EE5120 Linear Algebra: Tutorial Test 2, 05.09.18A

Give your answers in the space provided. No calculators or smartphones allowed. Please take a few minutes to read the questions carefully and answer only what is asked, keeping your answers brief.

Roll: No:__________________ NAME:____________________________ Time: 15 mins

[3] 1. Consider a vector space consisting of all possible upper triangular 3×3 matrices. Enumerate a set of basis vectors for this vector space.

Solution: The canonical basis can be formed by putting 1 in any one of the 6 upper triangular positions, i.e.:

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}, \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix}.$$

$[0.5 \times 6 = 3]$

[5] 2. The trace of a matrix is defined as the sum of its diagonal values. Consider the vector space from the above question, but with an added constraint: the trace of any matrix of this vector space is a fixed constant, say $c = 0$. Will the number of basis vectors of this new vector space increase, decrease or remain the same? Enumerate this new set of basis vectors [1+4].

Solution: Decrease. The diagonal entries get modified and give the following basis (each one of them has trace = c):

$$\begin{bmatrix}a&0&0\\0&b&0\\0&0&c-(a+b)\end{bmatrix}, \begin{bmatrix}a&1&0\\0&b&0\\0&0&c-(a+b)\end{bmatrix}, \begin{bmatrix}a&0&1\\0&b&0\\0&0&c-(a+b)\end{bmatrix}, \begin{bmatrix}a&0&0\\0&b&1\\0&0&c-(a+b)\end{bmatrix}.$$

$[1 + 4 \times 1]$

[2] 3. Fill in the blanks: The span of the following set of vectors forms a _____ dimensional sub-space of the vector space _____.

$$\mathbf{v}_1 = \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}1\\0\\0\\0\\-1\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}0\\1\\1\\1\\1\end{bmatrix}$$

Solution: Since $v_1 - v_2 = v_3$, answers are 2, $\mathbb{R}^5$.