Connection between GS & QR

GS

$(a, b, c) \longleftrightarrow (q_1, q_2, q_3)$

$a \longrightarrow q_1 = a/\|a\|$

$b \longrightarrow b' = b - (q_1^T b) q_1$

$q_2 = b'/\|b'\|$

QR

$a \longrightarrow \|a\| q_1 = (q_1^T a) q_1$

$b \longrightarrow (q_1^T b) q_1 + (q_2^T b) q_2$?

$b = b' + (q_1^T b) q_1$

$b = \|b'\| q_2 + (q_1^T b) q_1$

$q_2^T b = \|b'\| + 0$

$\Rightarrow b = (q_2^T b) q_2 + (q_1^T b) q_1$

— x —

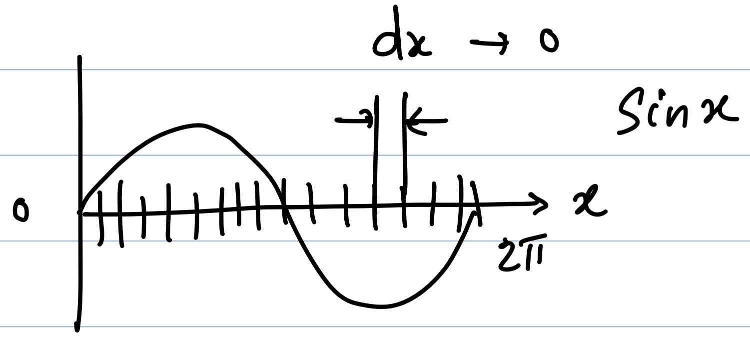
Hilbert Spaces.

Moving from $n = 2, 3, \ldots$ & so on upto ∞
space enlarges.

If we say $\|v\| < \infty$ finite length or energy.

- This defines a Hilbert space.

e.g. $f(x) = \sin x \; , \; 0 \leq x \leq 2\pi$



$v \in \mathbb{R}^n$

$[v_1, v_2 \ldots, v_n]$

$$\begin{bmatrix} f(0) \\ f(dx) \\ \vdots \\ f(n\,dx) \\ \vdots \end{bmatrix}$$

$$\|f(x)\|^2 = \int_0^{2\pi} |f(x)|^2 dx < \infty$$

↳ interval needs to be specified.
↳ Square integrable.

$v = \{1, 1, \ldots, 1, \ldots\} \longrightarrow$ ✗
$w = \{1, \frac{1}{2}, \frac{1}{4}, \ldots\} \longrightarrow$ ✓

Length of $w \to \|w\|_2$
Length of $\sin x \to \sqrt{\int_0^{2\pi} |f(x)|^2 dx}$

Function spaces → dealing with functions like we deal with vectors.

↳ Inner product of functions → f, g

$$(f, g) = \int_D f(x)\, g(x)\, dx = \langle f, g \rangle$$

↳ Orthogonality of functions

[complex conjugate if f, g are complex valued.]

are sin, cos orthogonal?

$$(f, g) = \int_0^{2\pi} \sin x \cos x \, dx = 0$$

↳ Fourier Series.

$$\underline{f(x)} = a_0 \cdot 1 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \cdots$$

All these fns are orthogonal to each other, i.e.

$$m \neq n \leftarrow \left.\begin{cases} (\cos mx, \sin nx) \\ (\cos mx, \cos nx) \\ (\sin mx, \sin nx) \end{cases}\right\} = 0$$

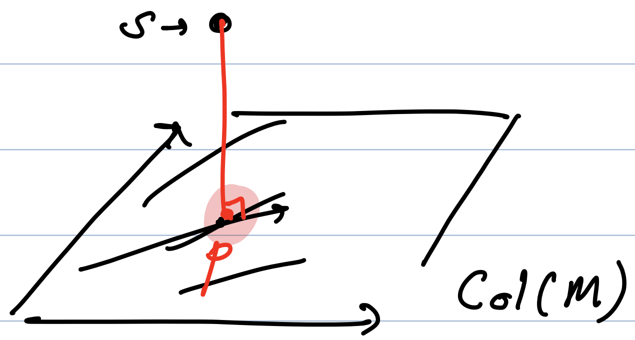
$$(\sin x, f(x)) = b_1 (\sin x, \sin x) = b_1 \pi$$

$$\Rightarrow \quad b_1 = \frac{(\sin x, f(x))}{(\sin x, \sin x)}$$

projection of f on $\sin x$.

$$\frac{a^T b}{a^T a}$$ earlier projection operator.

Connection with least squared error.



$$\boxed{Mz = s}$$

$$\begin{array}{cccc} m_0 & m_1 & & m_n \end{array}$$
$$\begin{bmatrix} | & | & & | \\ | & | & & | \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} s_1 \\ \vdots \\ s_n \end{bmatrix}$$

$$\rightarrow \quad m_i^T (s - p) = 0$$

$$\Updownarrow$$

$$(m_i, s - p) = 0$$

shift into Fourier series.

$$\begin{bmatrix} 1 & \cos x & \sin x & \cdots \end{bmatrix}^a \begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ \vdots \\ \vdots \end{bmatrix}_\infty = f(x)$$

Fourier Series statement.

$$\left(\cos x, f(x) - [a_0 + a_1 \cos x + \cdots + b_1 \sin x + \cdots]\right) = 0$$

$$(\cos x, f(x)) = (\cos x, \cos x)\, a_1$$

$$\boxed{a_1 = \frac{(\cos x, f(x))}{(\cos x, \cos x)}}$$

These are the comps which give a least squared error repr in terms of Fourier series.