

ELL212 - Tutorial 4, Sem II 2015-16

- 1) **Debye Screening:** In any system with mobile charges (e.g. plasmas like metals and semiconductors and even air), the electric field due to an externally introduced charge density is screened by the mobile charges – the mobile charges redistribute themselves so as to localise the impact of the introduced charge system. Typically, this results in a decay of the electric field due to the external charge system which is more rapid as compared to the corresponding decay in the absence of any mobile charges. In this problem, we analyze this effect for the simple case of a 1D plasma system.

Consider a plasma system with mobile electrons and positively charge ions (note that both positive and negative charges can move). In the absence of any external field, the number of electrons and ions per unit volume is denoted by n_0 . A small point charge Q is introduced inside this plasma at the origin. The presence of this charge pushes the positive ions away from the origin and pulls the negative ions closer to the origin, thereby resulting in the mobile charge densities being dependant on r . $n(r)$ and $p(r)$, which denote the number of negative and positive charges per unit volume respectively, can be modelled by the Boltzmann distribution:

$$n(r) = n_0 \exp(e\phi(r)/k_B T) \text{ and } p(r) = n_0 \exp(-e\phi(r)/k_B T) \quad (1)$$

where $\phi(r)$ is the electric potential at r . Now answer the following parts:

- a) Starting from the Poisson equation, show that for small Q , $\phi(r)$ approximately satisfies the following equation:
- $$\frac{1}{r} \frac{d^2}{dr^2}(r\phi(r)) = \frac{2e^2 n_0}{\epsilon_0 k_B T} \phi(r) \quad (2)$$
- b) Solve the above equation to obtain $\phi(r)$. [*Hint: Try to use the gauss law to incorporate the charge into your solution to calculate any arbitrary constants that you might get after solving the differential equation.*]
- c) For a typically achievable plasma density of $n_0 \sim 10^{19} \text{ cm}^{-3}$, estimate the *screening* length (think about how you might define this) at $T \sim 300 \text{ K}$.
- 2) **Polarisability of a H atom:** In class, you estimated the polarisability of a H atom (defined as the dipole moment per unit field created in the atom on the application of an external field) by assuming it to be a solid uniformly charged sphere. A more realistic model accounts for a variation in the charge density with r (distance from the center of the atom). Consider a H atom with its electron in the s orbital – the quantum mechanical wavefunction of the electron is given by:

$$\psi(r) = A \exp(-r/a) \quad (3)$$

where a is some (known) constant.

- a) Calculate the normalizing constant A by imposing the fact that the probability of finding the electron in all space is 1.
- b) Argue that the electron cloud can be modelled as a classical charge density $\rho(r) = e|\psi(r)|^2$. Calculate the electric field due to this charge density at all points in space using the Gauss' law.
- c) Obtain an expression for the polarisability of the H atom. You may assume that the electric field is small enough to cause only a very small separation between the positive and negative charges to simplify your calculations. Estimate this number for $a \sim 5 \times 10^{-11} \text{ m}$ and compare the result with the one that you obtained in class.