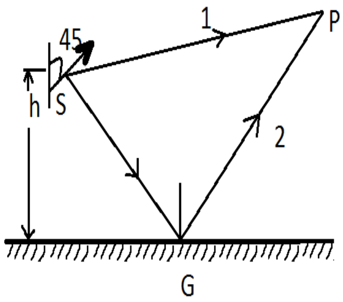
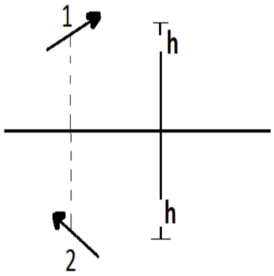
HW 2 Solns: EEL760 Antenna Theory & Design*

Q 1 (a)**[2+2]** (Method I): 'Physical' solution $\Rightarrow$ obtained by considering the reflectivity of the ground plane.



Let the ray SP have angle θ_1, and the ray SG have angle θ_2

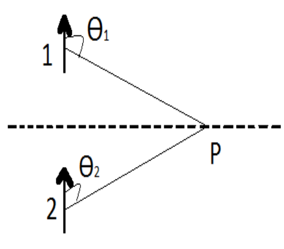
The total field is due to direct and reflected rays; i.e. $\overrightarrow{E} = \overrightarrow{E_1}(\theta_1) + R\overrightarrow{E_2}(\theta_2)$
where E_1 and E_2 can be simply obtained by the expressions for electric field of a dipole.
2 marks
(Method II) 'Image' Solution $\Rightarrow$ we apply the method of images to the problem and remove the ground plane. Resolve into vertical and horizontal dipoles and construct the image as:



The solution is now obtained as the superposition of the 2 dipoles
$\overrightarrow{E} = \overrightarrow{E_{1d}} + \overrightarrow{E_{2d}}$

(b)**[2]** The 2 methods are conceptually equivalent using the uniqueness theoram in the half space above the plane, the boundary conditions are identical -
1. Tangential fields are on the ground plane
2. Fields at ∞ go to 0 (radiation boundary condition)

More precisely, consider a vertical dipole we know that $E_\theta \propto \sin(\theta), E_r \propto \cos(\theta)$ (in any zone)



since $\theta_1 + \theta_2 = \pi$

$$at\ point\ P \begin{cases} E_{\theta_1} = & E_{\theta_2} \\ E_{r_1} = & -E_{r_2}, \end{cases}$$

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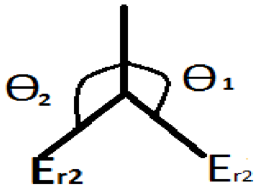
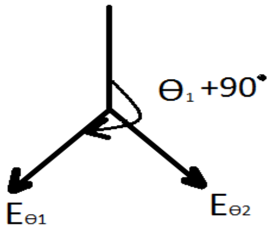
Radial components ⇒



Only normal components survive

Theta components ⇒



Again only normal components survive

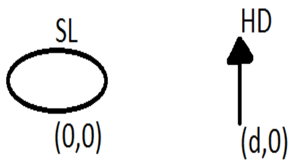
So the method of images also maintains the requirement that fields on the plane are purely normal. Similar arguments will hold for the horizontal dipole case.

Q 2 a)[**2**] For a SL, electric field is along $\hat{\phi}$

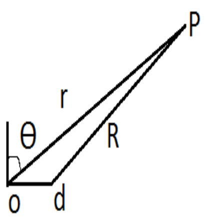
$$\overrightarrow{E}_{SL} = \frac{\eta k^2 S I_o \exp -jkr}{4\pi r} \sin(\theta)\hat{\phi} \tag{1}$$

Since the element patterns are along orthogonal directions (and hence different) the principle of pattern multiplication can't be used.

b) [**2**]



In the far-field, we can approximate $R \approx r$ (amplitude) $R \approx r - d\cos(\theta)$ (phase)



Total far field, then is

$$\overrightarrow{E} = \frac{\eta k^2 I_o \exp -jkr}{4\pi r} \sin(\theta)[ks\hat{\phi} + jl \exp jkd\cos\theta\hat{\theta} \tag{2}$$

c) [**2**] Beam forming is not possible in this case. this is since the electric fields are orthogonal to each other, there are no interference effects possible . See above , changing 'd' only changes E_θ. E_ϕ is unchanged.

d) [**4**] Yes we can use pattern multiplication now . 2 ways to think about it;

① A unit composed of 1 SL and 1 HD are arranged N times. Spacing = 2d

② 2 antenna arrays, each with N elements and spacing = d are being combined.
In both cases, there is an element pattern that can be taken common, hence pattern multiplication holds.

b) Using approach ① above, the far field is simply,

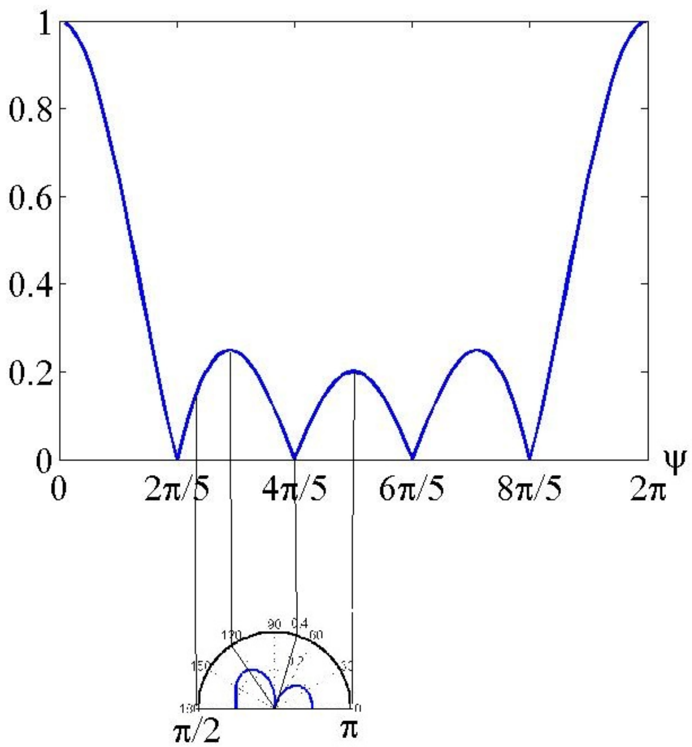
$$\overrightarrow{E} = \overrightarrow{E}_1 * \left(\sum_{n=0}^{N-1} \exp\left(j2d\cos\theta n\right) \right)$$

where $\overrightarrow{E}_1$ is obtained from eqn ① earlier

c) Yes, beam forming is possible in this case since multiple antenna patterns are interfering in each direction ($\hat{\theta}$ and $\hat{\phi}$). Note that the beam forming is limited in this case since the SL and HD beams are still in different directions.

Q 3 a)[**2**] The normalized array factor is $f(\psi) = \frac{\sin(5\psi/2)}{5\sin(\psi/2)}$, where $\psi = kd\cos(\theta)$ (see text for the simple derivation)

b)[**4**]



Here, $d = \lambda/8$, $\alpha = 3\pi/4$, i.e., radius of visible region $= kd = \pi/4$. Visible region is $\psi = \pi/2$ to π. Beam maxima at $\psi_0 = 3\pi/5 = \frac{\pi}{4}\cos\theta_0 + \frac{3\pi}{4}$. $\cos\theta_0 = -(\frac{4}{\pi})\, 3\pi(\frac{1}{20}) = -(\frac{3}{5})\theta_0 = 126.9^0$. Beam null at $\psi = 4\pi/5 = \frac{\pi}{4}\cos\theta' + \frac{3\pi}{4}$, $\cos\theta' = (\frac{4}{\pi})(\frac{\pi}{20}) = 1/5$, $\theta' = 78.5^o$.
Value of amplitudes at maxima $= \frac{\sin(5\psi_0/2)}{5\sin(\psi_0/2)} = 0.247$. Note that there are no nulls at $\theta = 0$ or π. One side lobe exists at $\theta = 0$. Value of side lobe amplitude, $\psi'' = \pi/4\ + 3\pi/4$, $f(\psi'') = \frac{\sin(5\pi/2)}{5\sin(\pi/2)} = 0.2$

c)[**1**] The radiation pattern resembles a broadside pattern since the beam maxima is closer to the broadside angle of $\theta = \pi/2$.

Q 4 (a)[**2**] We have $\mathcal{L}h_n(r) = \lambda_n h_n(r)$, which gives $L_{mn} = \langle h_m(r), \mathcal{L}h_n(r)\rangle = \lambda_n\langle h_m(r), \mathcal{L}h_n(r)\rangle$. Therefore, L is diagonal iff $\{h\}$ are orthogonal.

(b) [**3**]$\vec{f} = F\vec{a}$. The k^{th} basis function can be expressed in terms of H as $\vec{h}_k = H\vec{b}_k$. It is known that $L_{mn} = \langle f_m(r), Lf_n(r)\rangle$, which gives $l_{mn} = (Hb_m)^T L(Hb_n) = b_m^T(H^T LH)b_n$. But, due to the eigenvalue problem, $LH = H\Lambda$, where Λ is a diagonal matrix of eigenvalues. Thus, $L_{mn} = b_m^T(H^T H\Lambda)b_n$, where $H^T H$ is identified as the Gramian matrix.

(c) [**2**] $LH = H\Lambda$, and since H is a square non-singular matrix, $L = H\Lambda H^{-1}$. Thus, $L_{mn} = \langle f_m(r), Lf_n(r)\rangle = \langle f_m(r), H\Lambda H^{-1} f_n(r)\rangle = b_m^T H^T H\Lambda H^{-1} Hb_n = b_m^T(H^T H\Lambda)b_n$, which is the same as what we had obtained in (b).

Q 5 (a) [**3**] See class notes.

(b) [**3**] The solution involves a simple use of far field approximations of Hankel functions. See, for example, eqn (22) here: `http://web.iitd.ac.in/~uday/notes/fem2dprimer.pdf`.

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