ELL212 - Tutorial 5, Sem II 2015-16

1) **Faraday's disk:** A metal disk of radius a rotates with angular velocity ω about a vertical axis, through a uniform field $\vec{B}$, pointing up. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact, which touches the outer edge of the disk. Find the current in the resistor (Figure 1).

2) Unknown vector fields $\vec{P}$ and $\vec{Q}$ are expressed in terms of a known vector field $\vec{R}$ by the following relation: $\nabla \times \vec{P} + \beta\vec{Q} = \alpha\vec{R}$, where α, β are some constants. Now consider an interface between two regions. As a result of the interface, $\vec{P}$, $\vec{Q}$ may differ on either side. You may assume that the vector fields $\vec{P}, \vec{Q}$ are finite everywhere. The vector field $\vec{R}$ admits to a variable separable form as $\vec{R}(n,t) = \vec{R}_s(t)\delta(n)$, where $\delta(n)$ denotes a Dirac delta function, and n, t denote normal and tangential coordinates (to the interface at $n = 0$), respectively. Derive a boundary condition for the tangential component of $\vec{P}$, i.e. a relation between the tangential components of $\vec{P}$ across the interface, in terms of known quantities. Draw appropriate figure(s), state relevant vector calculus concepts, and express the final result in vector notation.

3) Imagine two concentric metallic shells, filled with an Ohmic material of conductivity σ. The inner one of radius a carries a charge $Q(t)$, while the outer of radius b carries charge $-Q(t)$. Prove that in this special case, a changing electric field does **not** lead to a time-varying magnetic field. *Hint:* Use Ohm's law, and consider the displacement current.

4) A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field $\vec{B}$, pointing into the page fills the entire region. (Figure 2).
(a) If the bar moves tot he right at speed v, what is the current in the resistor? What direction does it flow?
(b) What is the magnetic force on the bar? Direction?
(c) If the bar starts our with speed v_0 at time $t = 0$, and is left to slide, what is the speed at a later time t?
(d) Check that the energy delivered to the resistor is exactly $mv_0^2/2$.

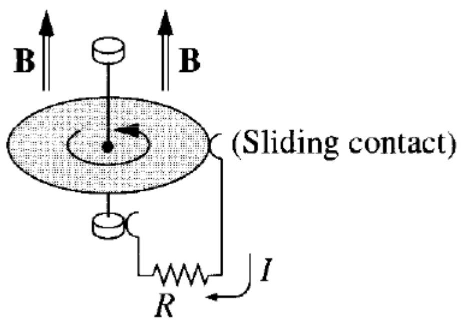

Figure 1: Faraday disk (Problem 1)

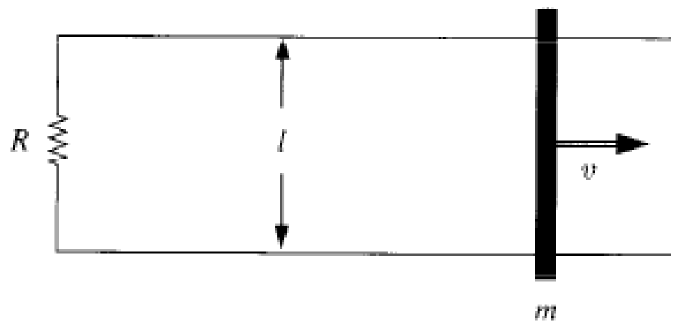

Figure 2: Sliding Bar (Problem 4)