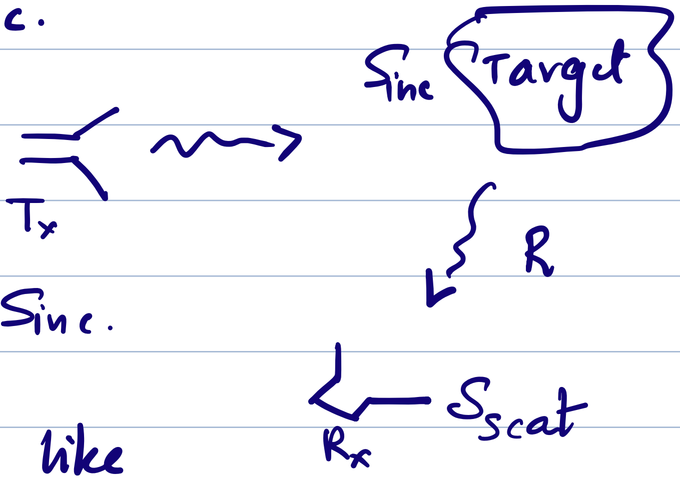
→ From here very easy to include scattering objects → Friis Radar Range eqn.

First, a <u>defn</u>: Radar cross-section.

Like the effective area of an ant:

Recall: $A_e = \dfrac{P_{load}}{S_{inc}}$.

This is the scene:



Here "P_{load}" $= \sigma \times S_{inc}$.

Then the target acts like an isotropic source,

So $$S_{scat} = \frac{\text{"}P_{load}\text{"}}{4\pi R^2} = \frac{\sigma S_{inc}}{4\pi R^2}$$

$$\Rightarrow \sigma = 4\pi R^2 \frac{S_{scat}}{S_{inc}}$$ <u>defn</u>

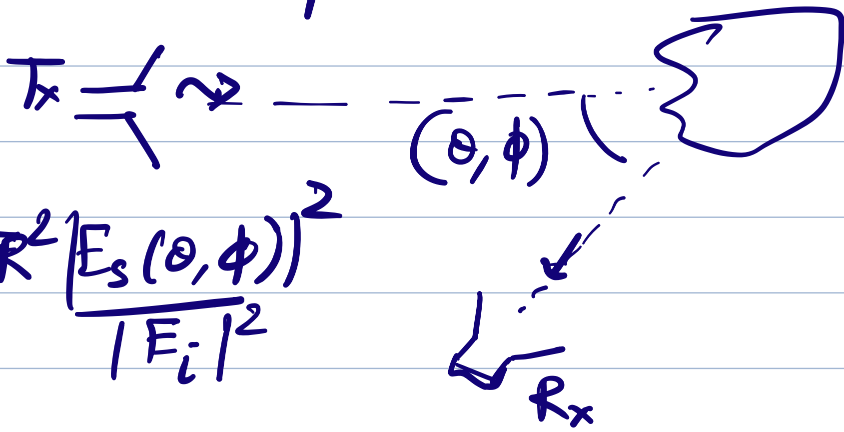
Actually, $$\sigma = \lim_{R\to\infty} 4\pi R^2 \frac{S_{scat}}{S_{inc}}$$

$$= \lim_{R\to\infty} \left[4\pi R^2 \frac{|E_{scat}|^2}{|E_{inc}|^2} \right]$$

Note: E_{scat} will itself be $\propto 1/R$, so

σ will not depend on R.

In general



$$\sigma(\theta,\phi) = \lim_{R\to\infty} 4\pi R^2 \frac{|E_s(\theta,\phi)|^2}{|E_i|^2}$$

If Tx & Rx co-located → monostatic RCS

If not co-located → bistatic RCS

<u>units?</u> $\sigma \to m^2$ or dBsm (dB per sq m)

<u>3 options</u> i.e. $10\log\left(\frac{\sigma}{1m^2}\right)$

or dB (RCS per λ^2)

i.e. $10\log\left(\frac{\sigma}{\lambda^2}\right)$

Some typical values: in dBsm

jumbojet	: 20
fighterjet	: 7–8
human	: 0
missile	: −3
drone	: −20
stealth jet	: −60

Typical radars, anywhere from 1 to 40 GHz

→ Scattering cross-section of a Rx antenna: (a timely topic)

Motivation: we have seen how in the case of an Rx antenna, apart from power delivered ($\frac{1}{2}|I|^2 R_L$) we also have re-radiated (or scattered) power ($\frac{1}{2}|I|^2 R_r$).

Q: What is the RCS of an Rx antenna?
Does it depend on → Antenna geometry?
→ Antenna load?

(1) First we need a bridge between Maxwell's eqns & ckt theory because in ME we do not talk of voltages, but the tx/Rx models of antennas have voltages.

Here comes the feedgap model: Antenna terminals modelled as a gap of width δ in the conductor, accross this gap a tangential E_{gap} can exist. So the "terminal voltage" is defined as

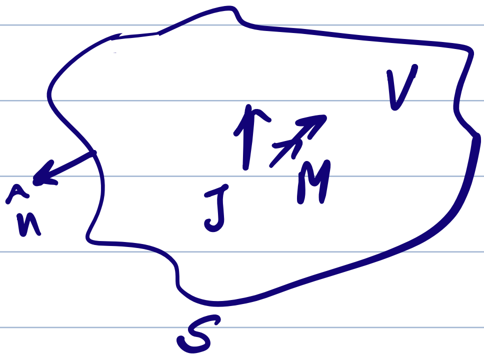
$$V = -\int_{\delta} \vec{E} \cdot d\vec{l}.$$

Everywhere else on the PEC surface of the antenna: $\hat{n} \times \vec{E} = 0$.

∴ In terms of B.C? We are specifying the value of $\boxed{n \times \vec{E}}$ everywhere.

(2) The second supplimentary concept is about the use of super position theorem in EM.

Consider a volume V, bounded by S with sources (impressed) J, M, material properties $\varepsilon_r(r)$, $\mu_r(r)$, and $\hat{n}\times\vec{E}$ / $\hat{n}\times\vec{H}$ specified everywhere on S. This is a complete setup to give unique fields everywhere in V.



(1) Maxwell's eqns have operators such as $\nabla\times$ and $\nabla\cdot$. These are linear. ⇒

$$\nabla\times(a+b) = \nabla\times a + \nabla\times b \quad \text{and}$$
$$\nabla\cdot(a+b) = \nabla\cdot a + \nabla\cdot b.$$

(2) Boundary conditions are also linear operators, as they are just $\boxed{\hat{n}\times}$

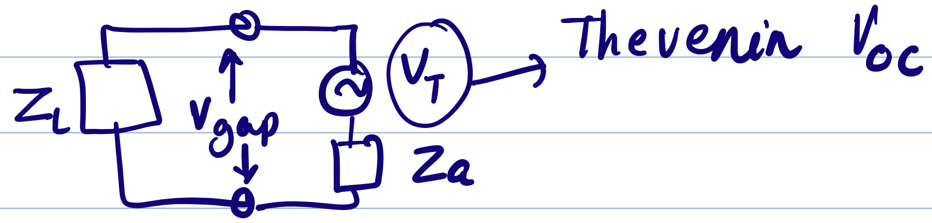
$$\Rightarrow \quad \hat{n}\times(\bar{a}+\bar{b}) = \hat{n}\times\bar{a} + \hat{n}\times\bar{b}$$

(3) Material properties are not linear operators since they appear as multiplications with E or H: $\varepsilon_r E$, $\mu_r H$.

→ Consider three problems in same V, S.

	(1)	(2)	(1) − (2)
Sources	J_1, M_1	J_2, M_2	J_1-J_2, M_1-M_2
Mat.	ε_r, μ_r	ε_r, μ_r	ε_r, μ_r
B.C.	$\hat{n}\times E_1(r) = \bar{f}_1(r)$	$\hat{n}\times\vec{E}_2(r) = \bar{f}_2(r)$	$n\times[E_1-E_2) = f_1 - f_2$
fields	$\vec{E}_1$, $\vec{H}_1$	$\vec{E}_2$, $\vec{H}_2$	E_1-E_2, H_1-H_2.

Superposition problem can have different B.C.



③ Now consider an antenna & 3 different problems on the same structure

ⓐ R_x mode, short ckted load; i.e. $Z_L = 0$.
boundary conds & sources?
→ $n \times \bar{E} = 0$, everywhere (including gap)
→ $\vec{J}_{inc}$ produces E_{inc}
→ Induced I_s.

total field outside: $E_a = E_{inc} + E_s(0)$ — Corresponds to $\frac{1}{2}|I|^2 R_r$.

$\downarrow$ indicates load

$I_s = V_T / Z_a$. $(\because Z_L = 0)$

ⓑ T_x mode with no $\vec{J}_{inc}$.
boundary conds?
→ $n \times E_b = 0$ everywhere on PEC
$n \times E_b = n \times \underbrace{E_{gap}}_{\text{we specify.}}$ in the gap

→ Some induced current I_t flows in ant

Total field outside: $E_b = E_t$, $(\propto I_t)$

ⓒ R_x mode, with a load Z_L across the terminals.

$\Rightarrow$ Current I_L flows & we get a gap voltage $V_L = I_L Z_L$, where
$V_L = -\int \vec{E}_{gap} \cdot \vec{dl}$

boundary condns?
→ $n \times E_c = 0$ on the PEC away from gap
→ At the gap $n \times E_{gap}$ with $V_L = -\int E_{gap} \cdot dl$
→ J_{inc}
→ Induced current I_L.

Total field $E_c = E_{inc} + E_s(Z_L)$

(4) Now the key thought expt: subtract case c & case a. Linearity of ME allows us to do this.

- $\Delta E = E_c - E_a$, $J_{inc} = 0$
- $\therefore$ Induced current $= I_L - I_s$
- $n \times \Delta E$? Outside gap $n \times \Delta E = 0$ (due to PEC)

Inside the gap? $n \times E_a = 0$
$n \times E_c \neq 0$ due to V_L

$\Rightarrow n \times \Delta E \neq 0$ which looks like V_L.

In summary :- no incidence
- impressed E field at gap
PEC b.c. everywhere else.

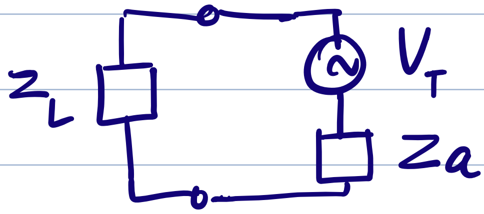
$\Rightarrow$ Doesn't this look like problem (b)?

YES! By uniquess thm, we can say the fields are same!

Any changes? Only the current.
In problem (b) → E_t came from I_t
In diff problem (c) − (a) → $I = I_L - I_s$

$$\Rightarrow \quad \Delta E = \frac{I_L - I_s}{I_t} E_t .$$

(5) Now let's come back to the main problem.



where Z_L : load
and $Z_a = (R_r + R_L) + jX_a$

<u>case 1</u>: No load at terminal : $Z_L = 0$
(short ckt) $I_s = \dfrac{V_T}{Z_a}$.

<u>Case 2</u> : Load at the terminal : Z_L

$$\Rightarrow \quad I_L = \frac{V_T}{Z_L + Z_a} = \frac{I_s Z_a}{Z_L + Z_a} .$$

Now the crucial step: Rewrite I_L and think of super position.

$$I_L = I_s + (I_L - I_s)$$

current flowing in ckt with Z_L

short ckt current, no Z_L

Balance current

$$I_L - I_s = I_s \frac{Z_a}{Z_a + Z_L} - I_s = \frac{-Z_L}{Z_L + Z_a} I_s.$$

$\Rightarrow$ fields due to these currents

$$E_s(Z_L) = E_s(0) + \frac{I_L - I_s}{I_t} E_t$$

$$E_s(Z_L) = E_s(0) - \frac{I_s}{I_t} \frac{Z_L}{Z_L + Z_a} E_t \quad \text{— (1)}$$

fields when s.c. load

fields when non s.c. load.

The above is one complete formulation as it separates into no load & load scattering terms.

(6) Another common way to peel the carrot: Say that we terminate the antenna with a load for optimal power tx, i.e. $Z_L = Z_a^*$. Then:

$$E_s(Z_a^*) = E_s(0) - \frac{I_s}{I_t} \frac{Z_a^*}{Z_a + Z_a^*} E_t \quad \text{— (2)}$$

Now substract (2) & (1)

$$E_s(Z_L) = \underbrace{E_s(Z_a^*)} - \underbrace{\frac{I_s}{I_t} E_t \left(\frac{Z_a^*}{Z_a + Z_a^*} - \frac{Z_L}{Z_L + Z_a} \right)}$$

Interpretation? First term: Structural scattering, doesn't depend on load, only on antenna geom.

Second term: Antenna mode scattering
- since proportional to E_t.
- depends on load.

<u>Note</u>: Vanishes when $Z_L = Z_a^*$.

From here, the usual defn of σ follows

$$\sigma = \lim_{R\to\infty} 4\pi R^2 \frac{|E_s(Z_L)|^2}{|E_i|^2}$$

and it clearly depends on both geometry & load.