Vector Spaces.

Advisor : Solve $Ax = b$.

Go solve $\rightarrow$ x_p.

$x_0 \neq 0$ advisor gave you this

property $Ax_0 = 0$

$R(A) \leftarrow x_p$ you gave

$x_p + x_0$ advisor gave

A $R(A) + N(A)$

eg 1 $A \rightarrow$
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$\rightarrow$ $C(A)$: Col space of A $\rightarrow$ The space spanned by the cols of A, i.e. LC of cols of A

2 dimensional space : $\boxed{\mathbb{R}^2}$

$b \in C(A)$ for $Ax = b$ to have a soln.

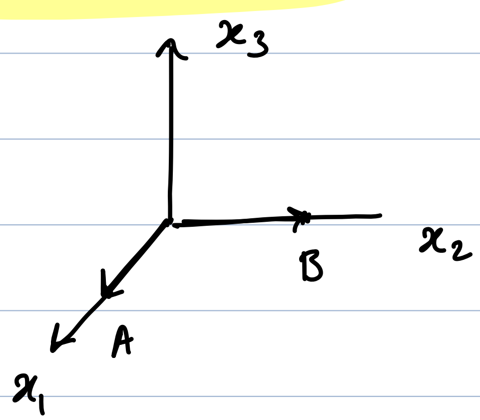
eg. 2.

A :
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$C(A)$? 2 dimensional, but is it $\mathbb{R}^2$? ✗

$C(A) \boxed{\in} \mathbb{R}^3$

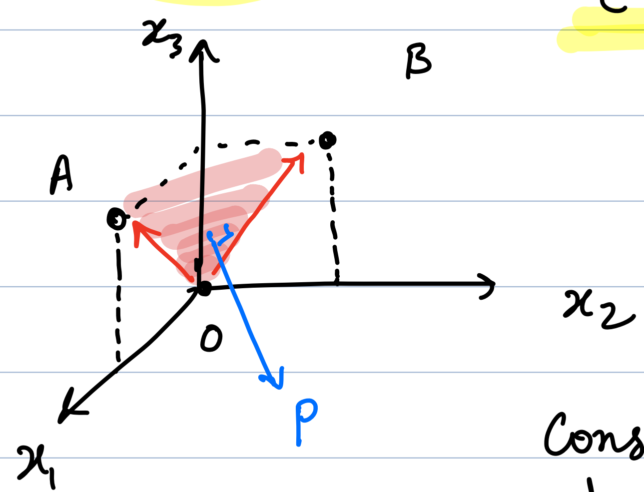
$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$?



e.g. 1 $R(A)$: Linear comb of rows of A.

$\in \mathbb{R}^3$ → actually 2 dim.

plane: OAB



OP ⊥ OAB.

Consider any scalar multiple of OP : y.

[dot prod of any vect in $R(A)$ with y = 0]

eg 1 A →

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Inner product : $r \cdot x$

if $x = y$, then $Ax = 0$

We have found $y \in \mathbb{R}^3$ s.t. $y \neq 0$ &
$Ay = 0$

Such a vector 'y' is said to belong to
the <u>Null space of (A)</u>: $N(A)$.

Row space of A + Null space of A = $\mathbb{R}^3$

Similarly; <u>left null space of A</u>

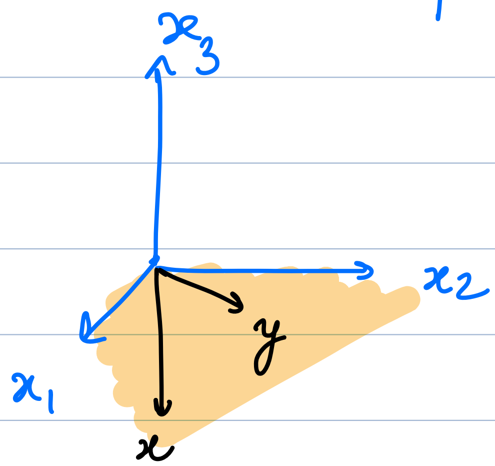
Col space of A + $\underbrace{\text{L.N.S}}_{N(A^T)}$ of A = $\mathbb{R}^3$ (for eg. 2).

formal: $\boxed{\text{Vector Space}}$

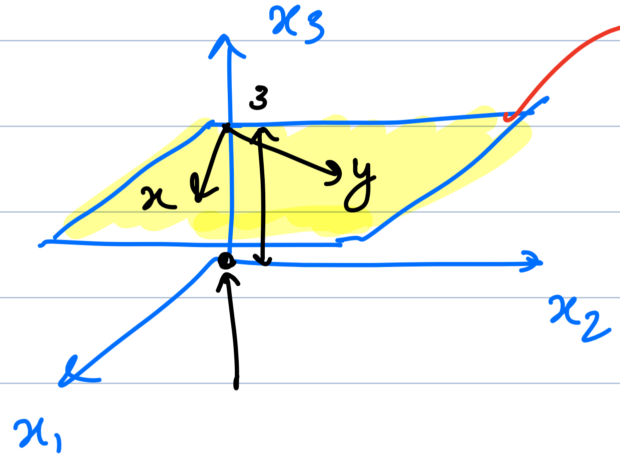
<u>Defn</u>: A real vector space is a set of vectors together with the rules of vector addn & multiplication by real nos.:
Linear operations <u>must produce</u> vectors in this space.

· eg. if $\underline{x, y \in V}$, then $\underline{\alpha x + \beta y} \in V$
for $\alpha, \beta \in \mathbb{R}$.

→ The 0 vector must belong to every vector space.



It is a v.s. because any LC of x, y belongs to the VS ($x_1 - x_2$ plane)



(An affine space.)

Not a VS because it does not contain 0.

$$\alpha x + \beta y \in V.$$

$$\text{set } \left.\begin{aligned} \alpha &= -\beta \\ x &= y \end{aligned}\right\} 0$$

$\therefore$ Not a VS $\because$ $\vec{0}$ not in it.

↳ Is $\mathbb{R}^n$ a v.s.? ✓

Is $\left(\underbrace{\mathbb{R}^{m\times n}}_{}\right)$ a v.s.? $\quad A, B \in \mathbb{R}^{m\times n}$

Real valued matrix of size $m \times n$.

$$\alpha A + \beta B \in \mathbb{R}^{m\times n} \text{ ? } ✓$$

$$\alpha, \beta \in \mathbb{R}.$$

$$\left.\begin{aligned} \alpha &= -\beta \\ A &= B \end{aligned}\right\} \Rightarrow 0^{m\times n} \quad ✓ \text{ is a VS.}$$

$\mathbb{R}^{3\times 3}$ $\qquad$ $\mathbb{R}^{9\times 1}$ $\qquad$ $\mathbb{R}^{mn\times 1}$

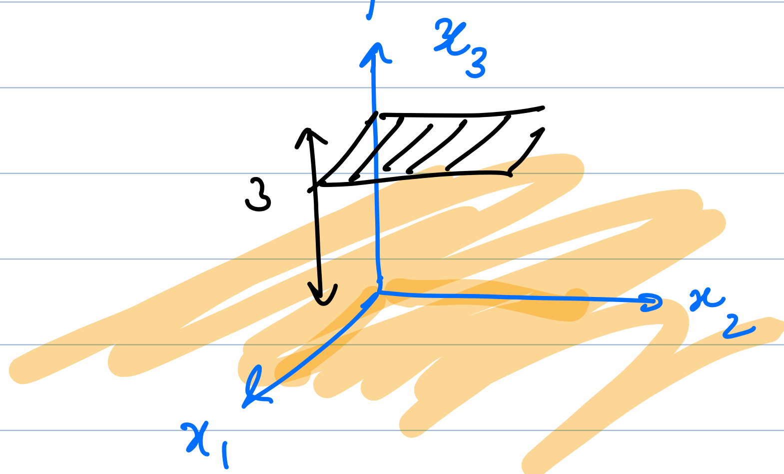
do upper triangular matrices form a v.s.?

→ "functional analysis"

6 <u>Vector subspace</u>.

A non empty subset which also satisfies the requirements of a vector space.

⇒ L.C.s of those elements remain in the same space.



V: $\mathbb{R}^3$

$\boxed{W}$: defined by $\vec{x}_1, \vec{x}_2$ live in V but they define a vector sub space.

——x——