$

$kh = 0.05, l/h = 500$

← HH →

$kh = 0.1, l/h = 20$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	10
5	-22.1	-22.3
25	-18.2	-18.5

$kh = 0.05, l/h = 500$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	10
5	-26.2	-24.6
25	-22	-21.8

← VV →

$kh = 0.1, l/h = 20$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	10
5	-18.8	-19
25	-12.6	-13

$kh = 0.05, l/h = 500$:

$\Delta mv \rightarrow$ $\downarrow mv$	0	10
5	-25.4	-24
25	-20.9	-20.4

Recall that all results are within a CI of 1 dB at 95% CL

$\Longrightarrow$ Tiny statistical significance to backscatter variation for smooth soils!

Inferences and implications – 1

1. Random rough surface: $s(x,\theta) = s_0(x) + \sum_{k=1}^{d} \sqrt{\eta_k} f_k(x) z_k(\theta)$

   $\rightarrow$ Large number ($d > 10$) of random variables for characterization

   $\rightarrow$ Surface randomness swamps out randomness in soil moisture

2. Radar backscatter sensitive only to average soil moisture

   $\rightarrow$ Sufficient to measure s.m. at a few points and average

   $\rightarrow$ Δmv not very significant, so ultra high accuracy not required

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Inferences and implications – 2

1. Only possibility of statistical difference between σ and $\bar{\sigma}$ is when surface is ultra smooth (i.e. small d or large l)
   $\rightarrow$ Unlikely that s.m. is the physical QoI in such cases

Estimating s.m. from SAR doable if effect of roughness can be undone!

2. Interesting future extension: What is the impact on radar backscatter if soil moisture is spatially inhomogeneous?
   Much larger number of random variables $\rightarrow$ might compete better with rough surface random variables!
   BUT, computationally intense.

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