Antenna Arrays

Uday Khankhoje

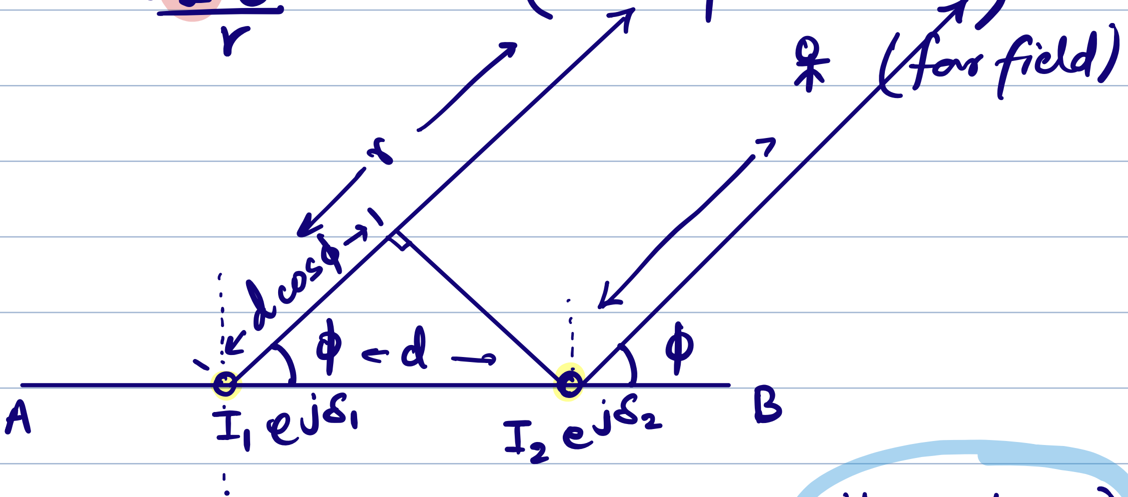
$$E_\theta = \frac{E_0 I_0 \sin\theta\, e^{-jkr}}{r} \qquad \text{(Hertz dipole) (Far field)}$$

$$E = E_0 I_0 f(\theta,\phi) \frac{e^{-jkr}}{r} \qquad \text{(a general antenna)}$$

E_0: constant. I_0: current. $f(\theta,\phi)$: element specific, <u>element pattern</u>. $\frac{e^{-jkr}}{r}$: spherical plane wave

For an isotropic antenna, we have:

$$E = K \frac{I e^{-jkr}}{r} \qquad \text{(isotropic antenna)}$$



$$E_1 = K I_1 e^{j\delta_1} \frac{e^{-jkr}}{r}, \qquad E_2 = k I_2 e^{j\delta_2} \frac{e^{-jk(r-d\cos\phi)}}{(r-d\cos\phi)}$$

Assumption: $d \ll r$, amplitude $r - d\cos\phi \approx r$

phase: $k(r-d\cos\phi)$

$$= \frac{2\pi r}{\lambda} - \frac{2\pi d\cos\phi}{\lambda} \quad \text{leave as is.}$$

Net Electric field: $E = E_1 + E_2$

$$= K I_1 e^{j\delta_1} \frac{e^{-jkr}}{r} + k I_2 e^{j\delta_2} \frac{e^{-jk(r-d\cos\phi)}}{r}$$

$$E = K\frac{e^{-jkr}}{r} \left\{ I_1 e^{j\delta_1} + I_2 e^{j\delta_2} e^{jkd\cos\phi} \right\}$$

What all can we play with?

① Phase difference, i.e δ_1 & δ_2

$$E = \frac{k e^{-jkr}}{r} e^{j\delta_1} \left\{ I_1 + e^{j(\delta_2-\delta_1)} e^{jkd\cos\phi} I_2 \right\}$$

$$|E| = \left|\frac{k}{r}\right| \underbrace{\left| I_1 + I_2 e^{j\overbrace{(\delta_2-\delta_1)}^{\delta}} e^{jkd\cos\phi} \right|}$$

Assume $d = \lambda/2 \Rightarrow kd = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi$

$$|E| = \left|\frac{k}{r}\right| \underbrace{\left| 1 + e^{j\delta} e^{j\pi\cos\phi} \right|} |I|, \quad \text{take } I_1 = I_2$$

$$|E| = \left|\frac{2kI}{r}\right| \cos\left(\underbrace{\frac{\delta + \pi\cos\phi}{2}}\right)$$

When this is $m\pi$
$|E|$ is max.

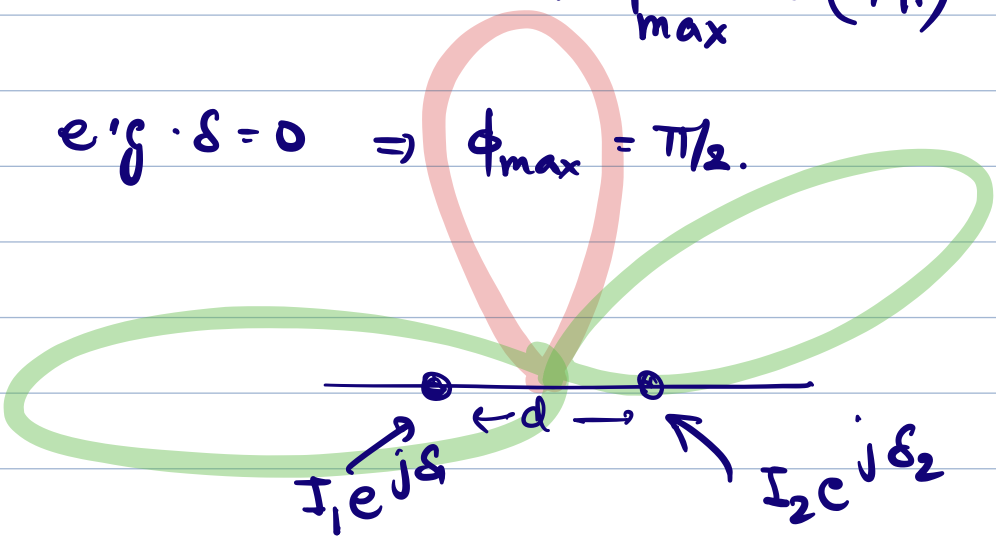
$$\frac{\delta + \pi\cos\phi}{2} = m\pi$$

Take $m = 0$,

$$\frac{\delta}{2} + \frac{\pi}{2}\cos\phi = 0 \Rightarrow \cos\phi = -\frac{\delta}{\pi}$$

$$\Rightarrow \phi_{max} = \cos^{-1}(\delta/\pi)$$

e.g. $\delta = 0 \Rightarrow \phi_{max} = \pi/2$.



phased array antenna.

② Change 'd', element spacing, assume $\delta = 0$

$$|E| = \left|\frac{2kI}{r}\right| \left|\cos\left(\frac{\pi d \cos\phi}{\lambda}\right)\right|$$

$\left(\frac{d}{\lambda}\right) \rightarrow$ We want $|E|_{max}$: $\frac{\pi d \cos\phi}{\lambda} = m\pi$

$$\phi_{max} = \cos^{-1}\left(\frac{m\lambda}{d}\right)$$

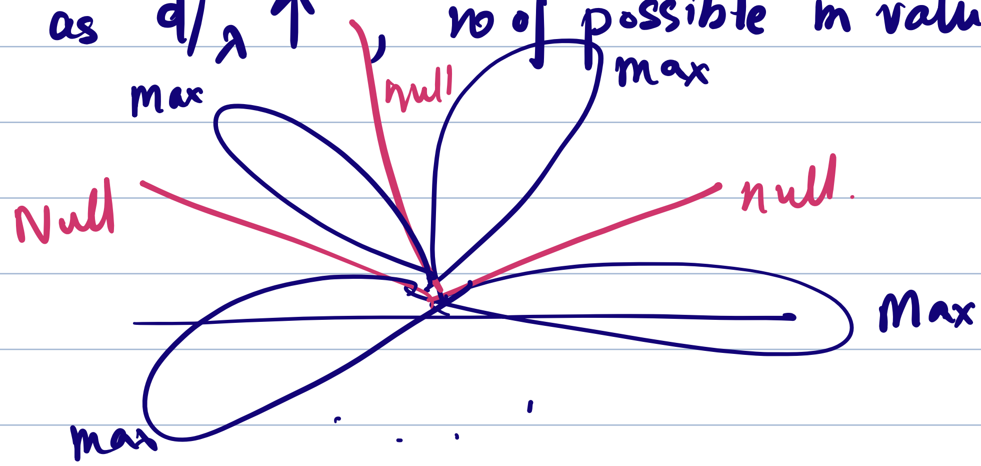
$|\,| \leq 1$

$\frac{m\lambda}{d} \leq 1$

$$\left[\text{Max allowed value of } m = \text{floor}\left(\frac{d}{\lambda}\right) + 1\right]$$

Suppose $\frac{\pi d \cos\phi_{null}}{\lambda} = (m + 1/2)\pi$

See as $d/\lambda \uparrow$, no of possible m values $\uparrow$



For single max $\frac{d}{\lambda} < 1$.

③ Effect of $I_1/I_2 \rightarrow$ amplitude ratio.

$$|E| = \left|\frac{kI_1}{r}\right| \left|1 + \frac{I_2}{I_1} e^{j(\delta + kd\cos\phi)}\right|$$

Useful for shaping beams.

$$E = E_0 I_0 f(\theta,\phi) \frac{e^{-jkr}}{r}$$

E_0 → lanl, I_0 → current, $f(\theta,\phi)$ → element specific, $\frac{e^{-jkr}}{r}$

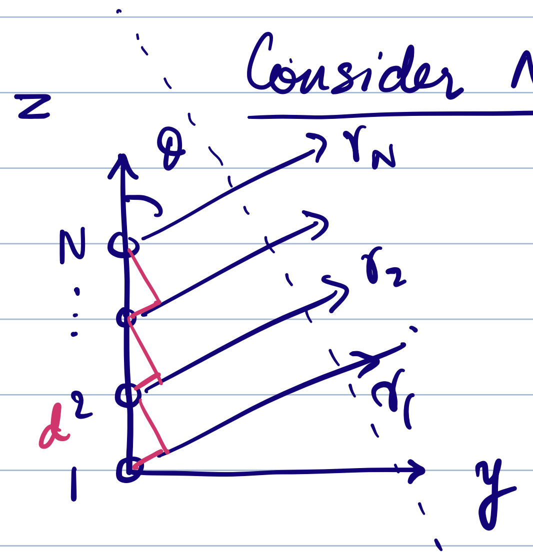
Say 2 identical antennas ⇒ $f(\theta,\phi)$ is the same.
⇒ E_0 is the same

$$\text{Net } E = E_0 I_1 e^{j\delta_1} f(\theta,\phi) \frac{e^{-jkr}}{r} + \cdots$$

$$E = \underbrace{\frac{E_0 e^{-jkr}}{r} I_1 f(\theta,\phi)}_{\text{single element}} \underbrace{\left\{ 1 + \frac{I_2}{I_1} e^{j\delta} e^{jkd\cos\phi} \right\}}_{\text{due to arrays.}}$$

Net rad pattern = (element pattern) × (Array factor.)

"Pattern multipliation"



Consider N-element array (isotropic elems)

$$I_n = I_0 e^{jn\beta}$$

progressive phase shift

$$r_1 = r_2 + d\cos\theta$$

$r_n = r_{n+1} + d\cos\theta$ & so on.

$$\therefore AF = 1 + e^{j\beta} e^{jkd\cos\theta} + e^{2j\beta} e^{j2kd\cos\theta} + \cdots$$

$$+ \cdots + e^{(N-1)j[\beta + kd\cos\theta]}$$

$$\Rightarrow AF = \sum_{n=0}^{N-1} e^{j(n\psi)} \qquad \psi = \beta + kd\cos\theta$$

$$= \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \qquad \text{Sum of a GP.}$$

$$\Rightarrow |AF| = \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{\psi}{2}\right)}$$

Implications:

① Beamforming. → If I want to send a beam to θ_0, when is AF max?

when $\psi = \beta + kd\cos\theta_0 = 0$

$$\Rightarrow \beta = -kd\cos\theta_0$$

With this expression we can compute all sorts of things, e.g. null locations, beamwidth, side lobe levels, etc

(2) **Gain** : At this max location, θ_0,

What is AF? % form, AF = N.

$\therefore$ Efield = N $E_{elements}$

$\Rightarrow$ Power = $N^2 P_{elem}$.

$$\Rightarrow \underbrace{P_{ar,dB}}_{\substack{\downarrow \\ \text{got current} \\ N I_0}} = \underbrace{20 \log N}_{} + \underbrace{P_{elem,dB}}_{\substack{\downarrow \\ \text{got current } I_0}}.$$

{careful here, as array/elem comparison not on equal footing as they are getting different powers.}

Inp power to array $\propto N I_0^2$

↓

to compare with a single element with same power → give $I' = \sqrt{N I_0^2} = \sqrt{N} I_0$.

$P_{elem,corr} \propto (I')^2 = N I_0^2$.

$$\therefore \frac{P_{elem,corr}}{P_{elem}} = N$$

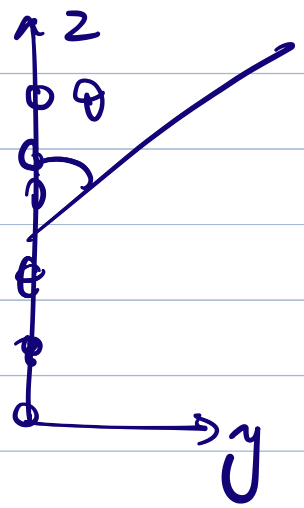
$$\therefore P_{array} = N^2 P_{elem} = N^2 \frac{P_{elem,corr}}{N}$$

$$= N \, P_{elem,corr}$$

$$\Rightarrow P_{ar,dB} = \underbrace{10 \log N}_{\text{Array gain}} + P_{elem,corr,dB}$$

Array gain.

(3)



2 extreme types of array configs:

① Broadside $\Rightarrow \theta_0 = 90^\circ$

② End fire $\Rightarrow \theta_0 = 0, 180^\circ$

↓ Terms are w.r.t. array axis.

Broadside $\overset{\psi}{=} \beta + kd\cos\theta_0 = 0 \Rightarrow \beta = 0$

$$|AF| = \frac{\sin N\psi/2}{\sin \psi/2}, \quad \psi = kd\cos\theta$$

if $d = \lambda$, $kd = \frac{2\pi}{\lambda}\lambda = 2\pi$

$$\Rightarrow |AF| = \frac{\sin(N\pi\cos\theta)}{\sin(\pi\cos\theta)}.$$

At $\theta = 0$? $0/0$

$$\therefore \lim_{\theta \to 0} |AF| = N, \quad \text{same as at } \theta = \pi/2 \,!$$

This is called a grating lobe.

$\therefore$ Broadside array $d_{max} < \lambda$

Turns out, for endfire array $d_{max} < \lambda/2$.

$\therefore$ Generally we keep $d \sim \lambda/2$.

<u>Directivity</u>

$$D_0 \approx 2N(d/\lambda)$$

$$D_0 \approx 4N(d/\lambda)$$

Arrays can be generalized to 2D, 3D.

Non ideality? <u>Mutual coupling</u>.

gets worse with $d \downarrow$.