ELL212 - Tutorial 2 Solutions, Sem II 2015-16

1) Assume the voltage and current at a point z on the transmission line to be of the form

$$V(z,t) = ({V_0}^{+}e^{-j\beta z} + {V_0}^{-}e^{-j\beta z})e^{j\omega t}$$
$$I(z,t) = \left(\frac{{V_0}^{+}}{Z_0}e^{-j\beta z} - \frac{{V_0}^{+}}{Z_0}e^{-j\beta z}\right)e^{j\omega t}$$

The unknowns V_0^+ and V_0^- can be evaluated using the boundary conditions at the two ends of the TL

$$V_s \exp(j\omega t) = V(0,t)$$
$$V(l,t) = R_L I(l,t)$$

Therefore

$$V_s = V_0^+ + V_0^-$$
$${V_0}^{-} = {V_0}^{+}\left(\frac{R_L - Z_0}{R_L + Z_0}\right)e^{-2j\beta l}$$

Also,

$$V_L = V(l,t)$$

Therefore,

$$V_L = \frac{R_L}{R_L\cos(\beta l) + jZ_0\sin(\beta l)}V_s$$

(Note that at low frequencies, $\beta l = \frac{\omega l}{c}$ is very small and thus $\cos(\beta l) \approx 1, \sin(\beta l) \approx 0$, $V_L \approx \frac{R_L}{R_L+0}V_S \approx V_S$ and the transmission line behaves like a short)

2) This circuit is equivalent to

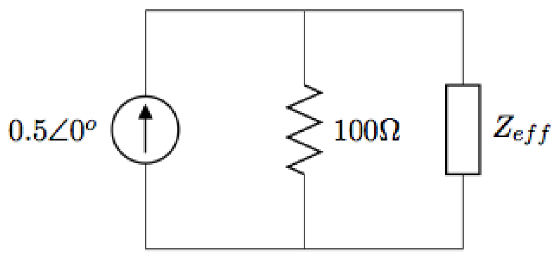

Fig. 1. Equivalent Circuit of **Q2**

$$Z_{eff} = 50\left(\frac{25 + j50\tan(360\times 2.6)}{50 + j25\tan(360\times 2.6)}\right) = (22.3186 - j7.3812)\Omega$$

The current through the 100Ω resistor (I_1):

$$I_1 = 0.5\left(\frac{22.3186 - j7.3812}{122.3186 - j7.3812}\right)A = (0.0927 - j0.02458)A$$

$$P_{R1} = |I_1|^2 \times 100 = 0.799W$$

The voltage, V_s across the current source can be calculated as:

$$V_s = I_1 \times 100 = 9.27 - j2.458$$
$$P_s = Re[V_s {I_s}^*] = 9.27 \times 0.5 = 4.635W \text{ and } P_{R2} = P_s - P_{R1} = 3.836W$$

3) The equivalent circuit is shown in Fig. 2:
Z_1 is the equivalent impedance of the short circuit stub attached between A and ground. Therefore

$$Z_1 = jZ_0\tan(\beta d_1) = j300\tan(\frac{2\pi}{100}10) = j217.963$$

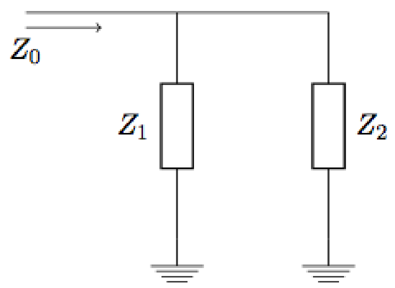

Fig. 2. Equivalent Circuit of **Q3**

Also

$$Z_0 = Z_1 || Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$
$$Z_2 = \frac{Z_1 Z_0}{Z_1 - Z_0} = \frac{300 \times j217.963}{j217.98 - 300} = (103.64 - j142.658)\Omega$$

Since,

$$Z_2 = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} \right)$$

Z_L can be calculated as:

$$Z_L = Z_0 \left(\frac{Z_2 - jZ_0 \tan(\beta d)}{Z_0 - jZ_2 \tan(\beta d)} \right)$$

4) Equivalent circuit

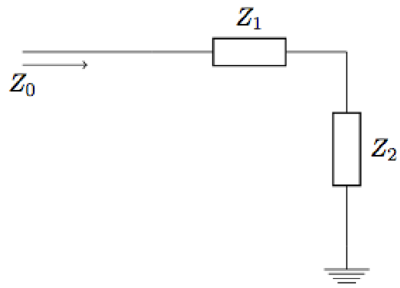

Fig. 3. Equivalent Circuit of **Q4**

$$Z_1 = jZ_0 \tan(\beta d_1)$$
$$Z_2 = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} \right)$$

Thus

$$Z_0 = jZ_0 \tan(\beta d_1) + Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} \right)$$

Separating the real and imaginary parts of this equation:

$$1 = \frac{Z_L Z_0 (1 + \tan^2(\beta d))}{{Z_0}^2 + {Z_L}^2 \tan^2(\beta d)}$$
$$\tan(\beta d_1) = \frac{(Z_L^2 - Z_0^2) \tan(\beta d)}{Z_0^2 + Z_L^2 \tan^2(\beta d)}$$

which can be solved to obtain

$$\tan(\beta d) = \pm \sqrt{\frac{Z_0}{Z_L}}$$
$$\tan(\beta d_1) = \mp \left(\sqrt{\frac{Z_0}{Z_L}} - \sqrt{\frac{Z_L}{Z_0}} \right)$$