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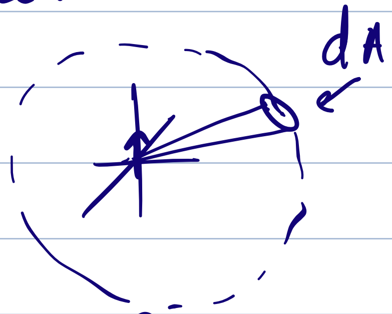
Antenna Descriptions

Starting point: Poynting vector (far field).
e.g. for a Hertz dipole:

$$\vec{S}_{rad} = \frac{\eta}{8}\left(\frac{I\Delta z}{\lambda}\right)^2 \frac{\sin^2\theta}{r^2}\,\hat{r}$$



From here, the total power is

$$W = \oint \vec{S}\cdot \underbrace{dA}_{r^2\sin\theta\, d\theta\, d\phi}\,\hat{r} =$$

Recall:
$$S = \frac{1}{2}\mathrm{Re}[E\times H^*]$$

Another way of seeing this integral:

$$\text{Solid angle} = d\Omega = \frac{dA}{r^2}$$

$$\therefore W = \oint r^2\vec{S}\cdot d\Omega\,\hat{r}$$

which is equivalent to $\oint \vec{S}\cdot dA\,\hat{r}$

Which is more convenient to compute and more practical for discussion?

Since $|S| \propto 1/r^2 \rightarrow$ I will need to know r to talk/compute $|S|$.

But not so for $|Sr^2|$ since in f.f $S \propto \frac{1}{r^2}$.

$\therefore$ More convenient qty is $U = r^2 S$
which is called radiation intensity.
units ? U: W/solid angle.
S: W/m^2

More precisely: $U = \frac{r^2}{\eta}|E|^2$

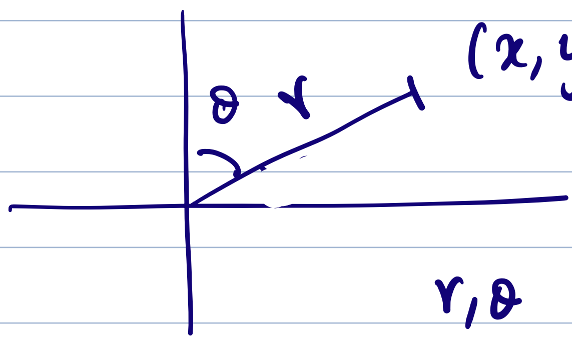
$$= \frac{r^2}{\eta}\Big[\underbrace{|E_\theta|^2 + |E_\phi|^2}_{}\Big]$$

depend on $\frac{1}{r}$.

e.g. for a Hertz dipole $U = U_0 \sin^2\theta$

Then total radiated power $= \int U \, d\Omega$
$= \iint U \sin\theta \, d\theta \, d\phi$

Also a useful qty to plot in polar coordinates.



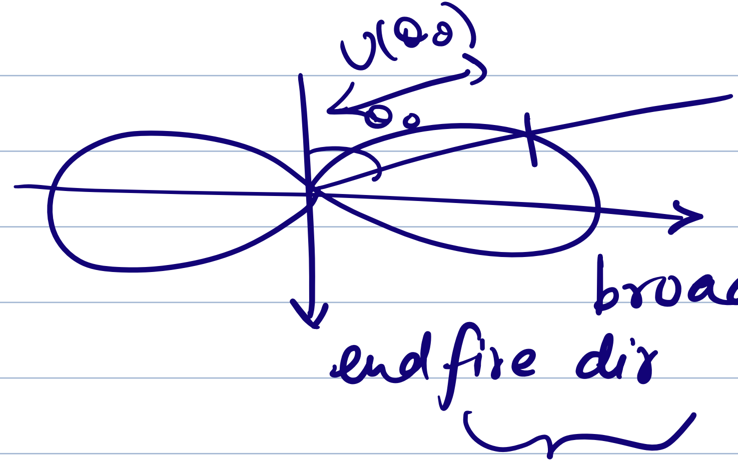
$(x, y) = r(\sin\theta, \cos\theta)$

r, θ plot $\rightarrow$ not the r, θ of the antenna but of the 2D plot.

So when we want to plot rad pattern of Hertz dipole

Map: $r \rightarrow U$
$\theta \rightarrow \theta$

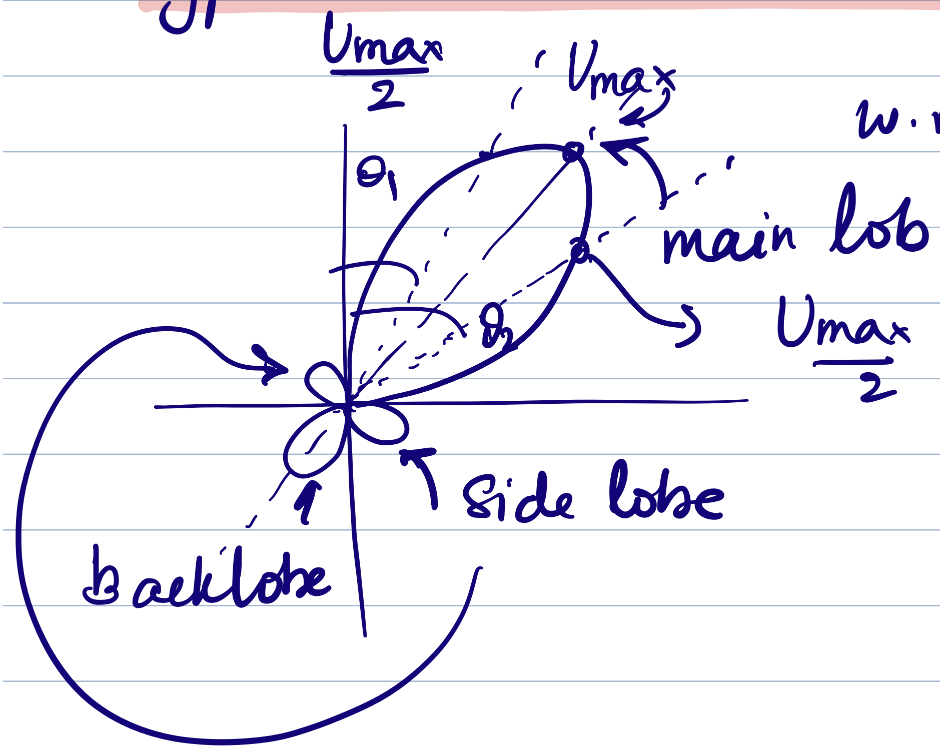
⇒



θ from 0 to 360.

Typical Radiation Patterns

w.r.t. antenna axis.



Half power beam width
$= \theta_2 - \theta_1$
$=$ HPBW

Directivity: Way to characterize the focussing power of an ant.

In general: $D(\theta,\phi) = \dfrac{U(\theta,\phi)}{\langle U(\theta,\phi)\rangle}$

where $\langle U(\theta,\phi)\rangle = \dfrac{\int U(\theta,\phi)\, d\Omega}{\int d\Omega}$

i.e. avg intensity.

Typically when antenna directivity is mentioned without qualification, it refers to $D(\theta,\phi)\big|_{max}$.

e.g. Hertz dipole $U = U_0 \sin^2\theta$

$$\Rightarrow \langle U \rangle = \frac{U_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, \sin\theta \, d\theta \, d\phi}{4\pi}$$

and $D_{max} = 3/2 . \longleftrightarrow 1.76 \text{ dB}$

an isotropic antenna would be $D = 1$
$\Rightarrow 0 \text{ dB}$.

Antenna gain : Takes ohmic losses into account.

$$P_{input} = W_{rad} + P_{loss}$$

$$\text{efficiency} = \eta_r = \frac{W_r}{P_i}$$

book uses e_{cd}.

$$\text{So gain} = G = \eta_r \times D$$

$$G_{dB} = \underbrace{10 \log \eta_r}_{<0} + D_{dB}$$

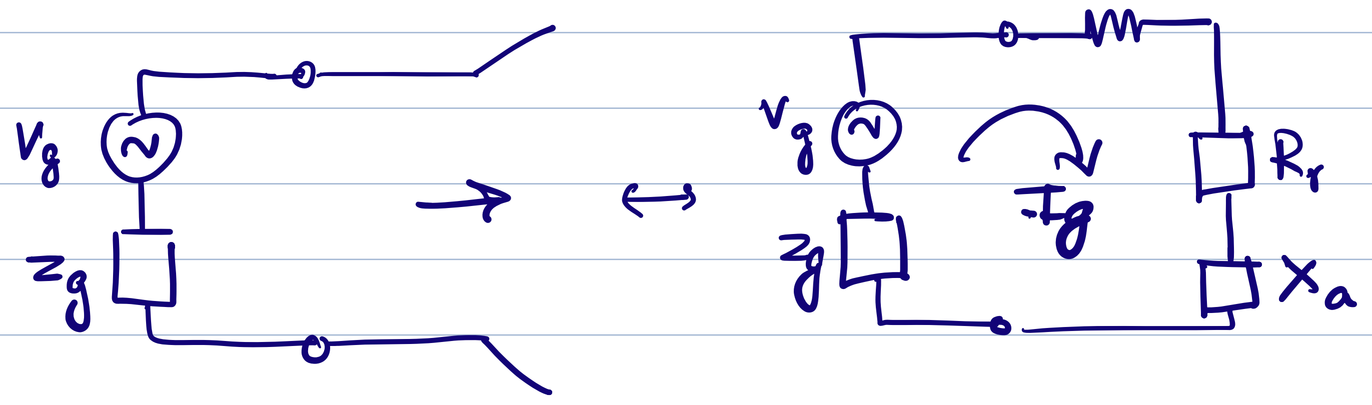
Another related term: 'Realized' gain

Above we assumed that input is matched to antenna impedance.

If not, $G_{re} = \underbrace{(1 - |\Gamma|^2)}_{\text{reflection loss}} \underbrace{\eta_r D}_{G}$

book uses e_r

↳ Antenna ckt models.



generator antenna

Z_a {
R_L : loss resistance
R_r : Rad resistance
X_a : Ant reactance

Max power corresponds to near field reactive energy.

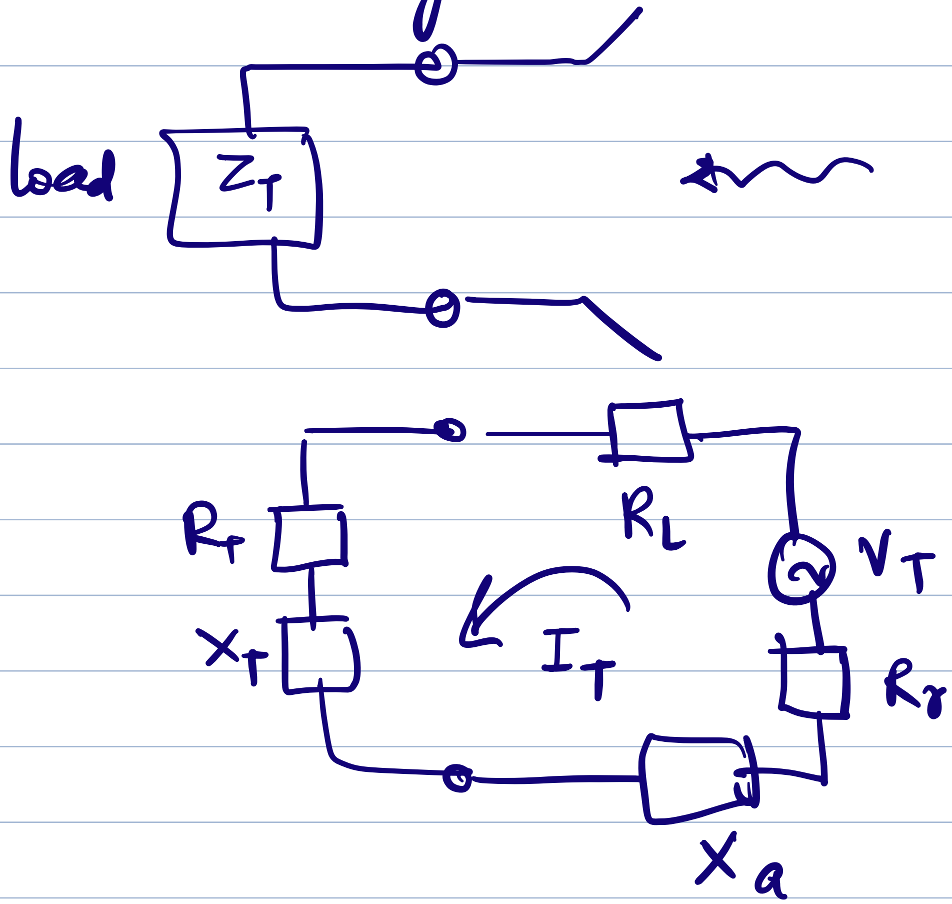
$Z_g = Z_a^*$, i.e.

$R_g = R_L + R_r$

$X_g = -X_a$.

Power radiated by ant: $P_r = \frac{1}{2}|I_g|^2 R_r$

Similarly in the R_x mode



In this case, collected power = $P_c = \frac{1}{2}|I_T|^2 R_T$

$P_T = \frac{1}{2}|I_T|^2 R_T \longrightarrow$ to load

$P_r = \frac{1}{2}|I_T|^2 R_r \longrightarrow$ scattered / re radiated

$P_L = \frac{1}{2}|I_T|^2 R_L \longrightarrow$ heat

Total $P_c = \frac{1}{2} V_T I_T^* \rightarrow$ collected power.

Equivalent Antenna areas.

Effective aperture area, $A_e = \dfrac{\text{Power avl at terminal}}{\text{inc power density}}$.

$$\Rightarrow A_e = \frac{\frac{1}{2}|I_T|^2 R_T}{S_i}.$$

$$= \frac{\frac{1}{2}|V_T|^2}{S_i}\left[\frac{R_T}{(R_L + R_r + R_T)^2 + (X_A + X_T)^2}\right]$$

Under conj matching, $A_{em} = \dfrac{1}{8}\dfrac{|V_T|^2}{S_i}\dfrac{R_T}{(R_L+R_r)^2}$

$$\left[\begin{array}{ll} X_A = & -X_T \\ R_L + R_r = & R_T \end{array}\right]$$

$$= \frac{1}{8}\frac{|V_T|^2}{S_i} \times \left(\frac{1}{R_L + R_r}\right)$$

↳ Let's put some numbers here to make sense of the implications.

Consider a Hertz dipole. ($\ell << \lambda$ e.g. $\frac{\lambda}{50}$)

$$U_{rad} = \frac{\eta}{8}\left(\frac{I\Delta z}{\lambda}\right)^2 \sin^2\theta, \quad P_{rad} = \int U_{rad}\, d\Omega$$

$$= \eta\frac{\pi}{3}\left|\frac{I_0 \ell}{\lambda}\right|^2$$

$$= \frac{1}{2}|I_0|^2 R_r$$

$$\Rightarrow R_r = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2$$

Assume lossless, so $R_L = 0$

$$\therefore \quad A_{em} = \frac{|V_T|^2}{8 S_i}\left[\frac{1}{R_r}\right]$$

What is V_T? Approx, $\frac{|V_T|}{\ell} = |E_i|$

And $S_i = \frac{1}{2\eta}|E_i|^2$

$$\Rightarrow \quad A_{em} = \frac{(E_i \ell)^2}{8\left(\frac{1}{2\eta}|E_i|^2\right)} \times \frac{1}{80\pi^2 (\ell/\lambda)^2}$$

$$= \frac{3\lambda^2}{8\pi} \approx 0.119\,\lambda^2 .$$

Compare this with the physical area of the antenna?

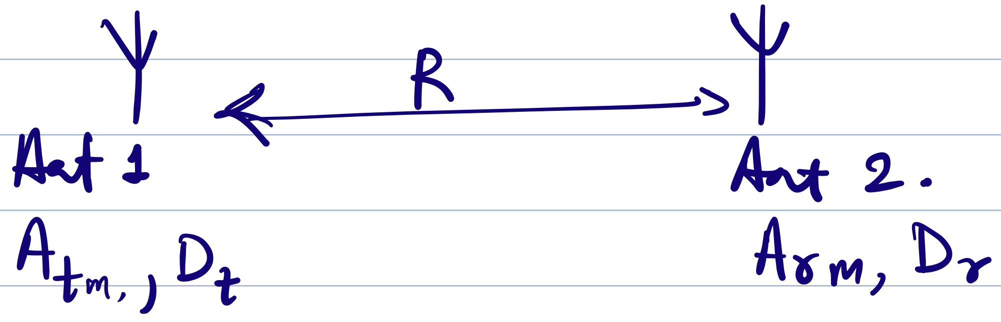
$$A_p = \ell \times w \approx \frac{\lambda}{50} \times \frac{\lambda}{300}$$

$$\Rightarrow \quad \frac{A_{em}}{A_p} \approx 1785$$

$\Rightarrow$ How an antenna appears physically & how it appears electrically can be very different!

↳ Next Q: Is there a rel$^{\underline{n}}$ bet$^{\underline{n}}$ D & A_{em}?

Simple thought experiment:



↳ Say that D_t is max value in the direction of Ant 2.

⇒ Due to Ant 1, Rx power density on Ant 2?

$$S_t = \frac{P_{tx} D_t}{4\pi R^2}.$$

Received power

By effective area defn, $A_{rm} = P_{rx}/S_t$

inc density

$$\Rightarrow A_r = \frac{P_{rx}}{P_{tx} D_t / 4\pi R^2}$$

$$\Rightarrow D_t A_r = \frac{P_{rx}}{P_{tx}}(4\pi R^2)$$

If we swap the power source & make Ant 2 as Tx and Ant 1 as Rx, then?

$$D_r A_t = \frac{P_{rx}}{P_{tx}}(4\pi R^2) \Rightarrow D_t A_r = D_r A_t$$

$$\Rightarrow \frac{D_t}{A_t} = \frac{D_r}{A_r} \xrightarrow{\text{generalize}} \frac{D_{ot}}{A_{tm}} = \frac{D_{or}}{A_{rm}}$$

max

Now the Q: we are free to choose Ant 2 the way we want! Tells you that the ratio D_t/A_t must be const!

$\therefore$ let's pick Ant 2 to be something we know and love! Hertz dipole!

$$D_{0r} = 3/2 \qquad A_{rm} = 3\lambda^2/8\pi$$

$$\Rightarrow \frac{D_{0t}}{A_{tm}} = \frac{3/2}{3\lambda^2/8\pi} = \frac{4\pi}{\lambda^2}$$

$\Rightarrow$ Max effective ant area

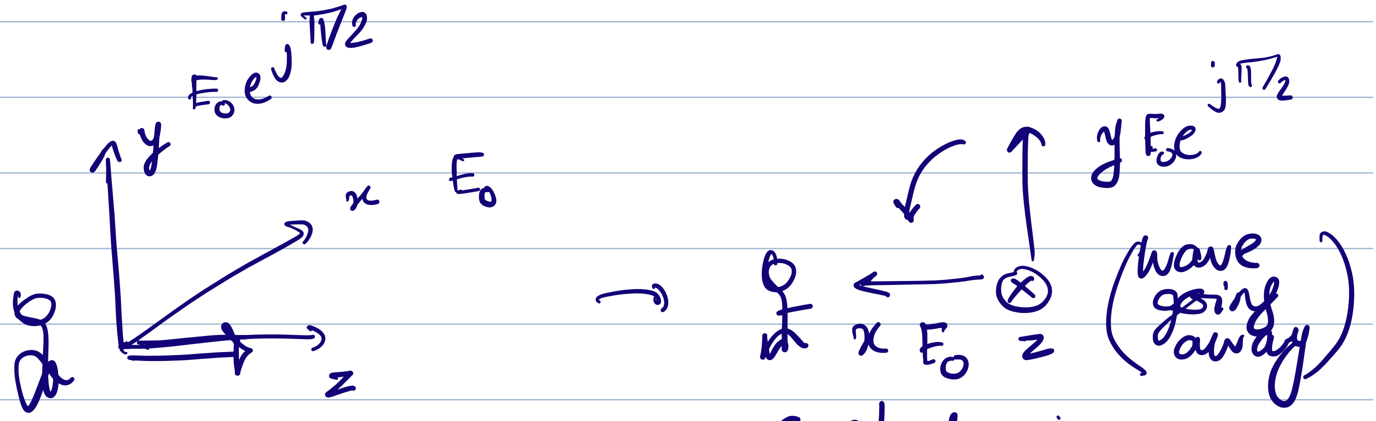
$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} D_0}$$

max D

$\Rightarrow$ The more directive the antenna, the better it's area.

Note: This is the best case scenario when polarizations of Tx & Rx are matched. What to do when they are not?

Recap on Polarization



c. clockwise
$\Rightarrow$ LH

$\rightarrow$ rotating from phase leading to phase lagging

Convention:

$\begin{cases} \text{phase leading} \Rightarrow 0 \text{ to } 180 \\ \text{phase lagging} \Rightarrow 180 \text{ to } 360 \end{cases}$

So, $\delta = \pi/2 \longrightarrow$ LHCP

$\delta = -\pi/2 \rightarrow$ RHCP

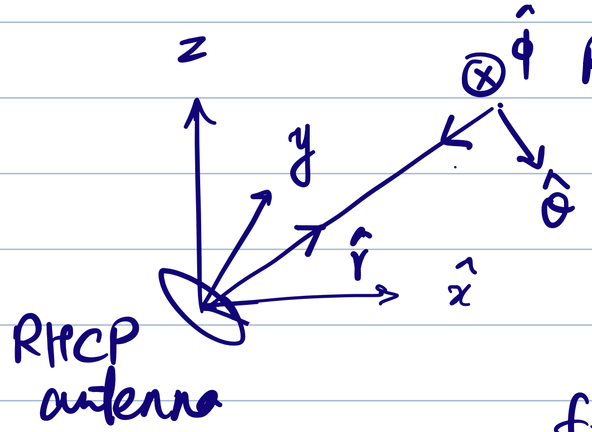
↳ How to deal with polarization mismatch between E_i & Rx ant?

Assume $E_i = \hat{\rho}_w E_i$ and the pol of the Rx antenna E-field written as:

$E_a = \hat{\rho}_a E_a$. From this the PLF, pol loss factor, is defined as

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 .$$ Best case: 1

But note $\hat{\rho}_a$ is defined for an ant as if it were in <u>tx mode</u> (IEEE defn) e.g.



Put right hand on $-\hat{r}$ direction.
from $\hat{\phi}$ to $\hat{\theta}$
from lead to lag.

$$\therefore \hat{\rho}_w = (\hat{\theta} + j\hat{\phi}) \frac{1}{\sqrt{2}}$$

What about $\hat{\rho}_A$? Put it in tx mode.
RH Thumb about $\hat{r}$, just reverse of above

$$\hat{\rho}_A = \frac{\hat{\theta} + j(-\hat{\phi})}{\sqrt{2}} = (\hat{\theta} - j\hat{\phi})\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1.$$

If LHCP antenna, $\hat{\rho}_a = (\hat{\theta} + j\hat{\phi})1/\sqrt{2}$

$$\Rightarrow PLF = 0$$

If $\hat{\theta}$ antenna (LP) $\hat{\rho}_a = \hat{\theta}$

$$\Rightarrow PLF = 1/2$$

—x—

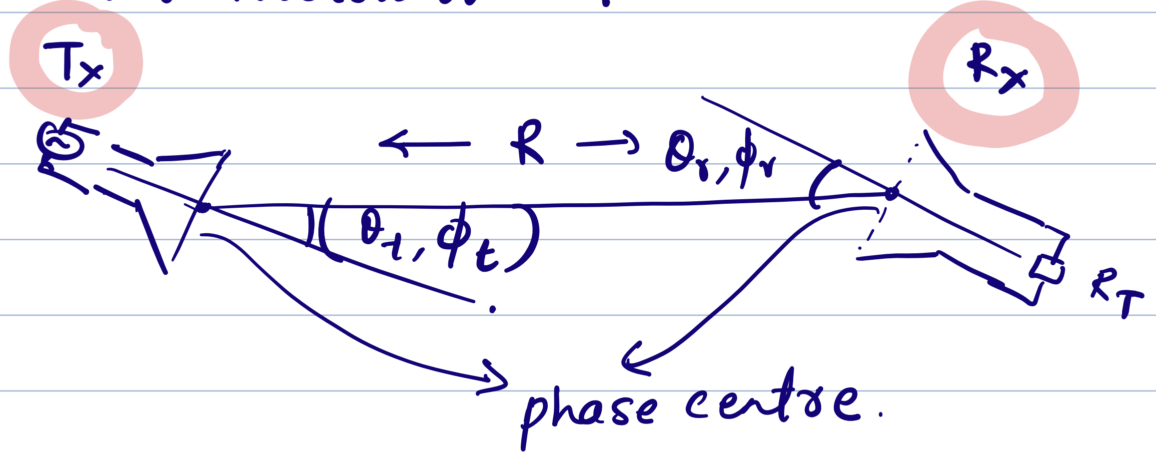
with all non idealities taken into acc:

$$\left[A_{em} = \underbrace{e_{cd}(1-|\Gamma|^2)}_{e_0} \left(\frac{\lambda^2}{4\pi}\right)(D_0)(PLF) \right]$$

Friis Transmission Eqn.

(Useful for link budget analysis in Comm)

Say we want to establish Comm between 2 antennas, e.g. Cube Sat & ground stn. How do I figure out how much power should I transmit at?



Power density at Rx? $\quad e_t D_t(\theta_t, \phi_t) \dfrac{P_t}{4\pi R^2}$

efficiency $\left[e_{cd} \times (1 - |\Gamma|^2) \right]$ $\quad$ directivity

or simply $S_{rx} = G_t(\theta_t, \phi_t) \dfrac{P_t}{4\pi R^2}$ — ①

Received power? $\quad P_r = A_r \times S_{rx}$ — ②

effective area

$$A_r = e_r D_r(\theta_r, \phi_r) \frac{\lambda^2}{4\pi}$$

$$\Rightarrow P_r = \left(\frac{\lambda}{4\pi R}\right)^2 e_r e_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) P_t$$

Most commonly done in log scale.

$$P_{r,dB} = G_r(dB) + G_t(dB) - \underbrace{20\log\left(\frac{4\pi R}{\lambda}\right)}_{\text{free space loss factor.}} + P_{t,dB}$$

(assuming PLF = 1)

Called the Friis transmission eqn.

→ Any receiver will require a min value of P_r in order to operate.

→ Moreover, a modulation format may require higher P_r, all related to SNR.

→ Remember this is ant to ant. Receiver electronics will further degrade the SNR, so that further budget needs to be added.