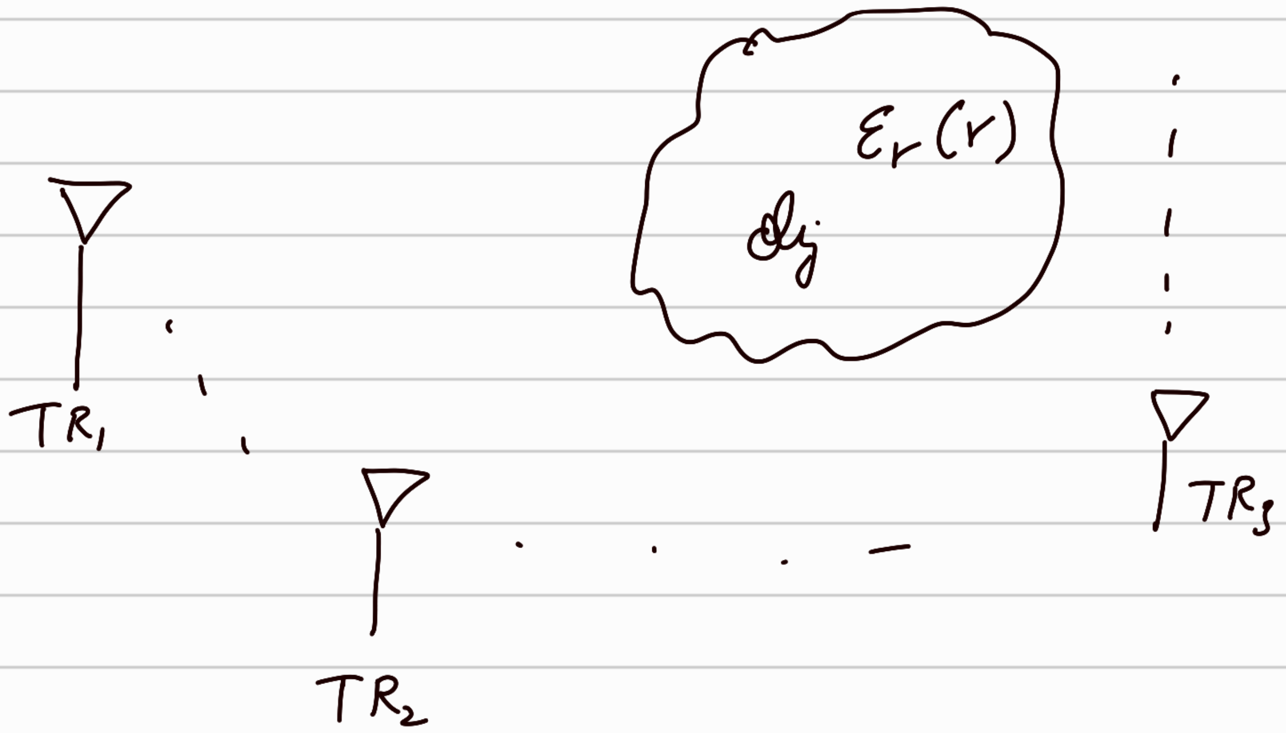
Lecture notes thanks to Charan

Microwave Imaging / Inverse Scattering



What do we want:

$\varepsilon_r(r)$ for microwave frequency

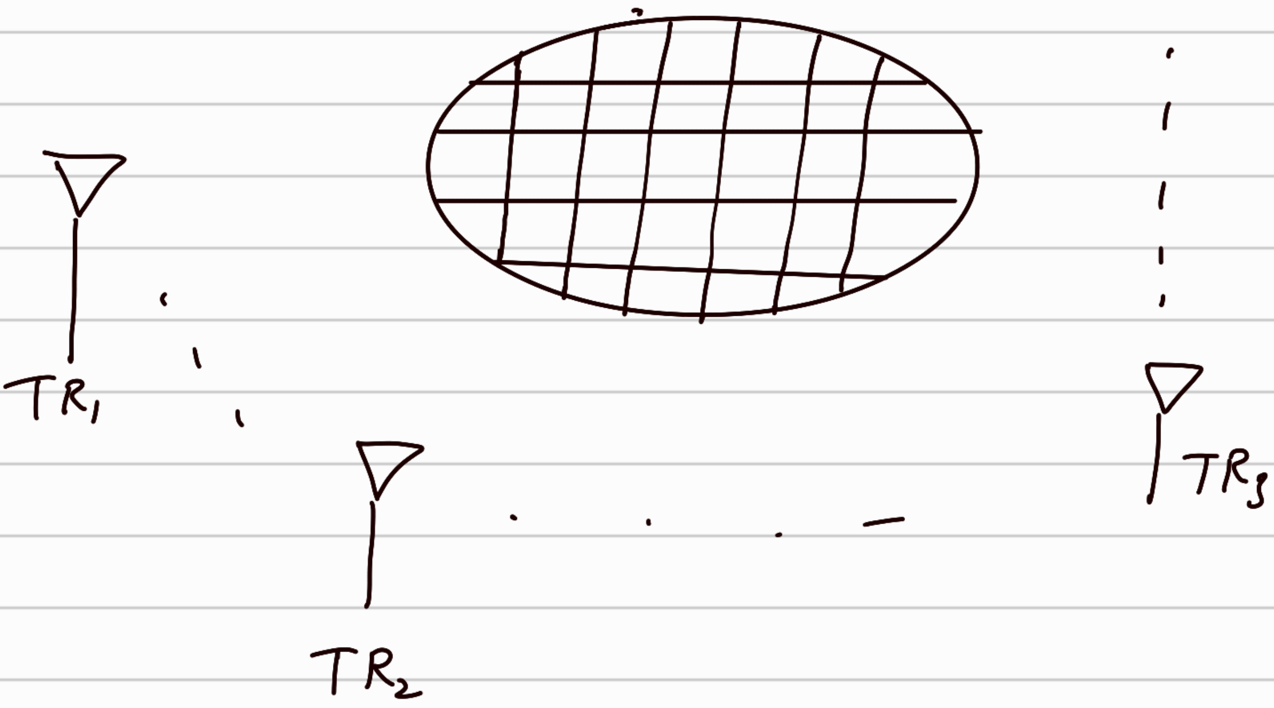
$\Downarrow$

for the object

Tradeoff between Resolution vs penetration.

$\Rightarrow$ High frequency $\uparrow$ Resolution $\downarrow$ penetration

$\Rightarrow$ Low " $\downarrow$ " $\uparrow$ "



Let one of the TR be a Tx and rest Rx

now when there's a change in ε_r there will be reflections.

We sequentially make each turn Tx and rest Rx and solve equations.

2D

$$\longrightarrow (x, y) \longrightarrow (E_z, H_x, H_y)$$

↳ one polarization

1) Step 1: No object

$$\underbrace{\nabla^2 E_i + k_0^2 E_i}_{\text{Inc field, no object}} = j\omega\mu \underbrace{J_z}_{\text{Tx currents}}$$

2) Step 2: With object

$$\nabla^2 E + k_0^2 \varepsilon_r(r) E = j\omega\mu J_z$$

Stays same

<u>Subtract:</u>

$$\nabla^2 \left(E(r) - E_i(r)\right) + k_0^2 \varepsilon_r(r) E(r) - k_0^2 E_i(r) = 0$$

$\Downarrow$ algebra

$$\nabla^2 (E - E_i) + k_0^2 (E - E_i) = -k_0^2 (\varepsilon_r - 1) E$$

<u>Green's</u>

$$\nabla^2 G(r, r') + k_0^2 G(r, r') = -\delta(r - r')$$

$$\nabla^2 f(r) + k_0^2 f(r) = -\phi(r)$$

$\phi(r')$ and integrate over r'

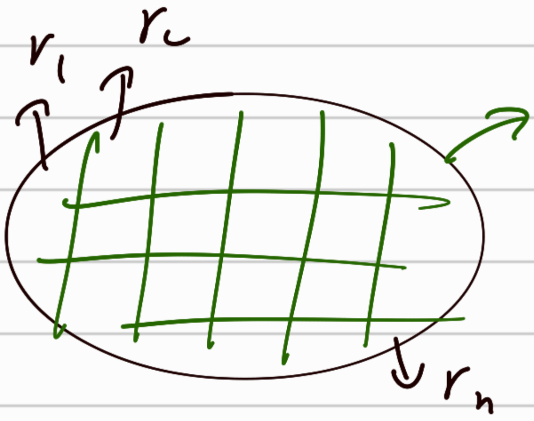
$$f(r) = \int G(r, r') \phi(r') dr'$$

$$\Rightarrow E(r) - E_i(r) = \int_{\Omega_{\text{object}}} G(r, r') k_0^2 (\varepsilon_r(r') - 1) E(r') dr'$$

Fredholm integral eqn of 2nd kind

We want to solve for $E(r)$

given $E_i(r)$ is known



Lets split the object into n sectors

$$E(r_1) - E_i(r_1) = \sum_{\substack{r'=r_i \\ i=[1:n]}} G(r_1, r')\, k_0^2 \,(\varepsilon_r(r') - 1)\, E(r')$$

$$E(r_2) - E_i(r_2) = \underbrace{\qquad\qquad}_{n \text{ terms}}$$

n equations n variables $\Rightarrow$ Assume $\varepsilon_r(r)$ known

Let $u = \begin{bmatrix} E(r_1) \\ E(r_2) \\ \vdots \\ E(r_n) \end{bmatrix}$ $\quad e = \begin{bmatrix} E_i(r_1) \\ E_i(r_2) \\ \vdots \\ E_i(r_n) \end{bmatrix}$

$$G_D = \begin{bmatrix} G(r_1,r_1) k_0^2 & G(r_1,r_2) & \cdots \\ \vdots & G(r_2,r_2) & \\ \vdots & & \end{bmatrix}$$

$n \times n$ $\quad$ $n \times 1$

$$u - e = G_D \begin{bmatrix} [\varepsilon_r(r_i) - 1] E[r_i] \\ \vdots \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_r(r_1) - 1 & 0 & 0 & 0 \\ 0 & \varepsilon_r(r_2) - 1 & & \\ 0 & 0 & \ddots & \\ 0 & 0 & & \ddots \end{bmatrix} u$$

Diagonal

$\Downarrow$

X

$$u - e = G_D X u$$

$$\boxed{u = (I - G_D X)^{-1} e} \quad - (1)$$

$\Downarrow$

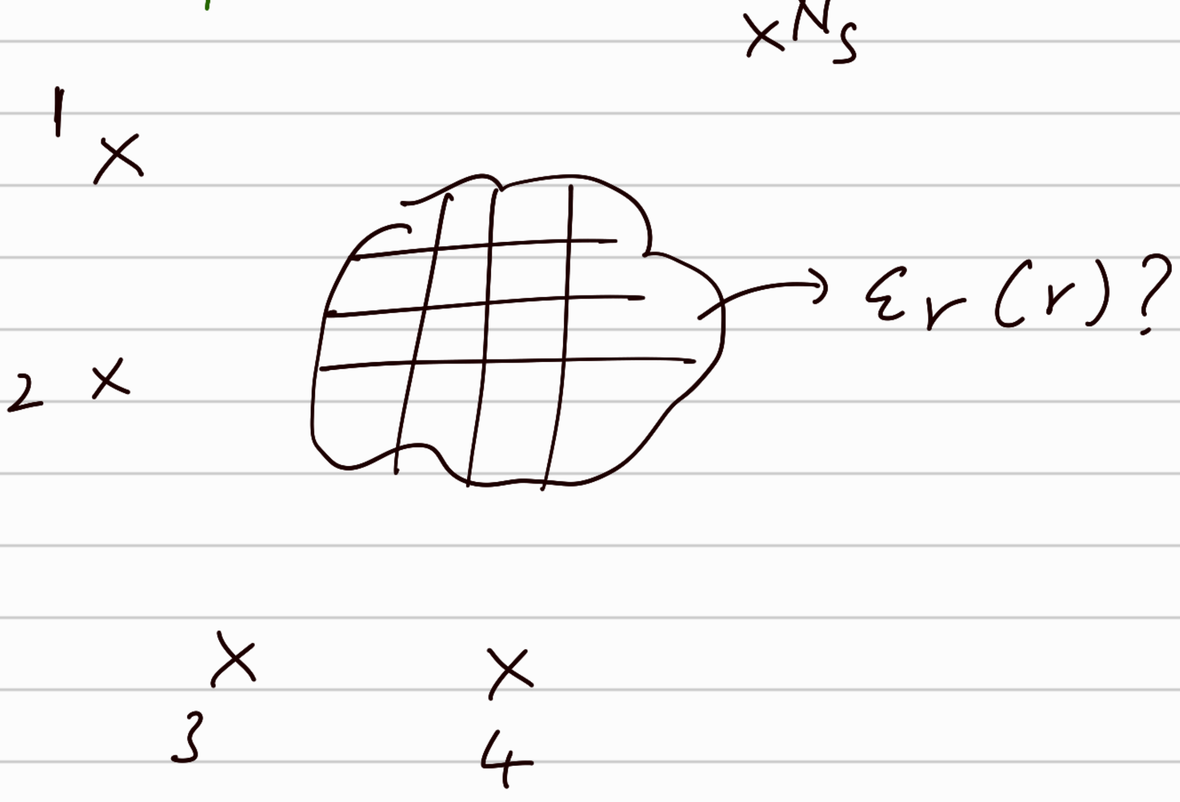
Now lets talk about some $r \notin \Omega$

Let $S = \begin{bmatrix} E_{scat}(r_1) \\ \vdots \\ E_{scat}(r_{N_s}) \end{bmatrix}$

$$S = G_s X u = G_s U x \quad -(2)$$

Note this will change

Forward problem done Now inverse problem:



Keep 1 Tx on collect N_s → scat field

$$\tilde{S} = G_s X u + \eta \rightarrow \text{noise}$$

$$\tilde{x} = \underset{x}{\operatorname{argmin}} \| \underset{\text{data}}{\underset{\uparrow}{\tilde{S}}} - \underset{\text{prediction}}{\underset{\downarrow}{G_s X u}} \|$$

→ 1^{st} approach $\| \tilde{S} - G_s X [I - G_d X]^{-1} e \|$

↳ non linea

→ Simplest approach $u \approx e$

→ Born iterative method

→ Pick a guess X_0 → Calc u_0

→ Use (3) with u_0 → Get x_1

→ $\min \left[\sum_j \| s^{(j)} - G_s X u^{(j)} \| \right]$

→ For different transmitters on ad different off.