ELL212 - Tutorial 6, Sem II 2015-16

P.1: Consider an electric field function in vacuum given by

$$\vec{E}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\left[\cos(kr-\omega t) - \frac{1}{kr}\sin(kr-\omega t)\right]\hat{e}_\phi$$

a) Show that this function satisfies the vector wave equation

$$\nabla^2\vec{E} = \frac{1}{v^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

Compute v. Also obtain the associated steady state magnetic field $\vec{H}(r,\theta,\phi,t)$. Approximate the expressions for $\vec{E}$ and $\vec{H}$ to the lowest single order for large r.

b) Compute the time averaged Poynting vector for large r.

c) Compute the time averaged power flowing through a sphere of radius R (R is very large) centred at the origin.

P.2: Consider a plane wave with the complex $\vec{E}$ field vector given by

$$\vec{E} = E_0(\hat{e}_x + \alpha\hat{e}_y)\exp\left(-\frac{\omega}{c}y\right)\exp\left(j\omega t - j\frac{3\omega}{c}x\right)$$

a) What is the complex wave vector $\vec{k}$ and the relative permittivity of the medium supporting the wave (assume it to be non-magnetic). Compute the value of α and derive an expression for the real electric field vector $\vec{E}$.

b) Compute the complex and real magnetic field vector $\vec{H}$.

c) Compute the time averaged Poynting vector.