EE2025 Engineering Electromagnetics: July-Nov 2019

Tutorial 2: Maxwell's equations & Plane waves

1. Prove that electric and magnetic fields obey the principle of super position. *Hint*: Use Maxwell's equations to show this.

2. In the class, we said that the 'lab quantity' $e(t)$ can be written in terms of it's phasor E as, $e(t) = \text{Re}(Ee^{j\omega t})$. However, the choice of $e^{j\omega t}$ over $e^{-j\omega t}$ is just a convention, i.e. we could have also chosen $e(t) = \text{Re}(Ee^{-j\omega t})$. Using this information, find out whether the following waves are forward traveling or backward traveling waves.

   (a) $e^{-j\omega t}$ convention, $E(z) = Ae^{-jkz}$ (b) $e^{j\omega t}$ convention, $E(z) = Ae^{-jkz}$

   (c) $e^{j\omega t}$ convention, $E(z) = Ae^{-jkz} + Be^{jkz}$

   where $A, B \in \mathbb{C}$ are constants.

3. Using the differential form of Maxwell's equations, derive the continuity equation ($\nabla.\vec{J} = -\frac{\partial \rho}{\partial t}$).

4. Given a plane wave travelling in the $+x$ direction in free space has the following phasor expressions for the electric and magnetic fields: $\vec{E} = \hat{y}E_y(x) + \hat{z}E_z(x)$ and $\vec{H} = \hat{y}H_y(x) + \hat{z}H_z(x)$. Draw a vector diagram showing all these components, and indicate which of the E's are related to which of the H's by a constant of proportionality.

5. If the electric and magnetic field in a medium are given by $\vec{E} = 3\sin(t - 5z)\,\hat{x}$ and $\vec{H} = 4\cos(t - 5z)\,\hat{y}$, then calculate (at $z = 0$) the
   a) the instantaneous power density,
   b) instantaneous power transmitted through a surface with an area of $5\,m^2$ at $z = 0$ and the normal pointing in $\hat{z}$ direction, and
   c) total energy carried by the wave through the given surface from $t = 0\,s$ to $t = 5\,s$.

6. In a non-magnetic material ($\mu_r = 1$) with dielectric constant $\epsilon_r = 4$, the electric field is given by $\vec{e}(t) = 20\sin(10^8 t - \beta z)\hat{y}$. Calculate the propagation constant β and the magnetic field $\vec{h}(t)$.

7. The magnetic field component of a plane wave in a lossless dielectric is $\vec{H} = 30\sin(2\pi \times 10^8 t - 5x)\,\hat{z}$ mA/m.

   (a) if $\mu_r = 1$, find ϵ_r (b) Calculate the wave length and wave velocity.

   (c) Determine the intrinsic impedance (d) Determine the polarization of the wave.

   (e) Find the corresponding electric field component. (f) calculate the Poynting vector

8. Consider a x-directed time varying electric field propagating in z direction $E_x(x, y, z) = f(x, y) + g(z)$ in source-free, free space. Using Maxwell's equations, prove that if it is a plane wave, then $f(x, y) = 0$.

9. What values of A and β are required if the two fields given below satisfy Maxwell's equations in a linear, isotropic, homogeneous medium with $\epsilon_r = \mu_r = 4$ and $\sigma = 0$?

   $$\vec{E} = 120\pi cos(10^6\pi t - \beta x)\hat{a_y} V/m$$

   $$\vec{H} = A\pi cos(10^6\pi t - \beta x)\hat{a_z} V/m$$

   Assume there are no current or charge densities in space.

10. Consider a point on the surface of a perfect conductor. The electric field intensity at that point is $\vec{E} = (500\hat{x} - 300\hat{y} + 600\hat{z})\cos 10^7 t$ and medium surrounding the conductor is characterized by $\mu_r = 5$ and $\epsilon_r = 10$ and $\sigma = 0$.
    (a) Find a unit vector normal to the conductor at that point of the conductor surface.
    (b) Find the surface charge density at the point.

11. Assume two regions are separated by $z = 0$ plane. Let $\mu_1 = 4$ μH/m in region 1 where $z > 0$, while $\mu_2 = 7$ μH/m in region 2 wherever $z < 0$. We are given $\mathbf{B}_1 = 2\mathbf{a}_x - 3\mathbf{a}_y$ mT, in region 1. Find $\mathbf{B}_2$ for both the cases (a) and (b). a) Let surface current be $\mathbf{J}_s = 80\mathbf{a}_x$ A/m on $z = 0$. b) Let surface current be $\mathbf{J}_s = 80\mathbf{a}_z$ A/m on $z = 0$.

12. Consider the result of superimposing left and right circularly polarized fields of the same amplitude, frequency and propagation direction, but where a phase shift of δ radians exists between two. What is the polarization of the resultant field? (consider the wave is travelling in +z direction)

13. We have seen in class that a good conductor is classified by the condition that $\epsilon''/\epsilon' \gg 1$, where the permittivity $\epsilon = \epsilon' - j\epsilon''$. For such a conductor, find a simple relationship for α and β. Further, derive the expression for phase velocity as a function of conductivity σ and frequency f. How does velocity vary with frequency, and how does it vary with conductivity?

14. A uniform plane wave is travelling in seawater. Assume that the x-y plane resides just below the sea surface and the wave travels in the +z direction into the water. The constitutive parameters of seawater are $\epsilon_r = 80$, and $\sigma = 4\ S/m$. If the magnetic field at $z = 0$ is $\boldsymbol{H}(0, t) = \hat{\boldsymbol{y}}\, 100\, cos(2\pi * 10^3 t + 15°)$ (m A/m),

    (**a**) obtain expressions for $\boldsymbol{E}(z, t)$ and $\boldsymbol{H}(z, t)$, and

    (**b**) determine the depth at which the magnitude of $\boldsymbol{E}$ is 1% of its value at $z = 0$.