Matrix Inversion

↳ Method: Gauss – Jordan method.
Similar to GE, $O(n^3)$

If A is sparse, $\underline{\underline{A^{-1}}}$ need not be sparse.

→ Matlab Online → Sign up for a mathworks account using smail ID.

Octave

→ Symmetric Matrix $A^T = A$

→ (AA^T) (A^TA)

A: m×n →

$$\begin{cases} A \times A^T \\ m\times n \quad n\times m \rightarrow m\times m \\ A^T \times A \rightarrow n\times n \end{cases}$$

$$(AB)^T = B^T A^T$$

$$(ABC)^T = C^T B^T A^T$$

If sym: $A = A^T$

$$\underbrace{\left(AA^T\right)} \xrightarrow{T} \left(AA^T\right)^T = \left(A^T\right)^T A^T$$
$$= \boxed{\underbrace{AA^T}}$$

equal.

Hermitian matrix

$$\left(A^T\right)_{ij} = A_{ji}$$

$$\left(A^H\right)_{ij} = A^*_{ji} \quad \text{conjugate transp.}$$

Inner & Outer products.

a, b	$\in$	$\mathbb{R}^{n\times 1}$	col vect
a, b	$\in$	$\mathbb{R}^{1\times n}$	row vect
A	$\in$	$\mathbb{R}^{m\times n}$	real
A	$\in$	$\mathbb{C}^{m\times n}$	complex.

$$a, b \in \mathbb{R}^{n\times 1}$$

1) $$\underbrace{a^T}_{1\times n}\underbrace{b}_{n\times 1} \rightarrow \mathbb{R}^1 = \sum_{i=1}^{n} a_i b_i$$

inner product.

2) $A = \boxed{a\, b^T} \rightarrow \mathbb{R}^{n\times n}$ outer product.

$n \times 1 \quad 1 \times n$

$n^2 \qquad a \qquad 2n$

$$A_{ij} = a_i b_j$$

In general: $\underbrace{A}_{n^2} = \sum_{i=1}^{n} \underbrace{a^i a^{i\,T}}_{n}$

Say $\underline{A = A^T}$. Spl form of LDU decomp
assume A is not singular.

$$A = LDU$$

$$A^T = U^T D^T L^T = \underbrace{U^T}_{\text{lower}} D \underbrace{L^T}_{\text{upper}} = L\, D\, U$$

By uniqueness $U^T = L$
& $L^T = U$

$$\underline{A = LDL^T}.$$

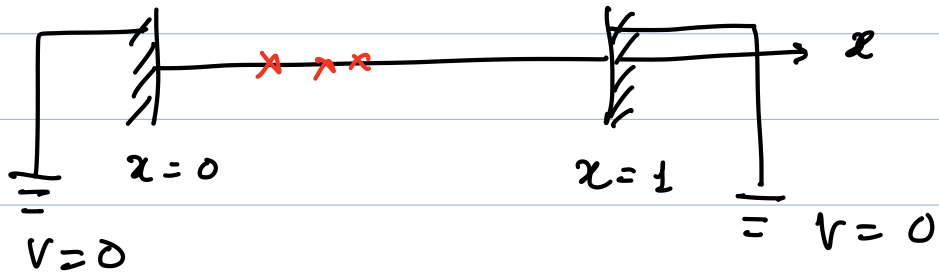
Special matrices . e.g. from PDE's or DEs.

Charge statics

$$\left.\begin{aligned} \bar{\nabla} \cdot \bar{E} &= \rho/\varepsilon_0 \\ \vec{E} &= -\nabla V \end{aligned}\right\}$$

$$\nabla^2 V = -\rho/\varepsilon_0 \quad \text{Poisson's eqn.}$$

<u>Make it 1D</u>. $\nabla^2 = \dfrac{d^2}{dx^2}$



Q: What is $V(x)$? $\rho(x)$

$$\frac{d^2 V}{dx^2} = -\frac{\rho(x)}{\varepsilon_0}.$$

$V(x)$ is a soln. $V(x) + bx + c$?

Also a soln. $\therefore$ I need 2 B.C's.

Here : $V(0) = 0, \quad V(1) = 0.$

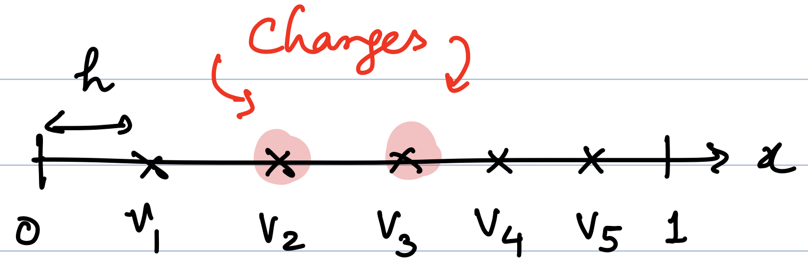
Finite differencing: $\dfrac{dV}{dx} = \lim\limits_{h \to 0} \dfrac{V(x+h) - V(x)}{h}$

$$= \lim_{h \to 0} \frac{\boxed{V(x + h/2)} - V(x - h/2)}{h} = V'(x)$$

$$\frac{d^2 V}{dx^2} = \frac{\left(\dfrac{V(x+h) - V(x)}{h}\right) - \left(\dfrac{V(x) - V(x-h)}{h}\right)}{h}$$

$$\rightarrow V'' = \frac{V(x+h) - 2V(x) + V(x-h)}{h^2}$$

$$v'' = -\rho/\varepsilon_0$$



pt # 1 :

$$v_2 - 2v_1 + 0 = 0$$

$$-2v_1 + v_2 = 0 \qquad - (1)$$

pt # 2 :

$$v_3 - 2v_2 + v_1 = -\frac{\rho_2}{\varepsilon_0} h^2$$

$$v_1 - 2v_2 + v_3 = \quad " \qquad - (2)$$

⋮

pt # 5

$$v_4 - 2v_5 + 0 = 0 \qquad - (5)$$

i^{th} row → $(i-1, \ i, \ i+1.)$ non zero

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 0 \\ \times \\ \times \\ 0 \\ 0 \end{bmatrix}$$ known

Max non zero entries per row: 3

PDE → Matrix eqn.

Tridiagonal matrix

Symmetric → $A = LDL^T$

Gaussian elim.

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3/2 & -1 & 0 & 0 \\ & & \left(\text{unchanged} \right) & & \end{bmatrix}$$

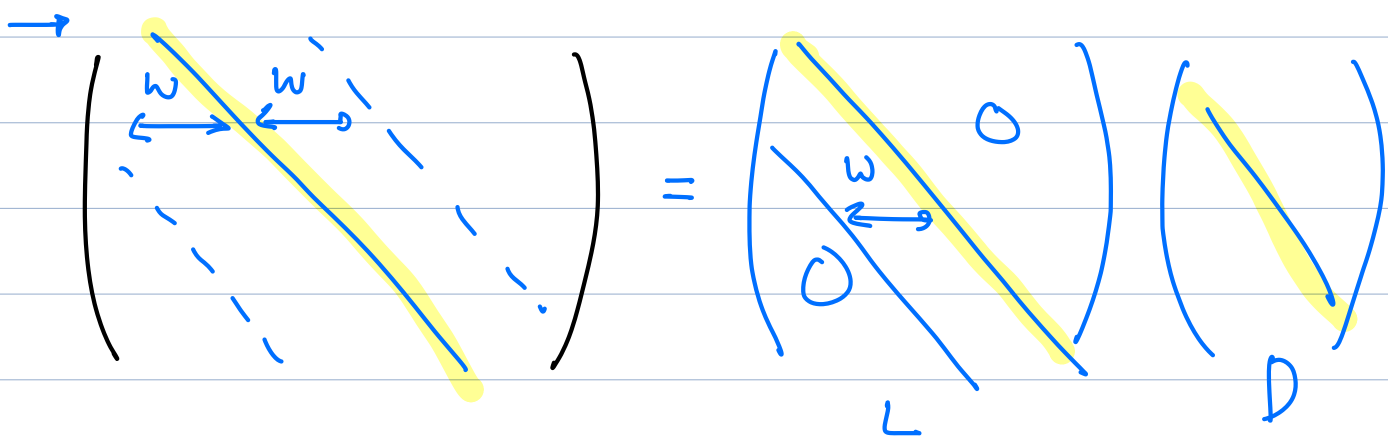
$R_2 = R_2 + \frac{1}{2} R_1$ only 1 op v/s $n-1$ op.

Whole matrix : $n-1$ opers.

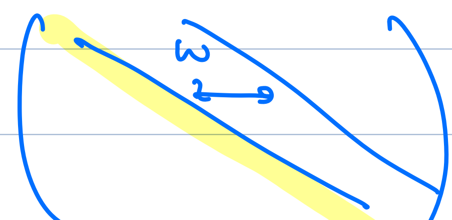
earlier $\sum_{k=1}^{n-1} k^2 - k \approx n^3$

LU decomp $\sim \underline{\underline{O(n)}}$

In general we have banded matrices



w: half bandwidth.



$\underline{w^2 x}$

v