ELL212 - Tutorial 1, Sem II 2015-16

1) Consider a monochromatic wave function given by
$$\psi(\vec{r}) = A(\vec{r})\cos(\omega t + \theta(\vec{r}))$$
Show that if $\psi(\vec{r})$ satisfies the wave-equation then so does
$$\xi(\vec{r}) = A(\vec{r})\exp[j\{\omega t + \theta(\vec{r})\}]$$
and vice-versa.

2) Consider the two dimensional wave equation in cylindrical coordinates:
$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial \psi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \psi}{\partial \phi^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$
Obtain an expression for an azimuthally symmetric (i.e. ϕ independant) cylindrical wave at frequency ω for large ρ. (Hint: Use the substitution $\psi(\rho, t) = f(\rho, t)/\sqrt{\rho}$). Compare with the case of a spherical plane wave.

3) Approximate distributed circuit models of (lossless) a lossless transmission operating in high frequency modes is shown in fig.1. Note that L has units H·m, C has units F·m, L_0 has units H/m and C_0 has units F/m. Obtain expressions for the propagation constant β and the characteristic impedance Z_0 of the line for both circuits at frequency ω.

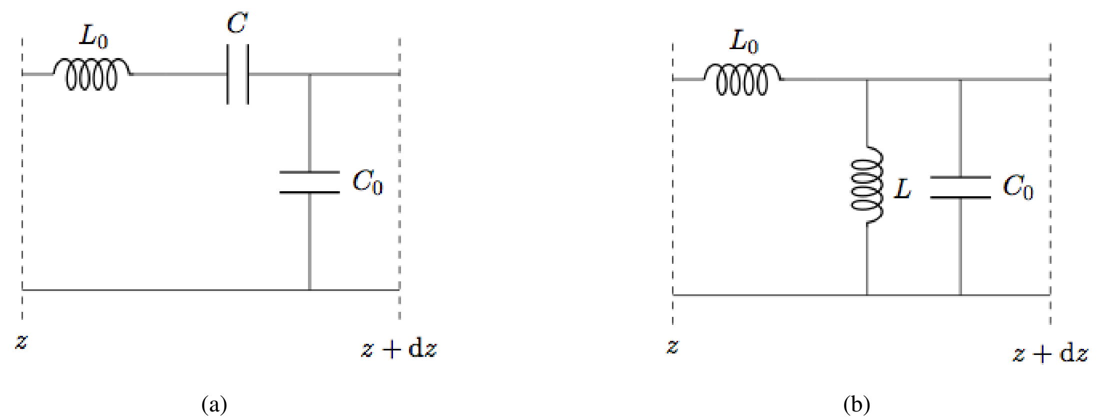

Fig. 1. Distributed circuit models for **Q3**

4) *Consider a one dimensional wave satisfying the wave equation:
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$
and the following boundary conditions:
$$\psi(x = 0, t) = \psi_0 \exp\left(-\frac{t^2}{\tau^2}\right) \text{ and } \left.\frac{\partial \psi(x,t)}{\partial x}\right|_{x=0} = \frac{\psi_0 t}{2v\tau^2}\exp\left(-\frac{t^2}{4\tau^2}\right)$$
Also assume that the function $\psi(x, t)$ is fourier transformable with respect to time t at every x. Obtain an expression for $\psi(x, t)$ for all x and t.