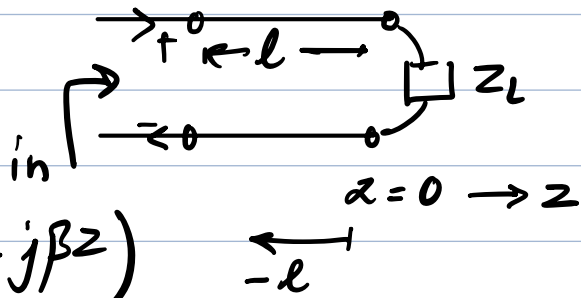


⑤ We saw that at the load end  $z_L = \frac{V(0)}{I(0)}$ .  
But we can define this  $z$  at any point, consider:



$$\therefore Z_{in} = \frac{V_0^+ (e^{-j\beta z} + \Gamma e^{+j\beta z})}{\frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{+j\beta z})}$$

Note: no need to memorize. Go term by term.

↳ Now subs  $z = -l$

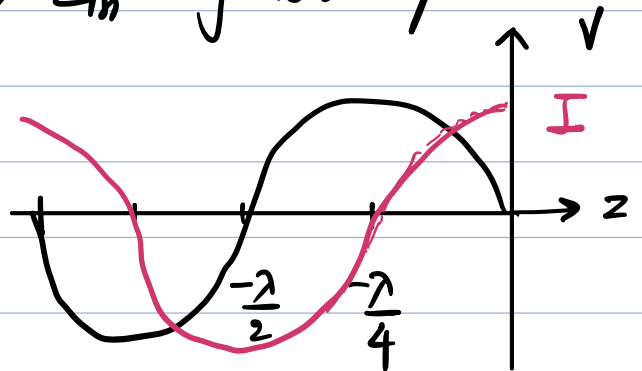
$$\frac{Z_{in}}{Z_0} = \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \quad \Leftrightarrow \quad \bar{Z}_{in} = \frac{\bar{Z}_L + j \tan \beta l}{1 + j\bar{Z}_L \tan \beta l}$$

where,  $\bar{Z} = z/Z_0$ .

2 special cases of practical use:

1) short ckt  $\Rightarrow Z_L = 0 \Rightarrow Z_{in} = jZ_0 \tan \beta l$ .

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

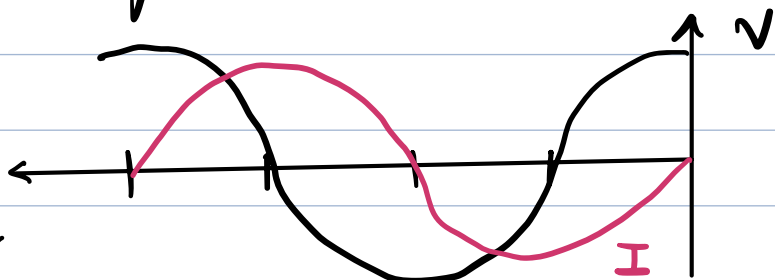


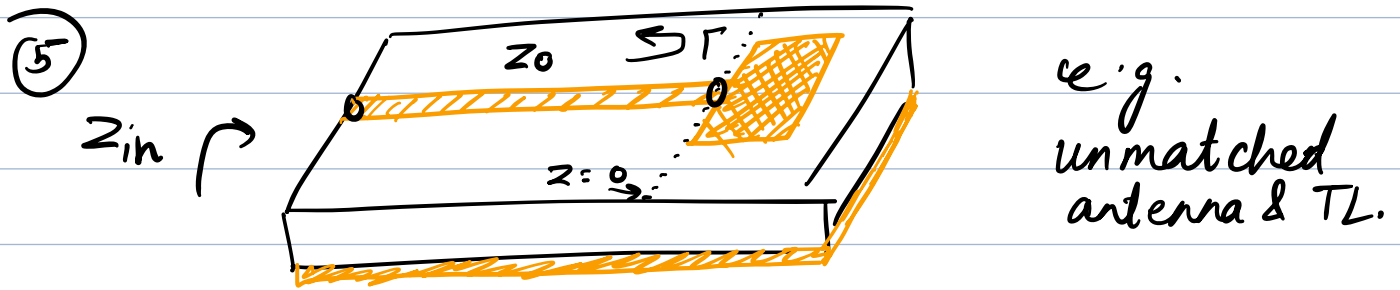
$\Rightarrow$  Inductive element  
from  $0 \leq l \leq \lambda/4$ .

2) Similarly for open ckt.,  $Z_L \rightarrow \infty$ ,  $\Gamma = 1$   
 $Z_{in} = -jZ_0 \cot \beta l$

$\Rightarrow$  Capacitive element for

$0 \leq l \leq \lambda/4$ .



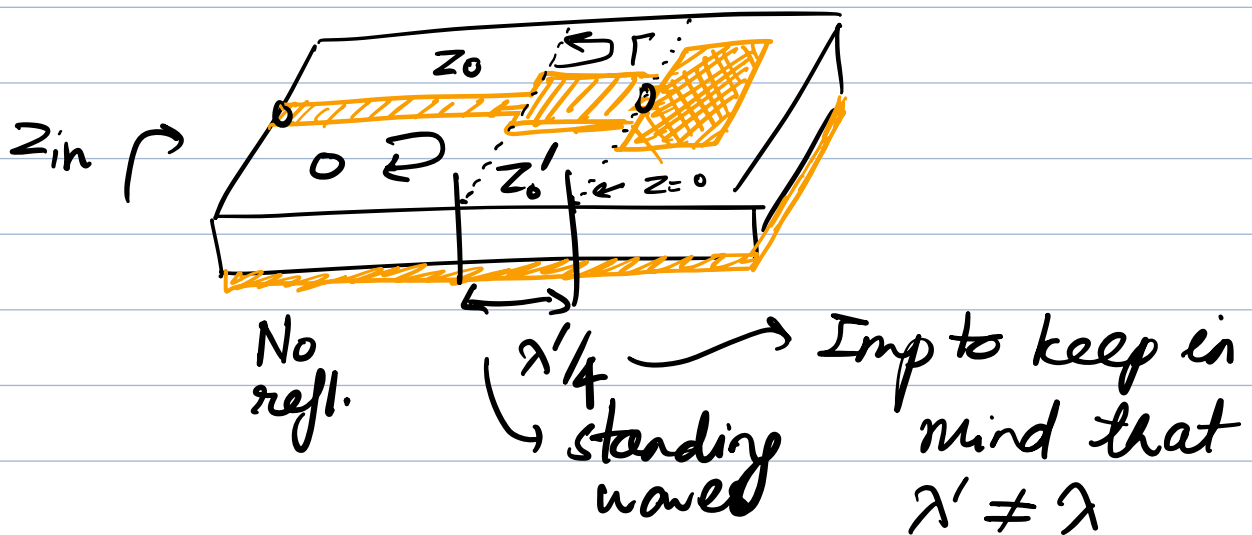


At  $z=0$  we have a load  $Z_L$ . eg.  $100\Omega$   
 we want  $\Gamma = 0$ . How?  $Z_0 = 50\Omega$ .

Consider a new TL,  $l = \lambda/4 \rightarrow \frac{Z_{in}}{Z_0'} = \frac{Z_0'}{Z_L}$ , i.e.  $Z_L$  is

transformed to  $Z_{in} = \frac{Z_0'^2}{Z_L}$

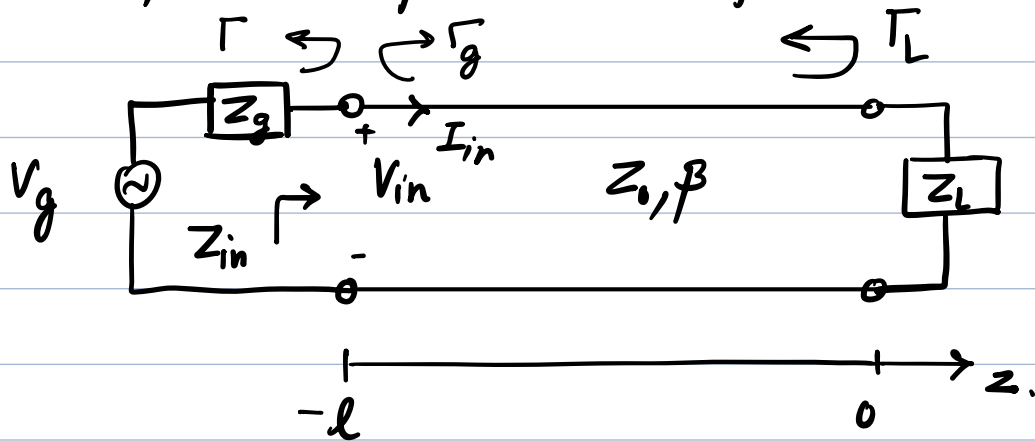
We want  $Z_{in} = Z_0 \therefore Z_0 = Z_0'^2 / Z_L$   
 $\Rightarrow Z_0' = \sqrt{Z_0 Z_L}$



Use new TL params.  $\rightarrow \lambda' = 2\pi/\beta'$   
 Called a quarter wave transformer.

$\rightarrow$  Complication:  $Z_L$  is complex! Put a stub in series/parallel sl.  $Z_L'$  is purely real. Then do QWT.  $\rightarrow$  Single stub matching.

⑥ Optimal power transfer, the full picture.



Q: Is it possible that under max power transfer to the load, there can be standing waves on the line?

↳ 2 sources of reflections:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ (load)}, \quad \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} \text{ (gen)}$$

↳ Power to load =  $P = \frac{1}{2} \operatorname{Re}[V_{in} I_{in}^*] \rightarrow$

with  $V_{in} = Z_{in} I_{in}$ ,  $P = \frac{1}{2} |V_{in}|^2 \operatorname{Re}\left[\frac{1}{Z_{in}^*}\right]$

$$V_{in} = \frac{V_g Z_{in}}{Z_g + Z_{in}}, \quad Z_{in} = Z_0 \left[ \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right]$$

$$= R_{in} + j X_{in}.$$

$$\operatorname{Re}\left[\frac{1}{R_{in} + j X_{in}}\right] = \frac{R_{in}}{R_{in}^2 + X_{in}^2}$$

∴ TL is lossless, all power goes to load

$$\Rightarrow P = \frac{1}{2} |V_g|^2 \left| \frac{Z_{in}}{Z_g + Z_{in}} \right|^2 \operatorname{Re} \left[ \frac{1}{Z_{in}} \right]$$

$$P = \frac{1}{2} |V_g|^2 \frac{R_{in}}{(R_{in} + R_g)^2 + (X_{in} + X_g)^2}$$

→ with  $Z_g$  as considered fixed, look at 3 cases. [ $Z_0$  is a lossless TL]

① Match the load to the line, i.e.  $Z_L = Z_0$

$$\Rightarrow Z_{in} = Z_0 \Rightarrow P = \frac{1}{2} |V_g|^2 \frac{Z_0}{(Z_0 + R_g)^2 + X_g^2}$$

Here  $\Gamma_L = 0$ .

② Match generator to loaded line  
 $Z_g$   $Z_{in}$

i.e.  $Z_{in} = Z_g$ . An overall coeff  $\Gamma = \frac{Z_{in} - Z_g}{Z_{in} + Z_g} = 0$ .

$$P = \frac{1}{2} |V_g|^2 \frac{R_g}{4(R_g^2 + X_g^2)}$$

Possible that ② is less than ①.

③ Conjugate Matching.

What if we vary  $Z_{in}$  for optimal power tx?  
 Then we can back calculate  $Z_L$ .

So, set  $\frac{\partial P}{\partial R_{in}} = 0$  &  $\frac{\partial P}{\partial X_{in}} = 0$

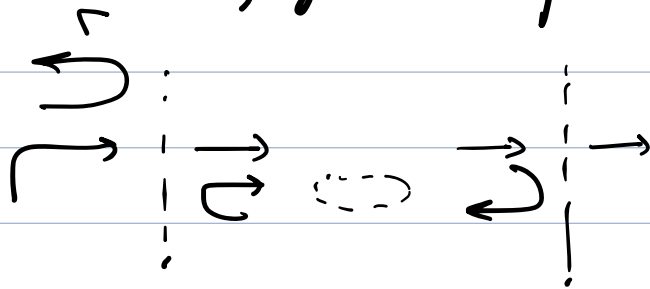
Simple math will give you  $R_{in} = R_g$ ,  $X_{in} = -X_g$ .

i.e.  $Z_{in} = Z_g^*$  and now  $P = \frac{1}{2} |V_g|^2 \frac{1}{4 R_g}$ .

greater than ① or ②.

This gives the max power  $P_x$  to the load for a fixed  $Z_g$ .

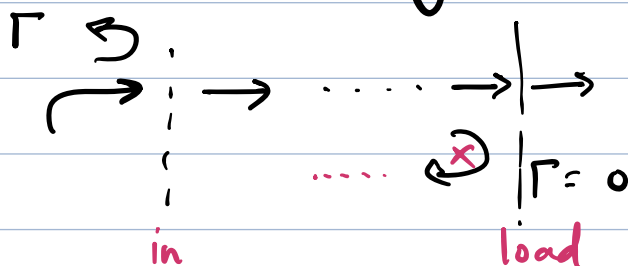
Here  $\Gamma_L, \Gamma_g, \Gamma$  may all be non zero.



Multiple reflections adding back in phase to give opt power.

Notes:

a) In ① will we have reflections?  $\Gamma_L = 0$ ,  $\Gamma_g \neq 0$ . But, there is no source of a backward wave since  $\Gamma_L = 0$ . Recall the more fundamental def:  $\Gamma = \frac{V_{ref}}{V_{inc}}$ .



② if  $X_g = 0$  then last 2 cases give same result.