Similarity Transforms (contd)

1) Seen than C.O.V. leads to

$$\underbrace{[y_1\ y_2\ \cdots\ y_n]}_{Y} = \underbrace{[x_1\ \cdots\ x_n]}_{X}\ \underbrace{M}_{\text{coefs of L.C. of } x}$$

2) A point v in X wrote as

$$v = X\alpha \quad \& \text{ in } Y, \quad v = Y\beta$$

$$= X\underbrace{M\beta}_{\alpha}$$

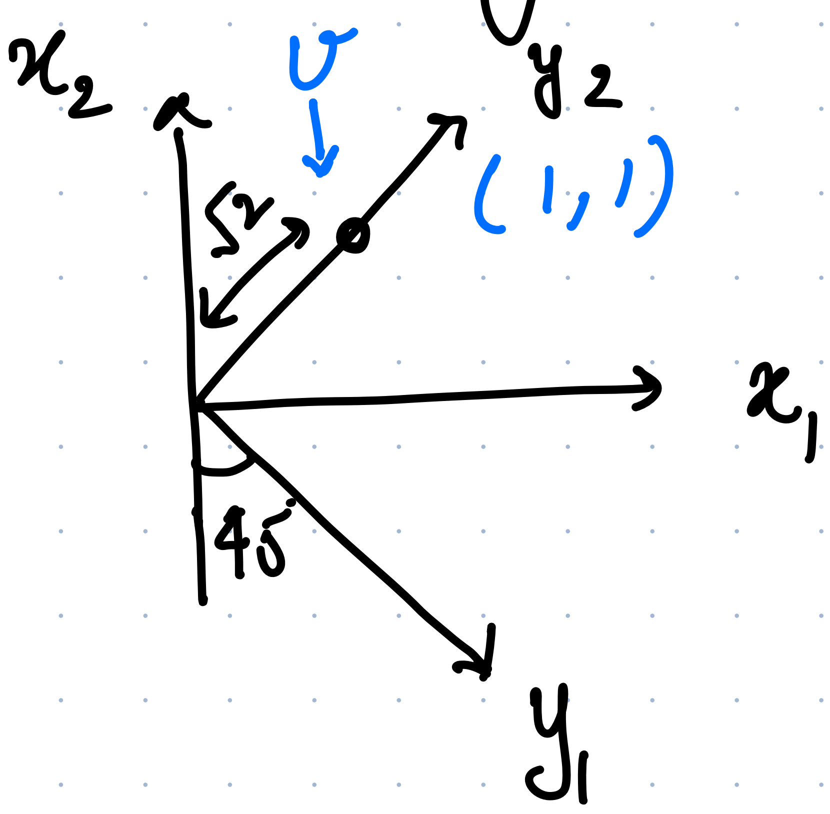
$\xrightarrow{\text{Repr in } X}$ α

$\xrightarrow{\text{Repr in } Y}$ β

$$\alpha = M\beta \quad \text{or}$$

$$\beta = M^{-1}\alpha.$$

3) A "default" quality of canonical basis



$$X \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Y \rightarrow [y_1 \;\; y_2] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

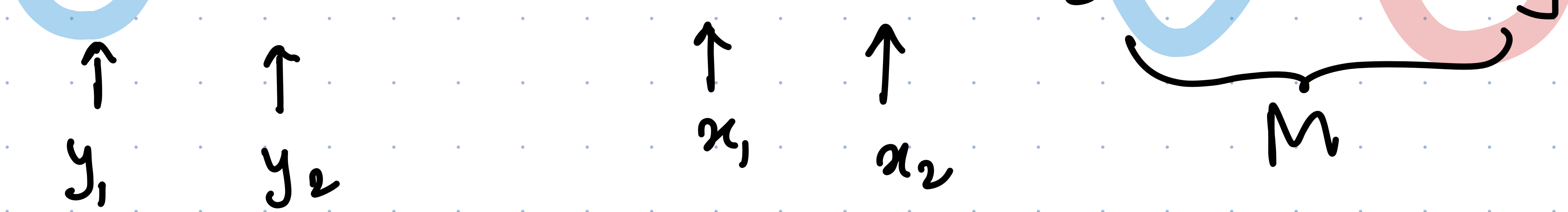
$$v = X\alpha = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\beta \rightarrow \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

$$v = \underline{Y\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}} = I \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

<u>Construct M</u>.

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$



Final check, $\alpha = M\beta$?

$$\begin{pmatrix}1\\1\end{pmatrix} \overset{?}{=} \begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ -1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}0\\ \sqrt{2}\end{pmatrix}$$

✓

4) Linear Transformations

$T: \underset{i/p}{V} \longrightarrow \underset{o/p}{W}$: $T(v_j) = [w_1 \cdots w_n]\begin{bmatrix}A_j\\ \updownarrow\end{bmatrix}$

$$T(v_j) = \sum_i w_i A_{ij}$$

$\therefore$ Matrix is A.

↳ Q: What kind of a transformation is a c.o.v ?

A: It is an identity transformation.

Matrix repr is <u>NOT</u> I.

a) $T_1 : X \to Y$, $T_1(x_j) = \sum_i y_i N_{ij}$ ← 1 col

$\Downarrow$

$T_1(X) = YN$ ← All cols.

Meaning?

Given α (repr in X) → get (repr in Y) : β

i.e. $\beta = N\alpha = M^{-1}\alpha$.

b) $T_2 : Y \to X$, $T_2(y_j) = \sum_i x_i M_{ij}$

$T_2(Y) = X\underline{M}$.

Meaning?

Given β, return α, i.e. $\alpha = M\beta$.

5) Say, we have an L.T. $X \xrightarrow{A} X$

Same L.T. $Y \xrightarrow{B} Y$. Reln betn A & B.

$T_1 \to LT \to T_2$

$x \qquad\qquad x$

$$\begin{tikzcd}
X \arrow[r, "M^{-1}", "T_1"'] \arrow[rrr, bend left, "LT: A"] \arrow[d] & Y \arrow[r, "B", "LT:B"'] \arrow[d] & Y \arrow[r, "M", "T_2"'] \arrow[d] & X \arrow[d] \\
\alpha & \beta & \beta' & \alpha'
\end{tikzcd}$$

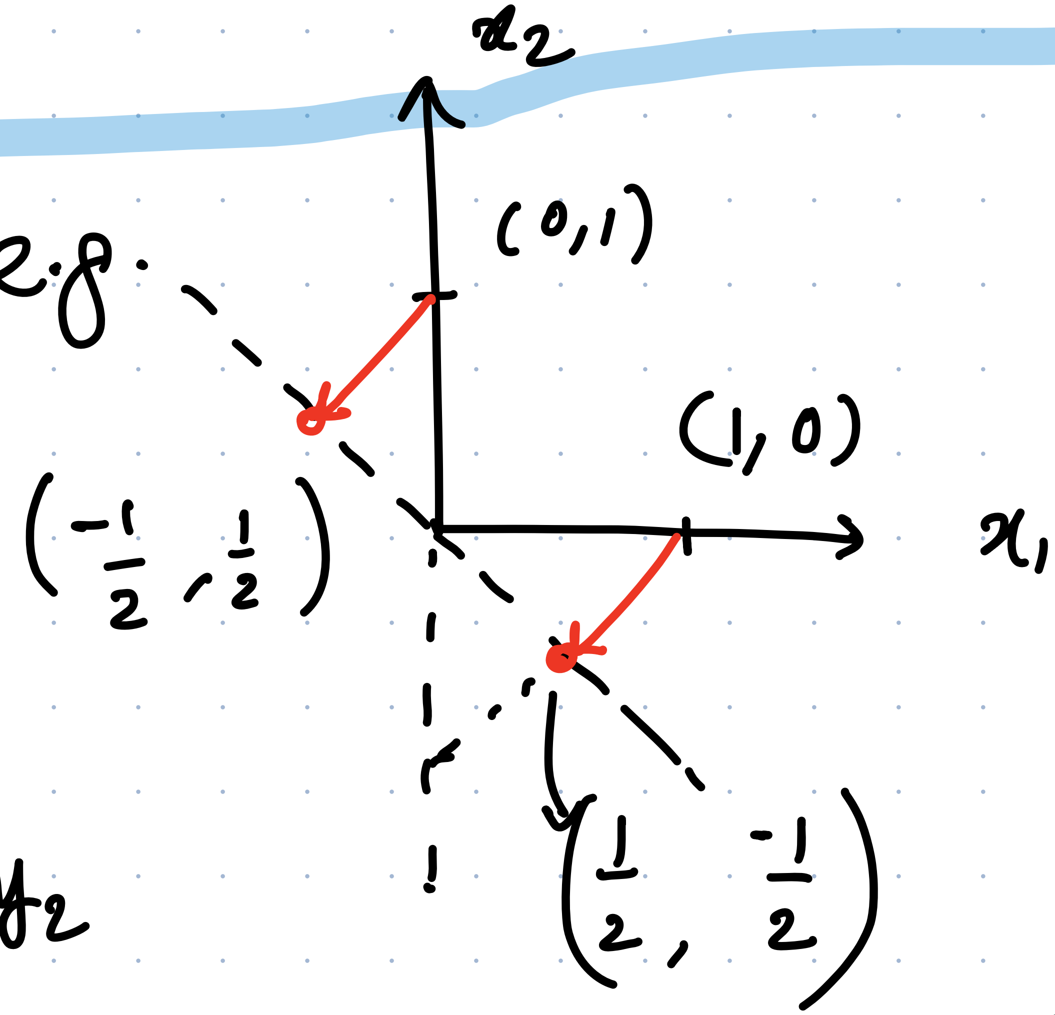
$$\alpha' = A\alpha$$

$$\alpha' = \underline{M}\,\underline{B}\,\underline{M^{-1}\alpha} \Rightarrow \underline{A = MBM^{-1}}.$$

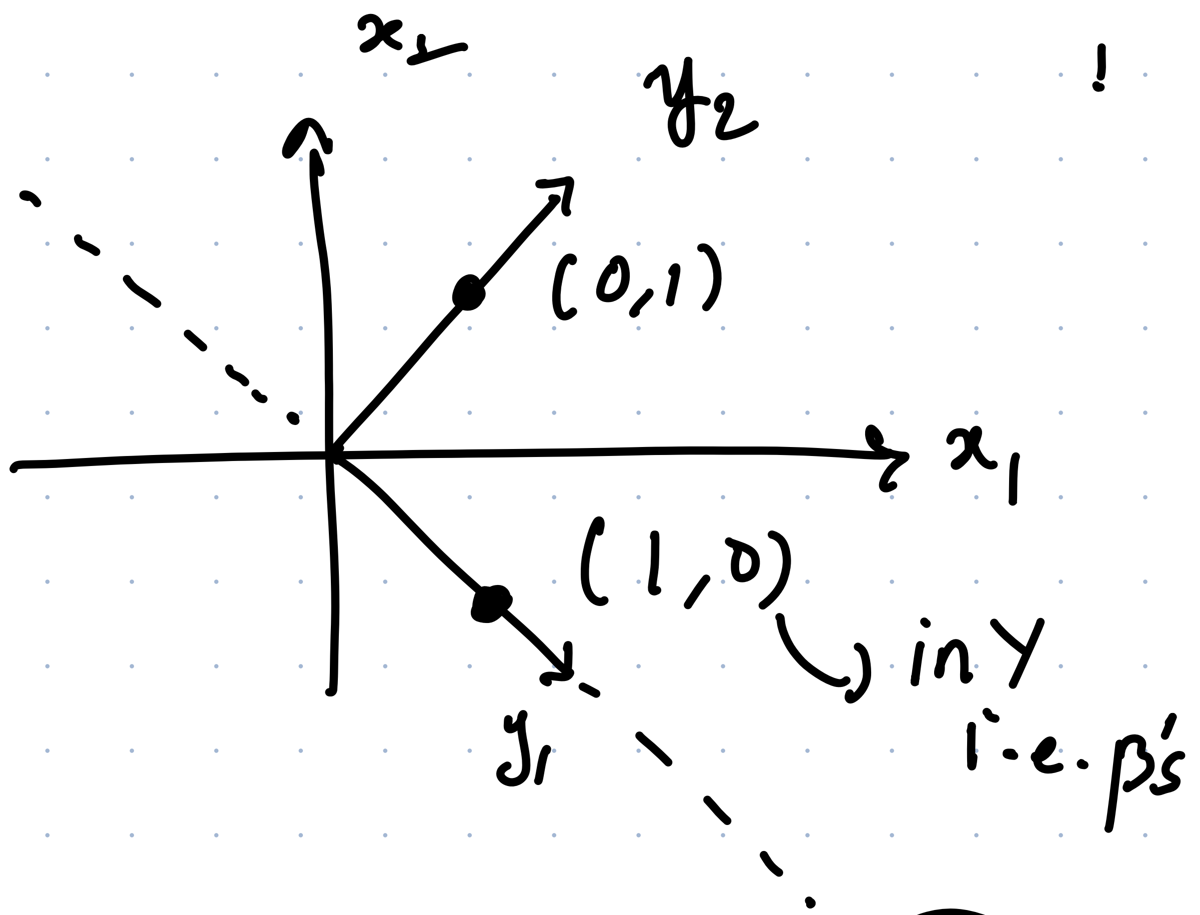
The matrices corresponding to a LT are similar.

6) Take an e.g.

projection



$$A = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$



$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$T(y_j) = \sum_i y_i B_{ij}.$$

$$A \overset{?}{=} M B M^{-1}$$

$AM \stackrel{?}{=} MB$

Schur Decomposition

For any square matrix A, it can be expressed as $A = BRB^{-1} = BRB^{H}$

where B is an orthogonal matrix and R is upper triangular.

(P1) FTA → gives that n^{th} order poly has atleast 1 complex root

⇒ A matrix A has atleast 1 eig value.

⇒ Matrix has atleast 1 eigvector.

(P2) (a) Matrix of a L.T. depends on the basis

(b) Length of basis vectors does not enter into the size of the L.T. Matrix. e.g. $T_k : V_k \to V_k$.

$$T_k\Big(\underbrace{[\,v_1 \;\; v_2 \;\; \cdots \;\; v_k\,]}_{k}\Big) = [\,\underbrace{v_1 \;\; v_2 \;\; \cdots \;\; v_k}\,] \begin{array}{c} k \\ \left[\begin{array}{ccc} & & \\ k & & \\ & & \end{array}\right] \end{array}$$