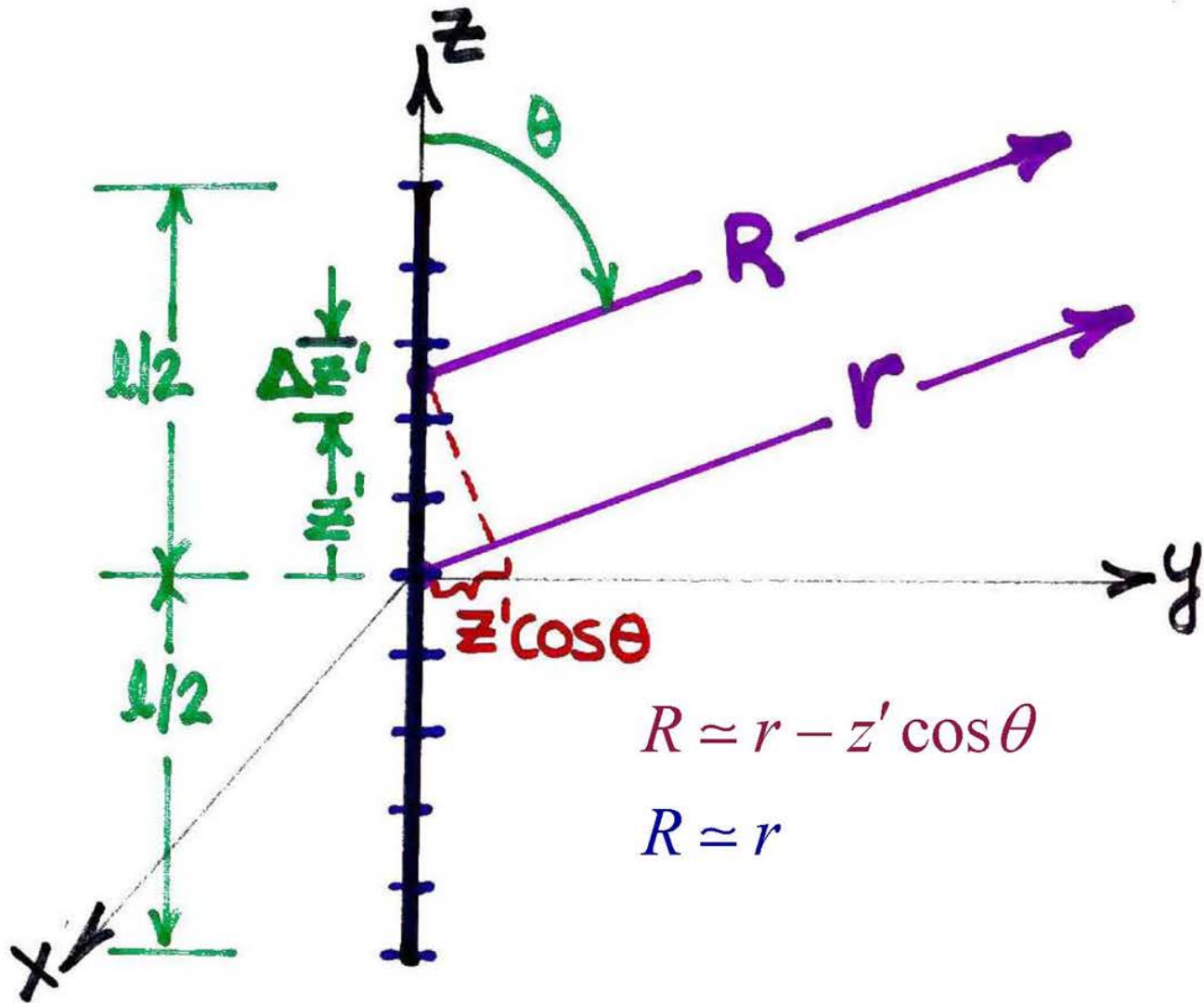


FINITE-SIZE LINEAR ANTENNAS: CURRENT DISTRIBUTIONS



$$R \approx r - z' \cos \theta$$

$$R \approx r$$

Ideal Sinusoidal Current Distribution

$$\underline{I}_e = \begin{cases} \hat{a}_z I_0 \sin \left[k \left(\frac{l}{2} - z' \right) \right] & 0 \leq z' \leq l/2 \\ \hat{a}_z I_0 \sin \left[k \left(\frac{l}{2} + z' \right) \right] & -l/2 \leq z' \leq 0 \end{cases}$$

(4-56)

Current Distributions Along the Length of a Linear Wire Antenna

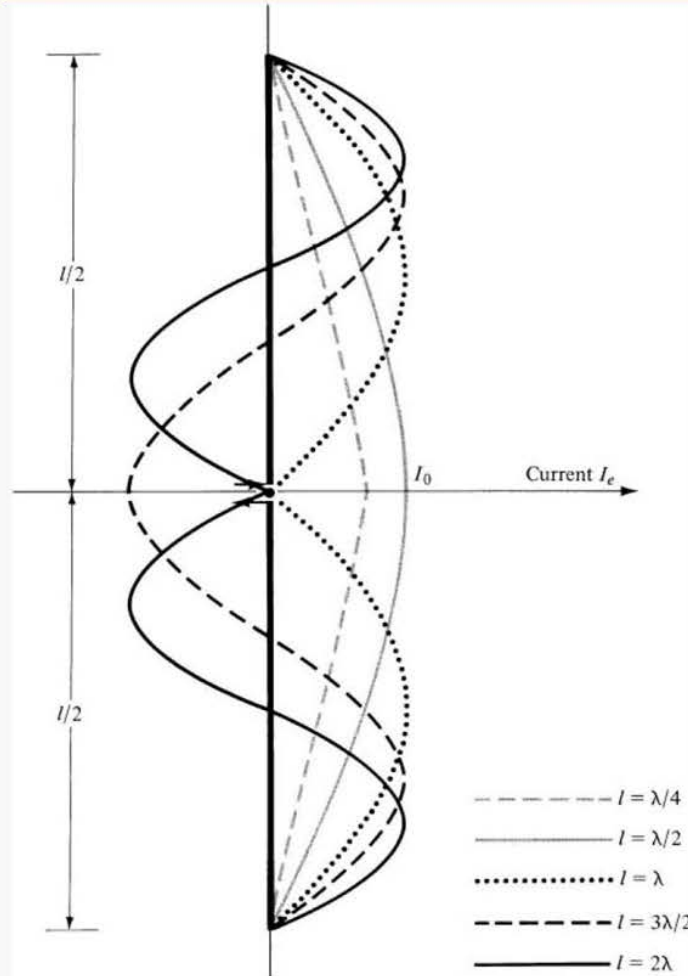


Fig. 4.8

FINITE-SIZE LINEAR ANTENNAS: RADIATION PATTERNS

$$U(\theta, \phi) = \eta \frac{|I_0|^2}{2\pi} F(\theta)$$

$$F(\theta) = \left[\frac{\cos\left(\frac{kl}{2} \cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2 \quad (4-73)$$

$$U(\theta = 0, \phi) = U(\theta = \pi, \phi) = 0$$

⇒ As expected, the radiation intensity along dipole axis is zero.

Elevation Plane Amplitude Patterns for a Thin Dipole with Sinusoidal Current Distribution ($l = \ll \lambda, \lambda/4, \lambda/2, 3\lambda/4, \lambda$)

HPBW

1. $l \leq \frac{\lambda}{50}$: HPBW = 90°
2. $l \leq \frac{\lambda}{2}$: HPBW = 74.93°
3. $l \leq \lambda$: HPBW = 47.8°

$$\frac{\lambda}{50} \leq l \leq \lambda$$

$$90^\circ \geq \text{HPBW} \geq 47.8^\circ$$

$$\Delta(\text{HPBW}) = 42.2^\circ$$

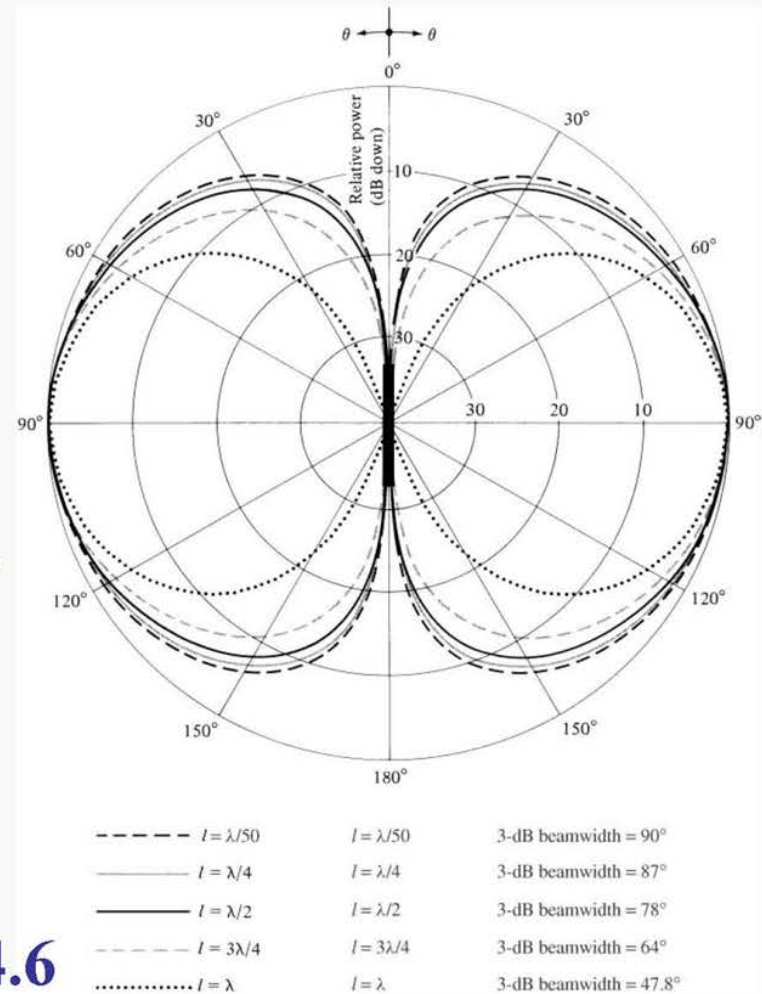
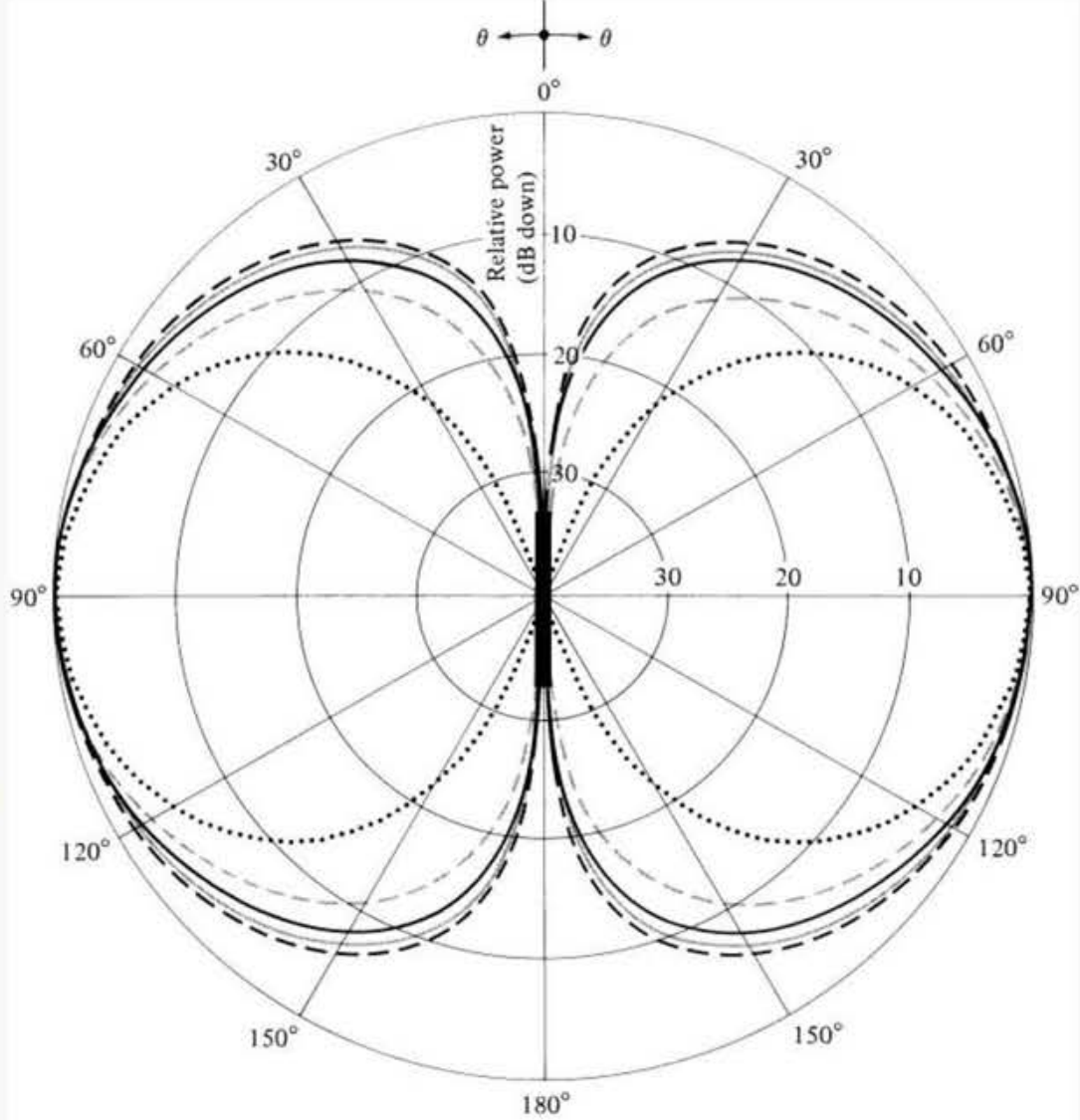


Fig. 4.6



-----	$l = \lambda/50$	$l = \lambda/50$	3-dB beamwidth = 90°
_____	$l = \lambda/4$	$l = \lambda/4$	3-dB beamwidth = 87°
—————	$l = \lambda/2$	$l = \lambda/2$	3-dB beamwidth = 78°
- - - - -	$l = 3\lambda/4$	$l = 3\lambda/4$	3-dB beamwidth = 64°
.....	$l = \lambda$	$l = \lambda$	3-dB beamwidth = 47.8°

Three-Dimensional Pattern ($l=1.25\lambda$)

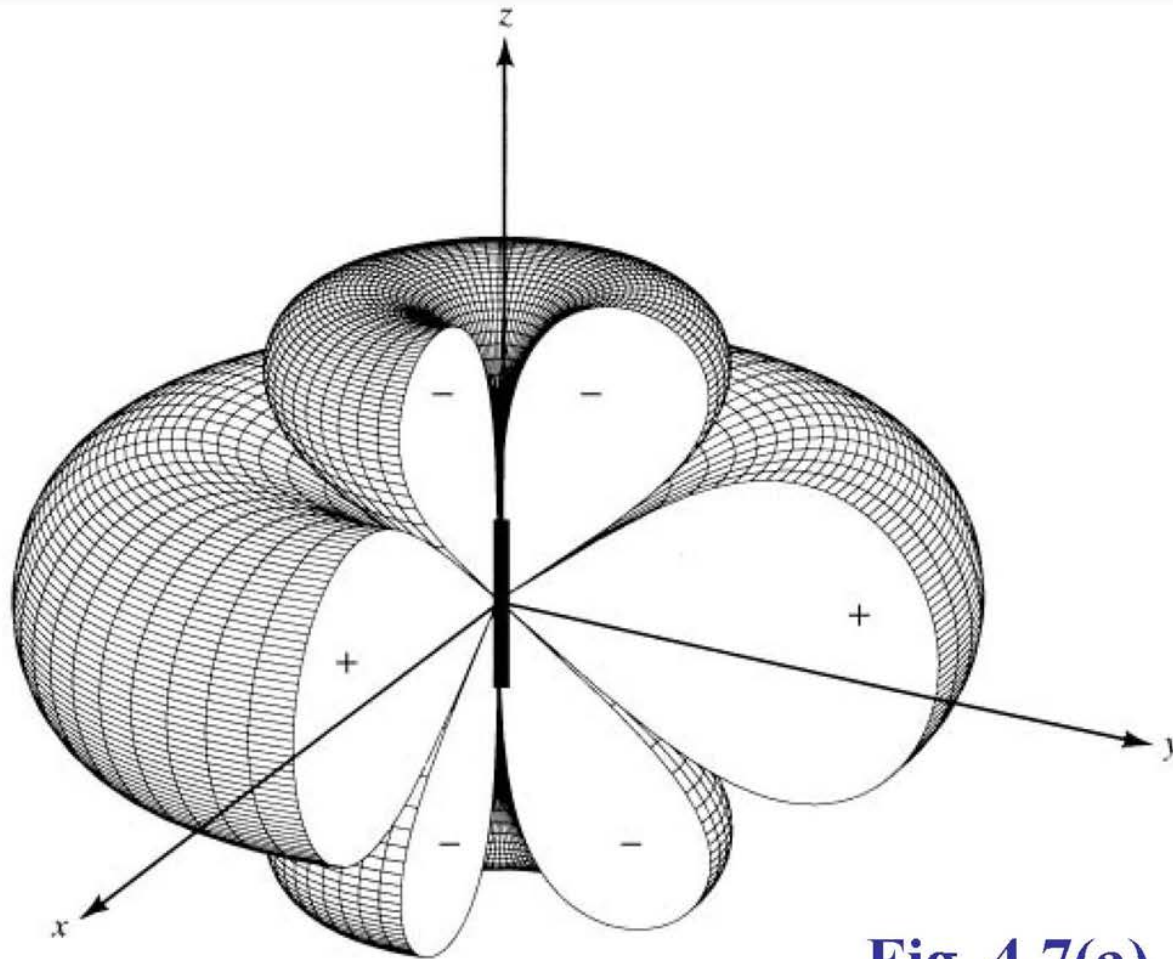


Fig. 4.7(a)

Two-Dimensional Pattern ($l=1.25\lambda$)

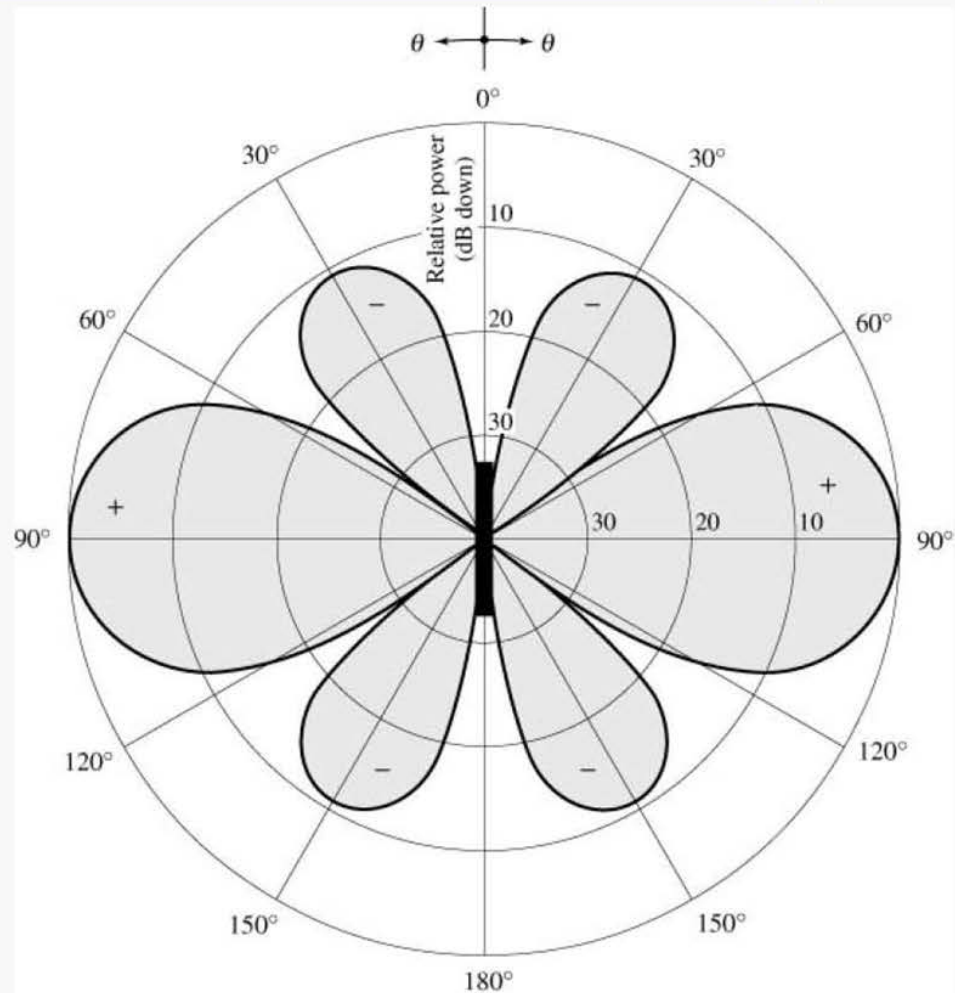


Fig. 4.7(b)

FINITE-SIZE LINEAR ANTENNAS: RADIATION RESISTANCE

$$P_{rad} = \frac{1}{2} |I_o|^2 R_r$$

$$R_r = \frac{2P_{rad}}{|I_o|^2}$$

$$\begin{aligned} P_{rad} &= \int_0^{2\pi} \int_0^{\pi} \underline{W}_{av} \cdot \hat{a}_r r^2 \sin \theta d\theta d\phi \\ &= \int_0^{2\pi} \int_0^{\pi} U_{rad} \sin \theta d\theta d\phi \end{aligned}$$

$$U(\theta, \phi) = \eta \frac{|I_0|^2}{2\pi} F(\theta)$$

$$F(\theta) = \left[\frac{\cos\left(\frac{kl}{2} \cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2 \quad (4-73)$$

$$U(\theta = 0, \phi) = U(\theta = \pi, \phi) = 0$$

⇒ As expected, the radiation intensity along dipole axis is zero.

Q

$$R_r = \frac{2P_{rad}}{|I_o|^2}$$

$$R_r = \frac{\eta}{2\pi} \left\{ \begin{aligned} & C + \ln(k\ell) - C_i(k\ell) + \frac{1}{2} \sin(k\ell) [S_i(2k\ell) - 2S_i(k\ell)] \\ & + \frac{1}{2} \cos(k\ell) \left[C + \ln\left(\frac{k\ell}{2}\right) + C_i(2k\ell) - 2C_i(k\ell) \right] \end{aligned} \right\} \quad (4-70)$$

$$X_m = \frac{\eta}{4\pi} \left\{ \begin{aligned} & 2S_i(k\ell) + \cos(k\ell) [2S_i(2k\ell) - S_i(2k\ell)] \\ & - \sin(k\ell) \left[2C_i(k\ell) - C_i(2k\ell) - C_i\left(\frac{2ka^2}{\ell}\right) \right] \end{aligned} \right\} \quad (4-70a)$$

$$C_i(x) = -\int_x^\infty \frac{\cos y}{y} dy = \int_\infty^x \frac{\cos y}{y} dy \quad (4-68a)$$

$$S_i(x) = \int_0^x \frac{\sin y}{y} dy \quad (4-68b)$$

$C = 0.5772$ (Euler's constant)

$$\begin{aligned} C_{in} &= \int_0^x \left(\frac{1 - \cos y}{y} \right) dy = \ln(\gamma x) - C_i(x) \\ &= \ln(\gamma) + \ln(x) - C_i(x) \end{aligned} \quad (4-69)$$

$$C_{in} = 0.5772 + \ln(x) - C_i(x)$$

$$\gamma = e^{0.5772} = 1.781$$

FINITE-SIZE LINEAR ANTENNAS: DIRECTIVITY

Directivity of Finite Length Dipole

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{16\pi^2 U_{\max}}{\eta |I_0|^2 Q(kl)}$$
$$= \frac{2F(\theta)|_{\max}}{Q(kl)}$$

$$\text{Dipole Length} \left(\frac{\lambda}{50} = 0.0\lambda \leq \ell \leq \lambda \right)$$

(Representative of Pattern Bandwidth)

Maximum Directivity D_o

$$1.5 \leq D_o \leq 2.411 \text{ (dimensionless)}$$

$$1.761 \text{ dB} \leq D_o \leq 3.822 \text{ dB}$$

Half-Power Beamwidth $HPBW$

$$90^\circ \geq HPBW \geq 47.8^\circ$$

Ideal Sinusoidal Current Distribution

$$\underline{I}_e = \begin{cases} \hat{a}_z I_0 \sin \left[k \left(\frac{l}{2} - z' \right) \right] & 0 \leq z' \leq l/2 \\ \hat{a}_z I_0 \sin \left[k \left(\frac{l}{2} + z' \right) \right] & -l/2 \leq z' \leq 0 \end{cases}$$

(4-56)

FINITE-SIZE LINEAR ANTENNAS: INPUT RESISTANCE

Current Distribution of a Linear Wire Antenna
When Current Maximum
Does not Occur at the Input Terminals

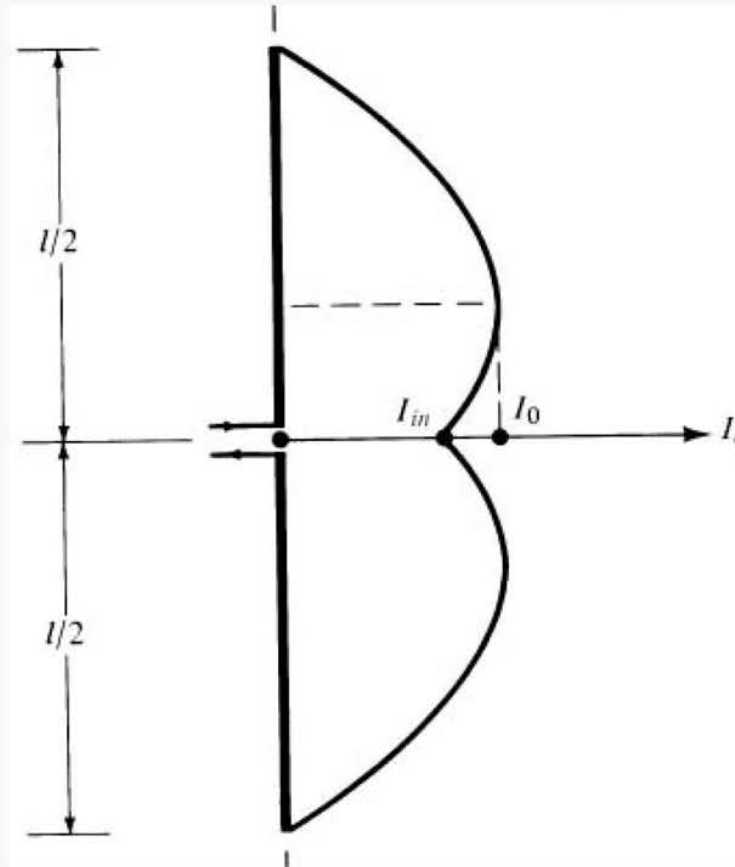


Fig. 4.10

Assume
Loss-less
antenna

$$P_{rad} = \frac{1}{2} |I_o|^2 R_r = \frac{1}{2} |I_{in}|^2 R_{in} \quad (4-77)$$

$$R_{in} = R_r \frac{|I_o|^2}{|I_{in}|^2} = R_r \left| \frac{I_o}{I_{in}} \right|^2 \quad (4-77a)$$

$$\underline{I}_e \Big|_{z'=0} = I_o \sin \left[k \left(\frac{\ell}{2} \pm z' \right) \right] \Big|_{z'=0} = I_{in}$$

$$I_{in} = I_o \sin \left(\frac{k\ell}{2} \right) \quad (4-78)$$

I_o = current maximum

I_{in} = current at input terminals

$$R_{in} = R_r \left| \frac{I_o}{I_{in}} \right|^2 = R_r \left| \frac{I_o}{I_o \sin\left(\frac{k\ell}{2}\right)} \right|^2$$

$$R_{in} = \frac{R_r}{\sin^2\left(\frac{k\ell}{2}\right)} \quad (4-79)$$

$$\text{Dipole Length} \left(\frac{\lambda}{50} = 0.02\lambda \leq l \leq \lambda \right)$$

(Representative of Impedance Bandwidth)

Radiation Resistance R_r (ohms)

$$0.0003 \leq R_r \leq 199$$

Input Resistance R_{in} (ohms)

$$0.079 \leq R_{in} \leq \infty$$

$$1. \ell = \lambda/2: \quad \sin^2\left(\frac{k\ell}{2}\right)\Big|_{\ell=\lambda/2} = \sin^2\left(\frac{2\pi}{2\lambda} \frac{\lambda}{2}\right) = \sin^2\left(\frac{\pi}{2}\right) = 1$$

$$R_{in} = R_r$$

$$2. \ell = \lambda: \quad \sin^2\left(\frac{k\ell}{2}\right)\Big|_{\ell=\lambda} = \sin^2(\pi) = 0$$

$$R_{in} = R_r / 0 = \infty$$

Directivity and Radiation/Input Resistance

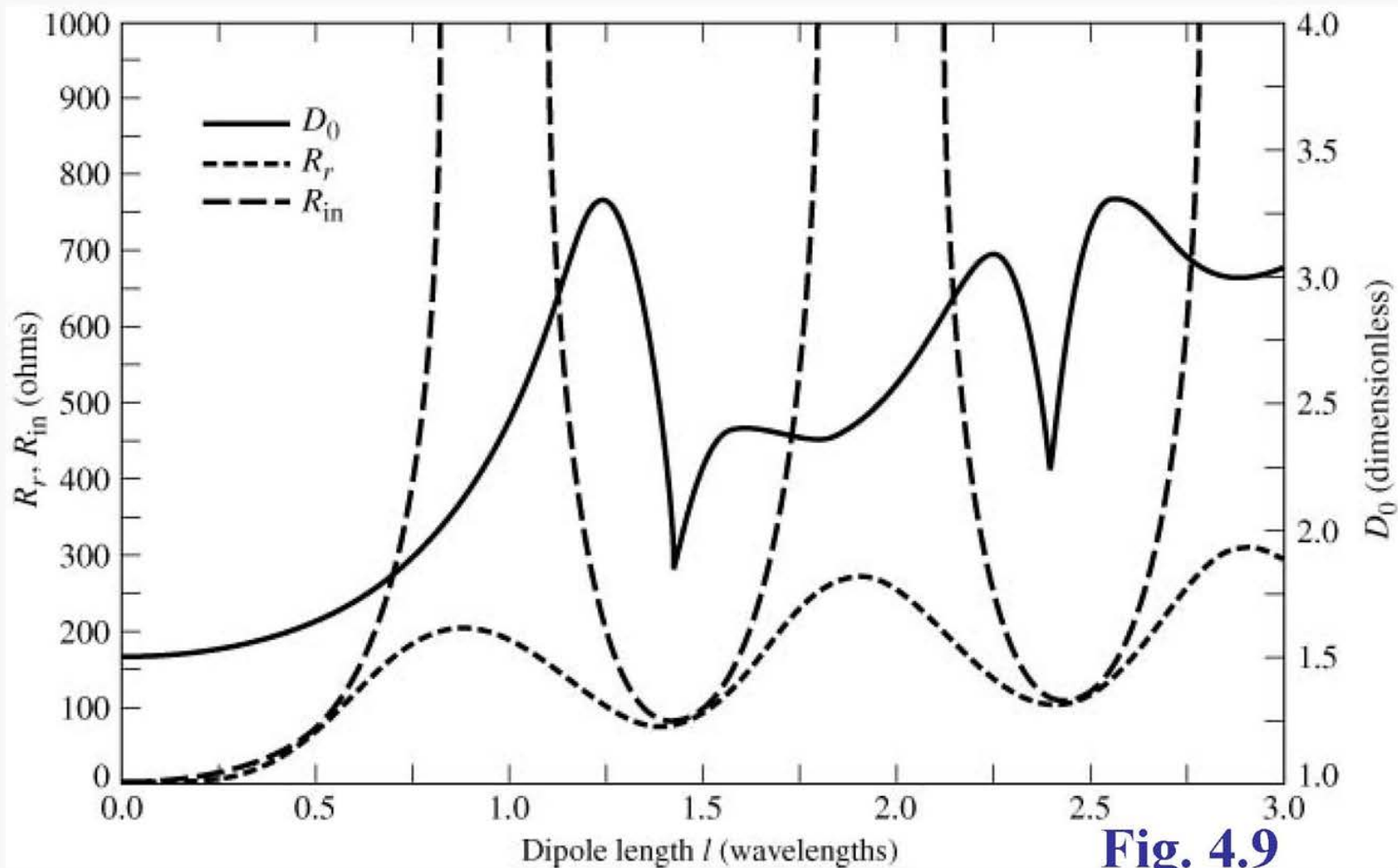


Fig. 4.9

FINITE-SIZE LINEAR ANTENNAS: HALF-WAVELENGTH DIPOLE

Half-Wavelength Dipole ($l = \lambda/2$)

$$E_{\theta} \approx j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\underbrace{\frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta}}_{\text{Field Pattern}} \right] \quad (4-84)$$

$$H_{\phi} \approx j \frac{I_0 e^{-jkr}}{2\pi r} \left[\underbrace{\frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta}}_{\text{Field Pattern}} \right] = \frac{E_{\theta}}{\eta} \quad (4-85)$$

$$W_{av} = \eta \frac{|I_0|^2}{8\pi^2 r^2} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right]^2}_{\text{Power Pattern}} \approx \eta \frac{|I_0|^2}{8\pi^2 r^2} \sin^3 \theta \quad (4-86)$$

$$U = r^2 W_{av} = \eta \frac{|I_0|^2}{8\pi^2} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right]^2}_{\text{Power Pattern}} \approx \eta \frac{|I_0|^2}{8\pi^2} \sin^3 \theta \quad (4-87)$$

$$P_{rad} = \eta \frac{|I_0|^2}{4\pi} \int_0^\pi \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} d\theta \quad (4-88)$$

$$P_{rad} = \eta \frac{|I_0|^2}{8\pi} \int_0^{2\pi} \left(\frac{1 - \cos y}{y}\right) dy = \eta \frac{|I_0|^2}{8\pi} C_{in}(2\pi) \quad (4-89)$$

$$\begin{aligned} C_{in}(2\pi) &= 0.577 + \ln(2\pi) - C_i(2\pi) \\ &= 0.577 + 1.838 - (-0.02) \approx 2.435 \quad (4-90) \end{aligned}$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = 4\pi \frac{U|_{\theta=\pi/2}}{P_{rad}}$$

$$= \frac{4}{C_{in}(2\pi)} = \frac{4}{2.435} = 1.643 \quad (4-91)$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} (1.643) = 0.13\lambda^2 \quad (4-92)$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} \frac{\eta}{4\pi} C_{in}(2\pi) \approx \frac{120\pi}{4\pi} (2.435) \approx 73.05 \quad (4-93)$$

Input Impedance

$(\ell = \lambda/2)$

$$Z_{in} = 73 + j42.5$$

(4-93a)

Infinitesimal Dipole

$$E_{\theta} = C_1 \underbrace{\sin \theta}_{\text{field pattern}} ; \quad U = C_2 \underbrace{\sin^2 \theta}_{\text{power pattern}}$$

$\lambda/2$ Dipole

$$E_{\theta} = C_1' \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]}_{\text{field pattern}} ; \quad U = C_2' \underbrace{\left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]^2}_{\text{power pattern}} \cong C_2' \sin^3 \theta$$

Three-Dimensional Pattern Of $\lambda/2$ Dipole

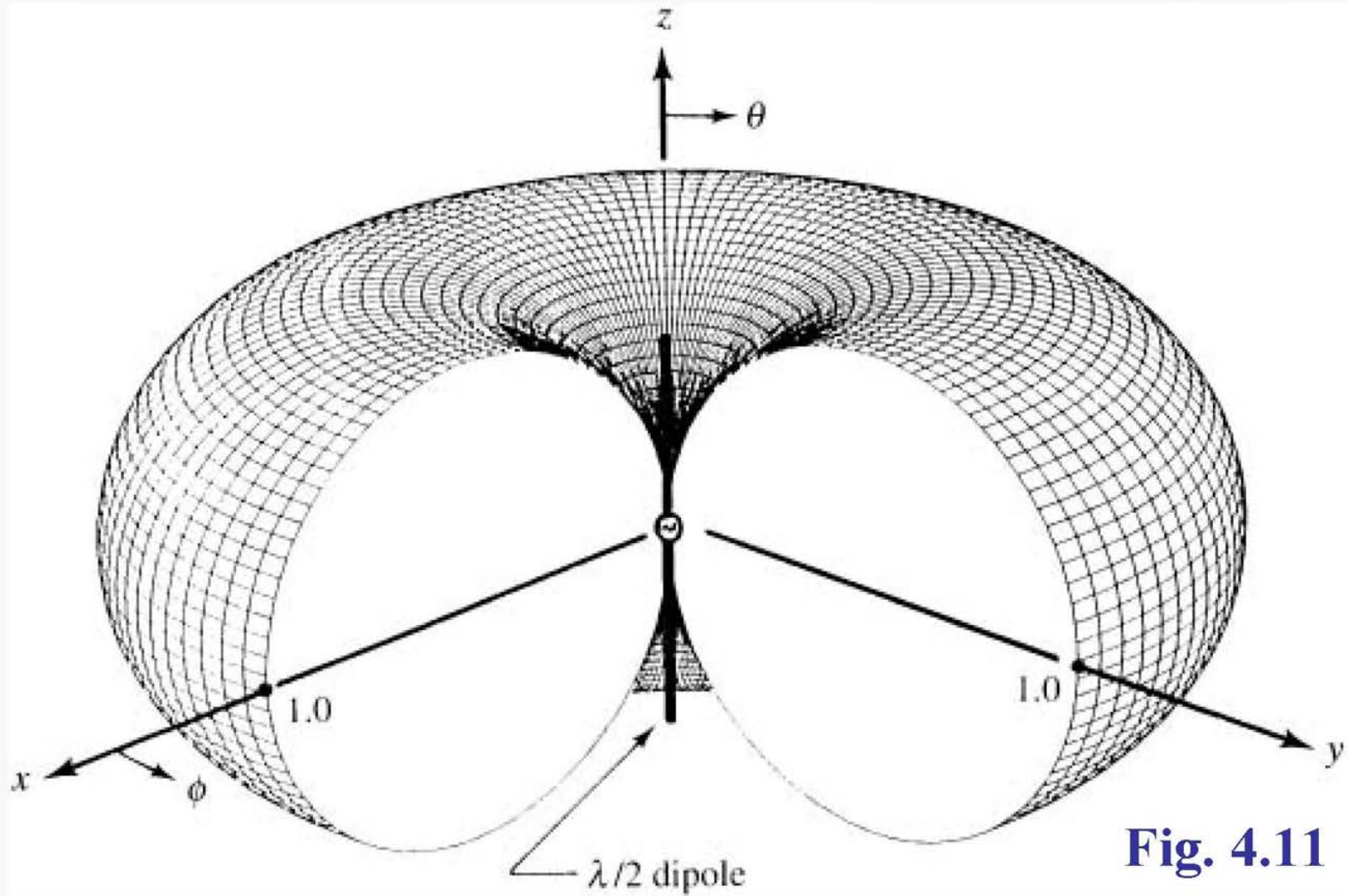


Fig. 4.11

Two-Dimensional Directivity Pattern of Isotropic Source and $\lambda/2$ Dipole

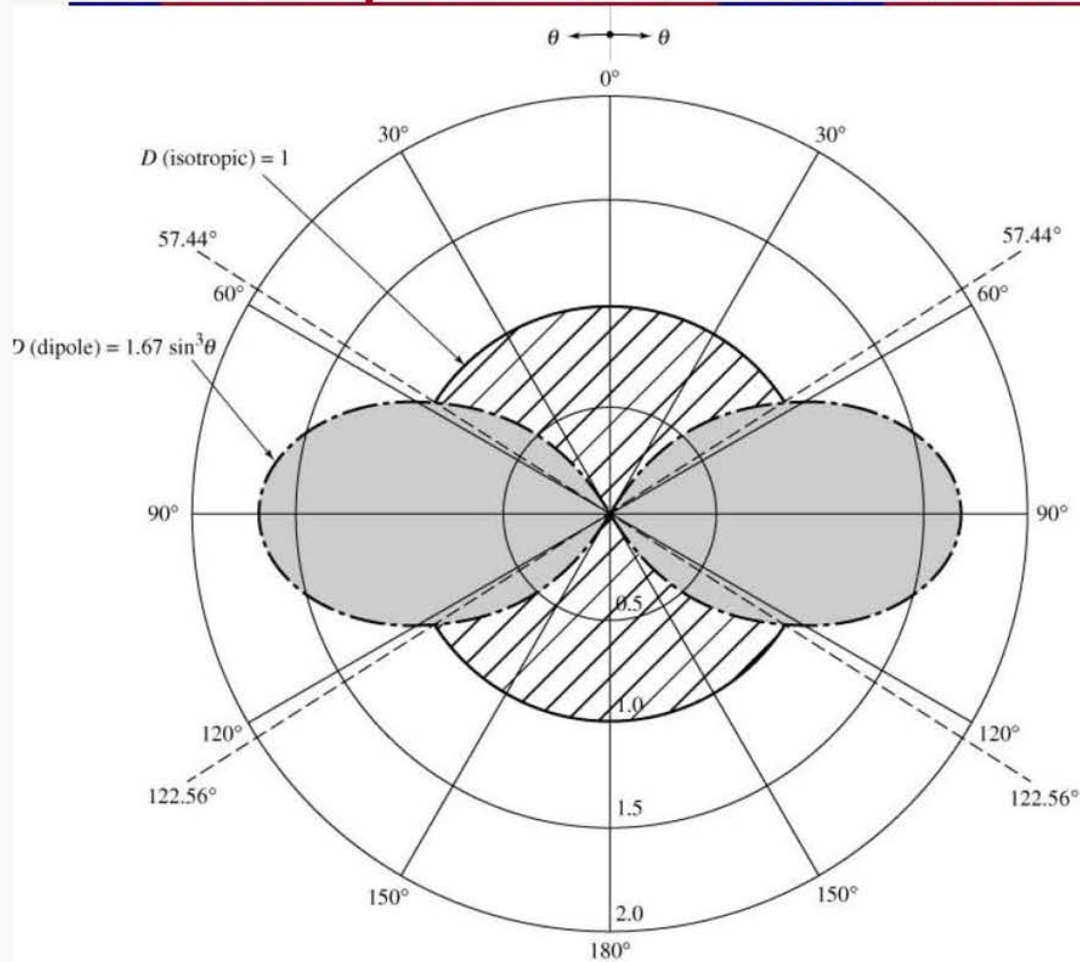


Fig. 2.13(a)

Three-Dimensional Directivity Pattern

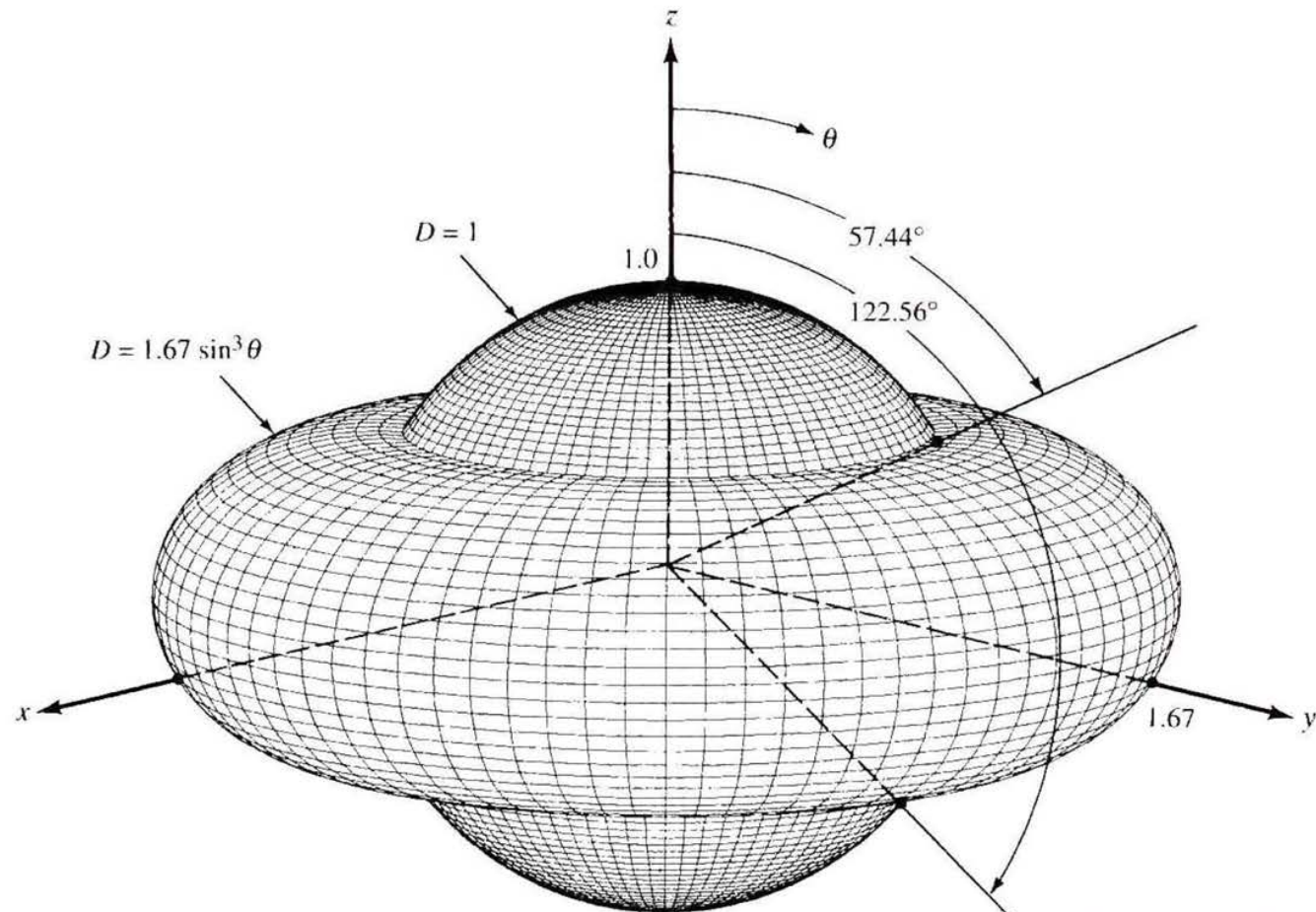


Fig. 2.13(b)