EE2025 Engineering Electromagnetics: July-Nov 2019

Tutorial 4: RKS sections 5.2-5.10

1. A uniform plane wave is incident from air onto glass at an angle from the normal of 30°. Determine the fraction of the incident power that is reflected and transmitted for (a) parallel polarization and (b) perpendicular polarization. Glass has refractive index $n_2 = 1.45$.

Solution: First we apply Snell's law to find the transmission angle using $n = 1$ for air to get,

$$\theta_2 = sin^{-1}(\frac{sin30^\circ}{1.45}) = 20.2^\circ$$

Now, for parallel polarization, using the formula for reflection coefficient:

$$\Gamma_p = \frac{\eta_2 cos20.2^\circ - \eta_1 cos30^\circ}{\eta_2 cos20.2^\circ + \eta_1 cos30^\circ} = -0.144$$

Therefore the fraction of incident power which is reflected is:

$$|\Gamma_p|^2 = 0.021$$

similarly for s-polarization, we have

$$\Gamma_s = \frac{\eta_2 sec20.2^\circ - \eta_1 sec30^\circ}{\eta_2 sec20.2^\circ + \eta_1 sec30^\circ} = -0.222$$

The reflected power fraction is thus

$$|\Gamma_s|^2 = 0.049$$

Thus the fraction of the power that is transmitted is

$$1 - |\Gamma_s|^2 = 0.951$$

[Umang]

2. The effect of total internal refection is observed by shining the green laser pointed (λ= 532 nm, $n_1 = 1.5$) under 45^o internal angle onto the base of the prism. (see figure 3) At what distance from the surface in the air is the amplitude of the evanescent wave $1/e$ of its value at the surface?

Solution:

The evanescent wave decreases exponentially from the interface between media.
The expression for the exponentially decaying term (in z-direction) for the evanescent wave is

$$\implies e^{-z\frac{2\pi}{\lambda}\sqrt{n_1^2 sin^2\theta_i - n_2^2}}$$

Here, λ is the wavelength of the light, θ_i is the angle of incidence of the light onto the interface between media with refractive indices n_1 and n_2.
The length at which the evanescent wave has decreased to an amplitude of 1/e of its original value

$$\implies e^{-z\frac{2\pi}{\lambda}\sqrt{n_1^2 sin^2\theta_i - n_2^2}} = e^{-1}$$
$$\implies z\frac{2\pi}{\lambda}\sqrt{n_1^2 sin^2\theta_i - n_2^2} = 1$$
$$\implies z = \frac{\lambda}{2\pi}(n_1^2 sin^2\theta_i - n_2^2)^{-1/2}$$
$$z = \frac{532\ nm}{2\pi}(1.5^2 sin^2 45^o - 1^2)^{-1/2}$$
$$\implies z = 239\ nm.$$

[Naresh]

3. An optical fiber is made up of a core, where light travels, made of glass of refractive index $n_1 = 1.5$ surrounded by another layer of glass of lower refractive index n_2.

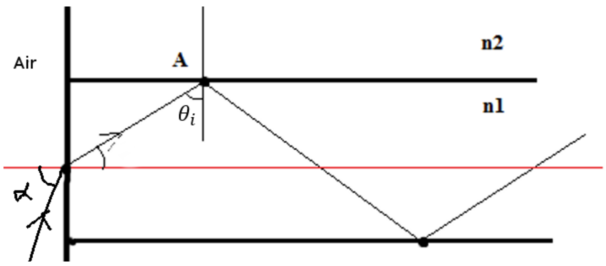


Figure 1: Optical fiber

a) Find the refractive index n_2 of the cladding so that the critical angle at the interface of core cladding is 80^o.

b) Let, α be the angle made by the ray with the axis of the fiber. For what values of α, the incident angle θ_i is larger than that of the critical angle found in part a) above.

Solution: a)

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\implies n_2 = 1.5 \times \sin 80^o$$
$$n_2 = 1.48$$

b)

$$\alpha + \theta_i = 90^o \implies \theta_i = 90^o - \alpha$$
$$for\ \theta_i > \theta_c$$
$$\implies 90 - \alpha > \theta_c$$
$$\implies \alpha < 90 - 80$$
$$\implies \alpha < 10^o$$

[Naresh]

4. Consider a circularly polarized plane wave incident on a medium at an angle θ_i.
(a) Derive a condition for the reflected wave to be circularly polarized.
(b) Derive a condition for the transmitted wave to be circularly polarized.
(*Hint:* Solve Example 5.8 in R.K.Shevgaonkar)

Solution: The incident wave is circularly polarized, which means that the parallel and the perpendicular components of the electric field, $E_{i,p}$ and $E_{i,n}$ respectively are equal in magnitude and 90° out of phase.

Without loss of generality let us take, $E_{i,p} = jE_{i,n}$. The reflection and transmission coefficients for parallel polarization, R_p and T_p are:

$$R_p = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} \qquad T_p = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

The reflection and transmission coefficients for perpendicular polarization, R_n and T_n are:

$$R_n = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad T_n = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

(a) Let the parallel and perpendicularly polarized components of the reflected electric field be $E_{r,p}$ and $E_{r,n}$ respectively.

We know, $E_{r,p} = R_p E_{i,p} = jR_p E_{i,n}$ and $E_{r,n} = R_n E_{i,n}$.

For the reflected wave to be circularly polarized, $E_{r,p}/E_{r,n} = \pm j$,

$$\begin{aligned}
\implies R_p &= \pm\, R_n \\
\implies \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} &= \pm \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \\
\implies \frac{\eta_1 \cos\theta_i}{\eta_2 \cos\theta_t} &= \pm \frac{\eta_2 \cos\theta_i}{\eta_1 \cos\theta_t}
\end{aligned}$$

This is possible only if $\eta_1 = \eta_2$. **Therefore, the reflected wave is never circularly polarized.**

(b) Let the parallel and perpendicularly polarized components of the transmitted electric field be $E_{t,p}$ and $E_{t,n}$ respectively.

We know, $E_{t,p} = T_p E_{i,p} = jT_p E_{i,n}$ and $E_{t,n} = T_n E_{i,n}$.

For the transmitted wave to be circularly polarized, $E_{t,p}/E_{t,n} = \pm j$,

$$\begin{aligned}
\implies T_p &= \pm\, T_n \\
\implies \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} &= \pm \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \\
\implies \eta_1 \cos\theta_i + \eta_2 \cos\theta_t &= \pm\,(\eta_2 \cos\theta_i + \eta_1 \cos\theta_t) \\
\implies \cos\theta_i &= \pm \cos\theta_t
\end{aligned}$$

Substituting in Snell's law, we get two cases

(i) $\eta_1 = \eta_2$ which means there is no interface, and
(ii) $\theta_i = 0°$, which implies normal incidence.
Since, only case (ii) is meaningful, **the transmitted wave is circularly polarized only for normal incidence.**

[Karteek]

5. A plane wave in region 1 is normally incident on the planar boundary separating lossless regions

1 and 2. If their relative permittivities and permeabilities are related as $\epsilon_1 = \mu_1^3$ and $\epsilon_2 = \mu_2^3$, find the ratio ϵ_2/ϵ_1 such that 20% of the energy in the incident wave is reflected at the boundary.

> **Solution:** Given, 20% of the energy in the incident wave is reflected.
>
> $\implies$ Reflection coefficient, $R = \pm\sqrt{0.2} = \pm 0.447$.
>
> $$\begin{aligned} R = \pm\, 0.447 &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \\ &= \frac{\eta_0\sqrt{\mu_2/\epsilon_2} - \eta_0\sqrt{\mu_1/\epsilon_1}}{\eta_0\sqrt{\mu_2/\epsilon_2} + \eta_0\sqrt{\mu_1/\epsilon_1}} \\ &= \frac{\sqrt{\mu_2/\mu_2^3} - \sqrt{\mu_1/\mu_1^3}}{\sqrt{\mu_2/\mu_2^3} + \sqrt{\mu_1/\mu_1^3}} \end{aligned}$$
>
> Further simplifying, we get $\pm 0.447 = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$, which gives $\mu_2/\mu_1 = 0.382 \quad \text{or} \quad 2.62$.
>
> Finally, $\epsilon_2/\epsilon_1 = (\mu_2/\mu_1)^3 = 0.056 \quad \text{or} \quad 17.9$.
>
> [Karteek]

6. A 10-MHz uniform plane wave having an initial average power density of $5W/m^2$ is normally incident from free space onto the surface of a lossy material in which $\epsilon_2''/\epsilon_2' = 0.05$, $\epsilon_{r2}' = 5$, and $\mu_2 = \mu_0$. Calculate the distance into the lossy medium at which the transmitted wave power density is down by $10dB$ from the initial $5W/m^2$:

> **Solution:** First, $\epsilon_2''/\epsilon_2' = 0.05 << 1$, we recognize region 2 as a good dielectric. Its intrinsic impedance is therefore approximated well
>
> $$\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2'}}(1 + j\frac{1}{2}\frac{\epsilon_2''}{\epsilon_2'}) = (377/\sqrt{(5)})[1 + j0.025] \tag{1}$$
>
> $\Gamma = -0.383 + j0.011$
>
> The fraction of the incident power that is reflected is then $|\Gamma|^2 = 0.147$, and thus the fraction of the power that is transmitted into region 2 is $1 - |\Gamma|^2 = 0.853$. Still using the good dielectric approximation, the attenuation coefficient in region 2 is
>
> $$\alpha = \omega\epsilon_2''/2\sqrt{\mu_0/\epsilon_2'} = 1.17 \times 10^{-2} Np/m \tag{2}$$
>
> Now, the power that propagates into region 2 is expressed in terms of the incident power through
>
> $$S_2(z) = 5(1 - |\Gamma|^2)e^{-2\alpha z} = 0.5W/m^2 \tag{3}$$
>
> in which the last equality indicates a factor of ten reduction from the incident power,as occurs for a 10 dB loss. Solve for z to obtain $z = 91.6m$

7. a) Consider a 100V/m, 3 GHz wave that is propagating in a dielectric material having $\epsilon_{r1} = 4$ and $\mu_{r1} = 1$. The wave is normally incident on another dielectric material having $\epsilon_{r2} = 9$ and $\mu_{r2} = 1$ as shown in the figure below. Find out the locations of maxima and minima of the electric field and the standing wave ratio in the Region 1.

   b)If region 2 is free space, at what angle of incidence will the wave in region 1 (dielectric) will undergo total internal reflection assuming parallel polarization and the propagation in x-z plane.

Will there be any electric field in the region 2 (free space) under TIR, if yes why?

c) What will be the standing wave ratio if the material in region 2 is a perfect conductor? Apply the boundary conditions to find out electric field expression in region 1.

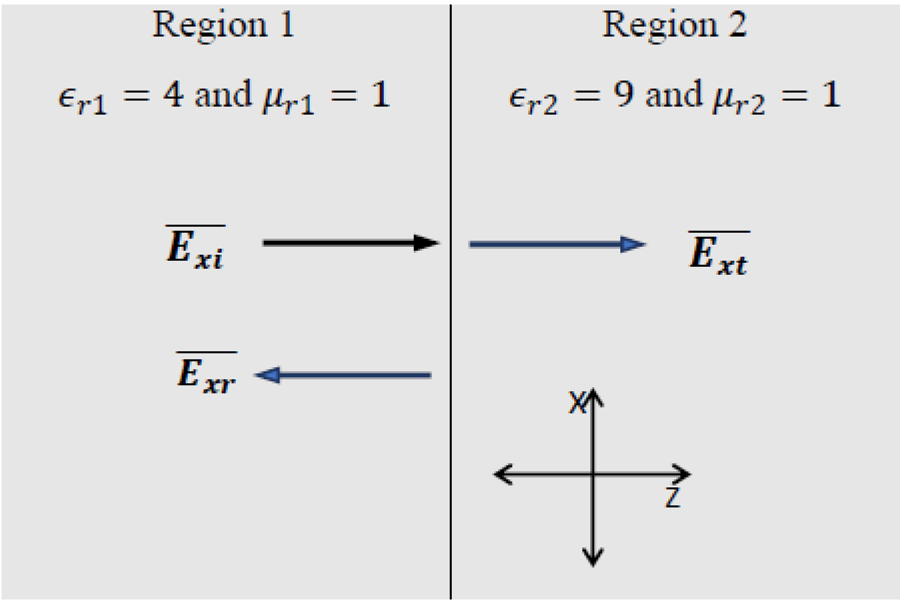


Figure 2: interface

Solution: (a) In region 1,

$$E = E_i + E_r$$

$$E_i = 100\, e^{-j\beta z}\hat{x}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1}{5}$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{3}{2}$$

$$E_r = \frac{1}{5}(100)(-\hat{x})e^{j\beta z}$$

$$E = 100e^{-j\beta z} - 20e^{j\beta z}(\hat{x}) = 100(1 - (1/5)e^{j2\beta z})e^{-j\beta z}(\hat{x})$$

$$E_{max} = |E|_{max} = (100)(1 + \frac{1}{5}) = 120V/m$$

$$2\beta z_{max} = \pm(2m + 1)\pi$$

$$z_{max} = \pm(m + \frac{1}{2})(\frac{\lambda}{2})$$

z is negative in region 1.

$$z_{max} = -\lambda/4,\ -3\lambda/4,\ -5\lambda/4, ...$$

$$2\beta z_{min} = \pm 2m\pi$$

$$z_{min} = -\frac{m\lambda}{2}$$

$$z_{min} = 0 - \lambda/2,\ -\lambda,\ -3\lambda/2, ...$$

λ=5 cm

$$z_{min} = 0,\ -2.5,\ -5,\ -7.5,\ .....cm$$

$$z_{max} = -1.25,\ -3.75,\ -6.25,\ .....cm$$

(b) For Total internal reflection,

$$\theta_i > \theta_c$$

So, if $\theta_i > 30^0$, there will be total internal reflection

$$\theta_i > \theta_c \implies sin(\theta_t) > 1 \implies cos(\theta_t)\, will\, be\, imaginary$$

$$cos(\theta_t) = \sqrt{1 - sin^2\theta_t} = \pm j\sqrt{\frac{\beta_1^2}{\beta_2^2} sin^2(\theta_i) - 1} = \pm j\sqrt{4\, sin^2\theta_i - 1}$$

$$\mathbf{E_t} = \tau_{11} E_{io} e^{-j(z\beta_2 cos(\theta_t) + x\beta_2 sin(\theta_t))}$$

using $\beta_1 sin(\theta_i) = \beta_2 sin(\theta_t)$

$$\mathbf{E_t} = \tau_{11} E_{io} e^{-j(x\beta_1 sin(\theta_i) \pm jz\sqrt{\beta_1^2\, sin^2\theta_i - \beta_2^2})}$$

$$\mathbf{E_t} = (\tau_{11} E_{io}) e^{-jx\beta_1 sin(\theta_i)} e^{-z\sqrt{\beta_1^2\, sin^2\theta_i - \beta_2^2}}$$

It can be seen that in z-direction, field exists for $z > 0$. These are called evanescent fields.Depending on the incident angle, the penetration of fields into region 2 varies.

(c) For a perfect conductor $\sigma = infinity$

$$Intrinsic\, impedance = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = 0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1$$

So,

$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} = infinity$$

At interface,

$$E_{xt} = E_{xi} + E_{xr}$$

$$E_{xr} = -E_{xi}$$

$$E_{xi} = E_0 e^{-j\beta z}$$

$$E_{xr} = -E_0 e^{j\beta z}$$

$$E_{xt} = E_{xi} + E_{xr}$$

$$E_{xt} = E_0(e^{-j\beta z} - e^{j\beta z}) = -2jE_0 sin(\beta z)$$

This represents a perfect standing wave in region 1 with standing wave ratio infinity
(Prajosh)

8. A linearly polarized wave is incident on an isosceles right triangle(prism) of glass as shown in Figure 3, and it exists as shown in figure. Assuming that the dielectric constant of the prism is 2.25, find the ratio of the exited average power density S_e to that of the incident S_i.

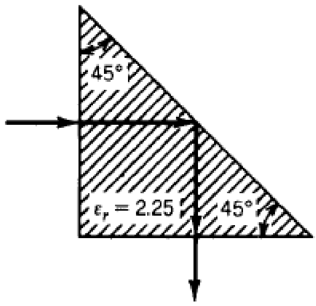


Figure 3: Prism

Solution: For the prism with a dielectric constant of 2.25, the critical angle is

$$\theta_c = sin^{-1}\sqrt{\frac{1}{2.25}} = 41.81^0$$

Therefore at the hypotenuse, the reflection coefficient $|\Gamma| = 1$ since the incident angle of 45^0 is greater than the critical angle of 41.81^0.

$$\Gamma_{21} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.2$$

$$\Gamma_{12} = -\Gamma_{21} = 0.2$$

$$\frac{S_{av}^{ex}}{S_{av}^{i}} = (1 - |\Gamma_{21}|^2)(1^2)((1 - |\Gamma_{12}|^2) = 0.9216$$

[prajosh]

9. You are asked to measure the distance from an antenna to a reflecting conducting surface. A plane wave is transmitted to the surface (normal incidence) and a zero (minimum reception) in the standing wave pattern is recorded using a second antenna at a distance 10m from the sending antenna, as shown in Setup diagram. The frequency of the wave is 100 MHz. Now, the frequency is decreased until the receiving antenna reads the next maximum in the electric field at the same location. If the frequency for the maximum reading is 99.9 MHz, calculate the distance between the transmitting antenna to the conducting surface. Use the properties of free space without attenuation.

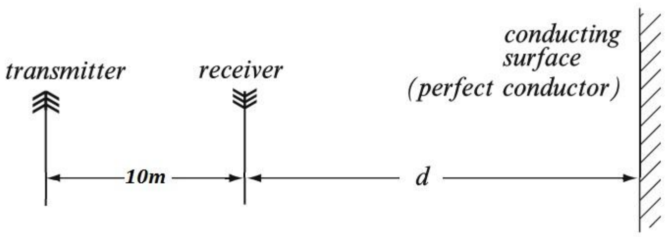


Figure 4: Setup diagram

Solution: For frequency = 100 MHz,assume there are n minimas between the wall and the receiver.Hence

$$d = \frac{n\lambda_1}{2} \implies \frac{nc}{2f_1}$$

Now as the frequency is decreased,wavelength increases and hence the distance between minima will increase.So now minimum at the receiver has now moved to the left.

Therefore,ther are now $(n-1)$ minimas in the standing wave pattern between the wall and the receiver plus the distance between a minimum and the maximum.Hence

$$d = \frac{(n-1)\lambda_2}{2} + \frac{\lambda_2}{4} \implies \frac{(n-1)c}{2f_2} + \frac{c}{4f_2}$$

Solving the above equations,we find the value of n which comes out to be 500.Substitute the value of n to get the value of d ,which is 750 meters. Therefore the total distance between transmitter and the wall is 750+10 = 760 meters.

[Aggraj]

10. Consider an air-medium interface. Determine the value of n of the medium for which Brewster's angle is equal to the critical angle.

Solution:

If we consider the critical angle and Brewster's angle for the wave travelling from denser to rarer medium

$$\theta_C = sin^{-1}(\frac{n_2}{n_1}) = \theta_B = tan^{-1}(\frac{n_2}{n_1}) \tag{4}$$

$$\theta_C = sin^{-1}(\frac{1}{n}) = \theta_B = tan^{-1}(\frac{1}{n}) \tag{5}$$

$n = 0$. This case is not interesting as $n = 0$ is the solution

Now consider the case where critical angle is considered for the wave travelling from denser to rarer medium and Brewster's angle for the wave travelling from rarer to denser. (i.e Critical angle for internal reflection and Brewster's angle for an external) $n_1 = 1$

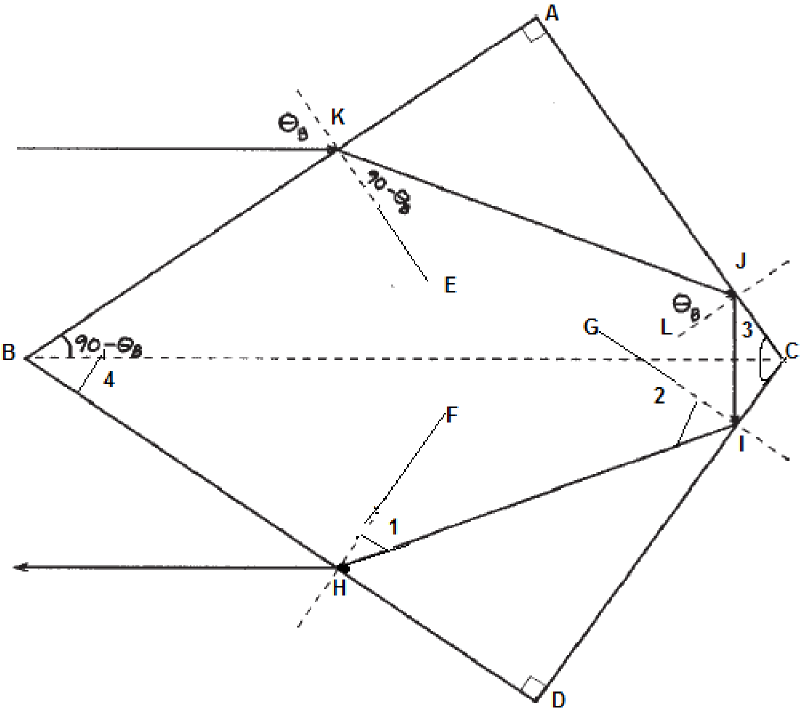
$$\theta_C = sin^{-1}(\frac{1}{n}) = \theta_B = tan^{-1}(n) \tag{6}$$

$$n^2 = \sqrt{1+n^2} \tag{7}$$

n^2 is golden ratio. $n^2 = \frac{1+\sqrt(5)}{2}$

$n = 1.272$

11. Show how a single block of glass can be used to turn a parallel polarized light through 180^0 (i.e change the direction of propagation by 180 degrees), with the light suffering (in principle) zero reflective loss. The light is incident from air and returning beam (also in air) may be displaced sideways from the incident beam. Specify all pertinent angles and use n=1.45 for glass. (More than one design is possible here).



Solution:

Consider the figure. Let incoming light enters at Brewster angle θ_B (so that p-polarized light completely transmits) at one side.

$$tan(\theta_B) = n \implies sin(\theta_B) = \frac{n}{\sqrt{1+n^2}} \implies \theta_B = 55.5^0$$

From snells law,

$$1.sin(\theta_B) = n.sin(\theta_t) = \implies \theta_t = sin^{-1} = 34.6^0$$

from geometry, $\angle KJL = \theta_B$
Make it undergo total internal reflection. For this, $\theta_B > \theta_c$ (glass air interface)

$$\theta_c = sin^{-1}(1/1.45) = 43.6^0 \implies \theta_B > \theta_c$$

So, TIR takes place.
From figure, ∠FHI should be Brewster angle to avoid reflection of p-polarized light.

$$\angle FHI = tan^{-1}(1/n) = 90 - tan^{-1}(n) = 90 - \theta_B$$

$$\angle HID = 90 - \theta_B$$

$$\angle JIC = \angle IJC = 90 - \theta_B$$

$$\angle JCI = 2\theta_B \implies \angle ABD = 180 - 2\theta_B$$

(Prajosh)

12. Given three materials fiberglass ($n = 1.6$), rain erosion paint ($n = 1.6$, thickness=$\lambda_3/16$) and primer ($n = 2.56$, thickness=$\lambda_2/2$). Where n is the relative refractive index of the material.

Design a cascade of these three materials (propose an arrangement and the thickness of the fiberglass), such that overall transmission coefficient for a normally incident wave is unity. The wave propagates through the air ($n = 1$) into the cascade and then leaves back into the air. Suppose now you use this configuration for a radome which must transmit at least 95% of the incident signal power. Find the value of n for the atmosphere (n of atmosphere varies with height).

Assume that the air between the aircraft antenna and the radome walls has $n = 1$ always, i.e air at sea level.

Solution:

The following configuration can be used to attain transmission coefficient equal to unity:

Air ($n = 1$)	Fiber-glass ($n_1 = 1.6$)	Primer ($n_2 = 2.56$)	Erosion Paint ($n_3 = 1.6$)	Air ($n = 1$)
	$7\lambda_1/16$	$\lambda_2/2$ **Fixed**	$\lambda_3/16$ **Fixed**	

Explanation: The Fiber-glass + Primer + Erosion Paint cascade has effective transmission coefficient equal to 1 and thus the primer in the middle can be ignored simplifying the system to the following:

Air ($n = 1$)	Fiber-glass ($n_1 = 1.6$)	Erosion Paint ($n_3 = 1.6$)	Air ($n = 1$)
	$7\lambda_1/16$	$\lambda_3/16$	

$$\lambda_1 = \lambda_3 = \lambda$$

$$total\ thickness = \lambda/2$$

This system has air on both sides and slab length is $\lambda/2$, thus has an effective transmission coefficient of unity.

Solve for the transmission coefficient when the air on the right has a variable n to get the following formula for the effective transmission coefficient. Setting length of slab equal to $\lambda/2$:

$$|T| = \frac{t_{12}t_{23}}{1 + r_{21}r_{23}}$$

substituting values in terms of n's ($n_{air} = n_1$, $n_{glass/paint} = n_2$, $n_atm = n$):

$$|T| = \frac{\frac{4n_1n_2}{(n_1+n_2)(n_2+n_3)}}{\frac{n_1n_2+n_1n_3+n_2^2+n_2n_3+n_1n_2+n_2n_3-n_2^2-n_1n_3}{(n_1+n_2)(n_2+n_3)}}$$

$$\Rightarrow |T| = \frac{2n_1}{n_1 + n_3}$$

For transmission of 95% of incident power, setting $|T|^2 = 1$ gives $n \approx 1.052$

[Umang]

13. A 1 GHz parallel polarized plane-wave is incident from medium 1 onto the medium 1 and 2 interface with an angle greater than critical angle as shown in Fig.5. Given the thickness of medium 2 is 5

cm. (*Info:* Materials are non-magnetic ($\mu_r = 1$) and magnetic fields are in +ve y-direction).

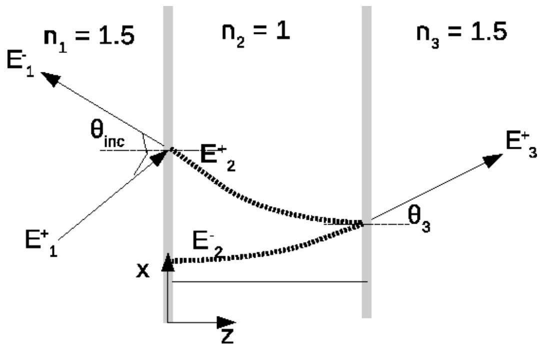


Figure 5: There will be infinite multiple reflections at each interface. For simplicity we show the steady state fields.

(a) Derive the expressions for fields in all the three medium when $\theta_{inc} = 60^o$.

(b) What is the average power density reflected and transmitted from medium 1 and medium 3 respectively when the incident average power density is $5W/m^2$.

(c) Is law of conservation of energy satisfied? If not, why?

Solution: (a) (Recap of formulas for parallel polarization for wave coming from medium 1 to 2):

$$E_{is} = E_{io}(\cos\theta_i \mathbf{a_x} - \sin\theta_i \mathbf{a_z})e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$
$$E_{rs} = E_{ro}(-\cos\theta_i \mathbf{a_x} - \sin\theta_i \mathbf{a_z})e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$
$$E_{ts} = E_{to}(\cos\theta_t \mathbf{a_x} - \sin\theta_t \mathbf{a_z})e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

The fields in each interface are given below using Fig.**??**:

$$E_1^+ = E_0(\cos\theta_1 \mathbf{a_x} - \sin\theta_1 \mathbf{a_z})e^{-j\beta_1(x\sin\theta_1 + z\cos\theta_1)}$$
$$E_1^- = A(-\cos\theta_1 \mathbf{a_x} - \sin\theta_1 \mathbf{a_z})e^{-j\beta_1(x\sin\theta_1 - z\cos\theta_1)}$$
$$E_2^+ = B(\cos\theta_2 \mathbf{a_x} - \sin\theta_2 \mathbf{a_z})e^{-j\beta_2(x\sin\theta_2 + z\cos\theta_2)}$$
$$E_2^- = C(-\cos\theta_2 \mathbf{a_x} - \sin\theta_2 \mathbf{a_z})e^{-j\beta_2(x\sin\theta_2 - z\cos\theta_2)}$$
$$E_3^+ = D(\cos\theta_3 \mathbf{a_x} - \sin\theta_3 \mathbf{a_z})e^{-j\beta_3(x\sin\theta_3 + z\cos\theta_3)}$$

Snell's Law:

$$1.5\sin 60^O = 1\sin\theta_2 = 1.5\sin\theta_3 \implies \theta_2 = 90 - 43.27i(\text{Complex}) \text{ ,and } \theta_3 = 60^o$$
$$\sin\theta_1 = \frac{\sqrt{3}}{2}, \sin\theta_2 = \frac{3\sqrt{3}}{4}, \sin\theta_3 = \frac{\sqrt{3}}{2}, \cos\theta_1 = \frac{1}{2}, \cos\theta_2 = \frac{-\sqrt{11}j}{4}, \cos\theta_3 = \frac{1}{2}$$

So, Total Internal Reflection happens in medium 1 and there will be evanescent wave in medium 2 and in medium 3 wave is going out at 60^o. Is it not surprising that a evanescent wave in medium 2 becomes as propagating wave in medium 3!
Apply Boundary conditions at z=0 and z=d as shown below:
At $z = 0$:

$$E_0\cos\theta_1 - A\cos\theta_1 = B\cos\theta_2 - C\cos\theta_2$$
$$E_0 + A = \frac{\eta_1}{\eta_2}(B + C)$$

At $z = d$:

$$B\cos\theta_2 e^{-j\beta_2 d\cos\theta_2} - C\cos\theta_2 e^{j\beta_2 d\cos\theta_2} = D\cos\theta_3 e^{-j\beta_3 d\cos\theta_3}$$
$$Be^{-j\beta_2 d\cos\theta_2} + Ce^{j\beta_2 d\cos\theta_2} = \frac{\eta_2}{\eta_3} De^{-j\beta_3 d\cos\theta_3}$$

Solving above equations we will get A, B, C, D in terms of E_0.

$$E_0 = \sqrt{2\eta_1 P_{inc}} = 50.13V/m$$
$$A = -24.17 + i33.08\, V/m$$
$$B = 29.45 + i47.22\, V/m$$
$$C = 9.50 + i2.41\, V/m$$
$$D = 4.45 + i28.55\, V/m$$

(b) Average Power Reflected in medium 1 = $|A|^2/(2\eta_1) = 3.34W/m^2$
Average Power transmitted to medium 3 = $|D|^2/(2\eta_3) = 1.66W/m^2$
(c) Law of conservation energy is satisfied as $P_{incident} = P_{reflected} + P_{transmitted}$.
If you use only the first order reflections then due to this approximations the power will not match.

A matlab code will be shared where you can change the 'd' to check when 99.9% power is reflected back. Practically $n_{core} = 1.4475$ and $n_{cladding} = 1.444$ operating at 1550nm. Typically $d = 125\mu m$. You can use these values to find the same. Typical core thickness is $9\mu m$. [Yaswanth]