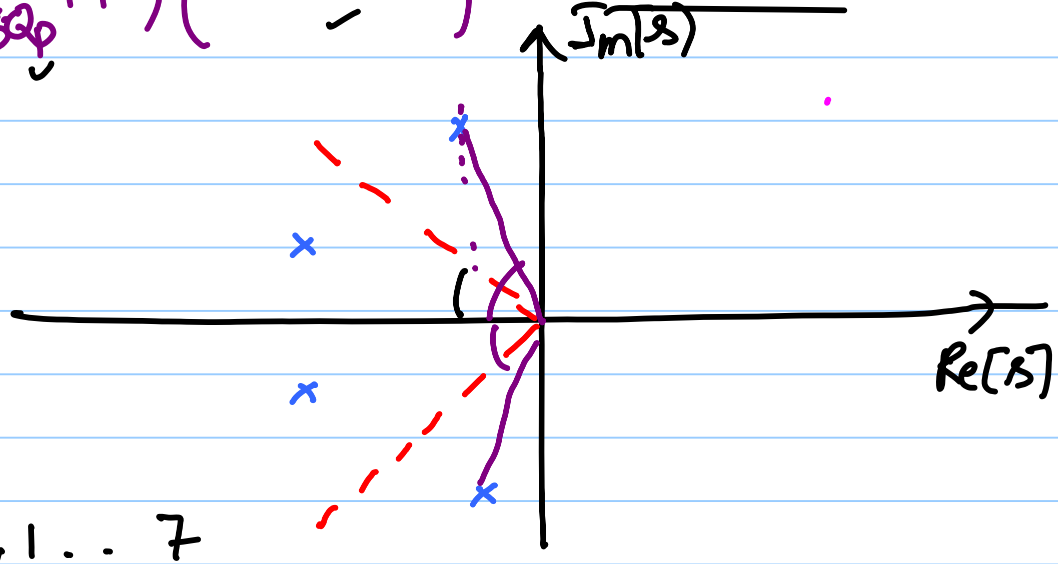
Note Title 06-04-2022

# Lecture #36

## Butterworth filter

$$H(s) = \frac{1}{\quad} = \frac{1}{\left(\frac{s^2}{\omega_p^2} + \frac{s}{\omega_p Q_p} + 1\right)\left( \quad - \quad \right)}$$

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$




$$s^{2n} = -1 = e^{j(2m\pi+\pi)}$$

$$s^{2\times 4} = e^{j(2m\pi+\pi)}$$

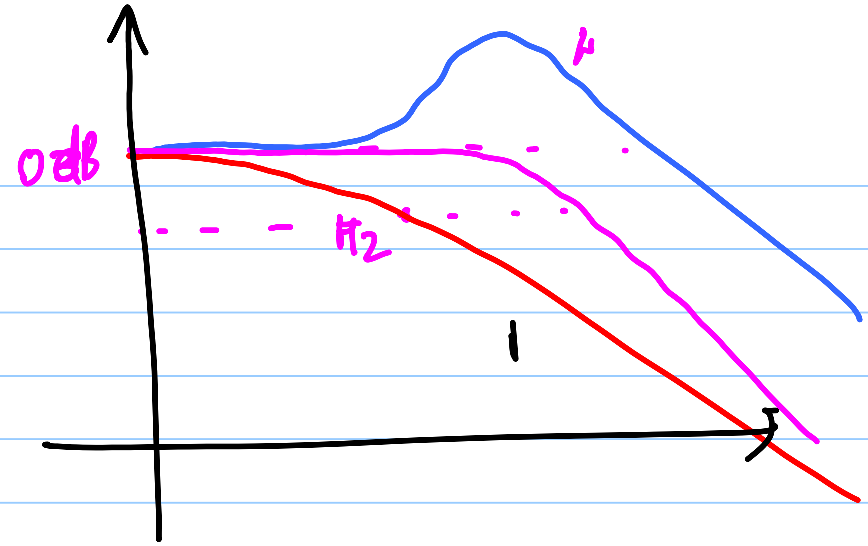
$$s = e^{j\left(\frac{2m\pi}{8}+\frac{\pi}{8}\right)} \qquad m = 0, 1 \ldots 7$$

$$s = e^{j\pi/8},\ e^{j3\pi/8},\ e^{j5\pi/8},\ e^{j7\pi/8},\ e^{j9\pi/8},\ e^{j11\pi/8},\ e^{j13\pi/8},$$

$$e^{j15\pi/8}$$

$$\pi - \frac{3\pi}{8}$$

$$\frac{1}{(s^2+\sqrt{2}s+1)}$$




$\omega_p = 1$ , $Q_p = \frac{1}{\sqrt{2}}$

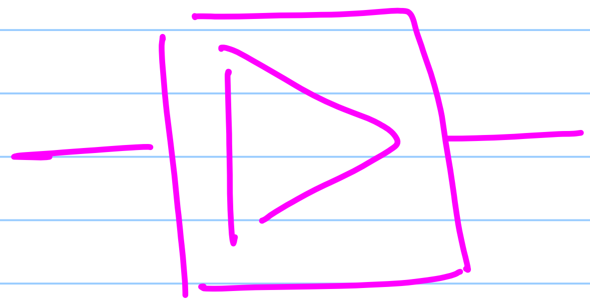
$H(s) = H_1 \times H_2$

$\downarrow$ poles closer to real axis

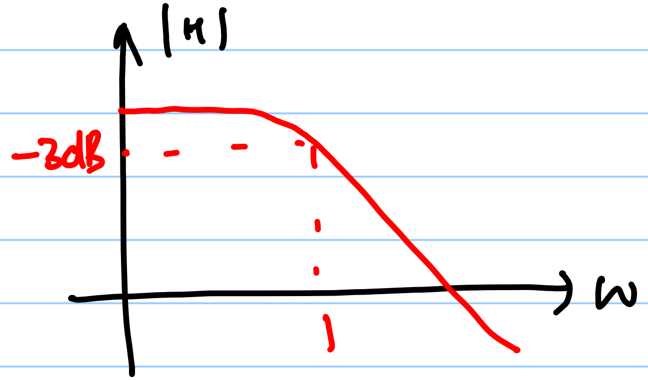
$= H_1$

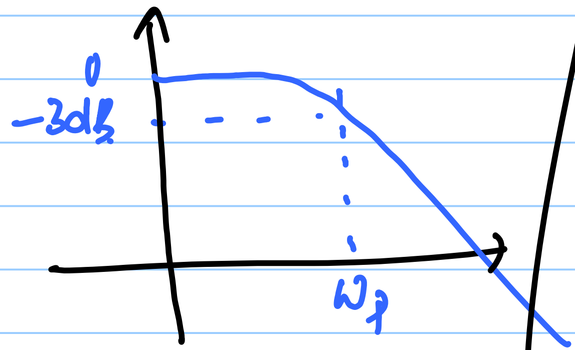
$\downarrow$

poles closer to $j\omega$ axis



$$H(s) = \frac{1}{s^2 + \frac{s}{Q_p} + 1} \xrightarrow{s \to \frac{s}{\omega_p}} \frac{1}{\frac{s^2}{\omega_p^2} + \frac{s}{\omega_p Q_p} + 1}$$



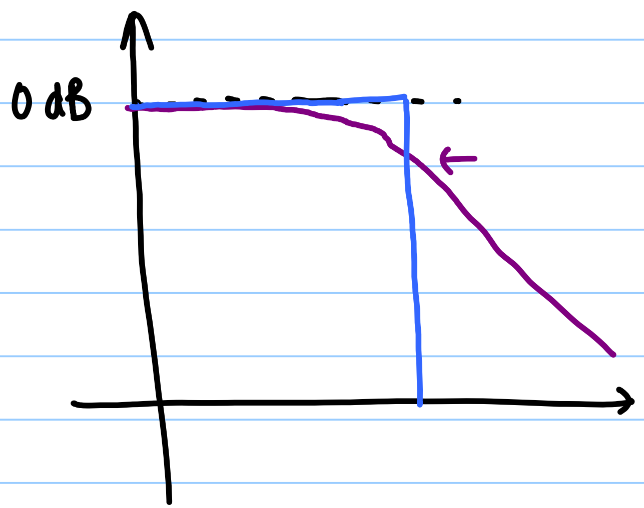




$$H(s) = \frac{1}{\frac{s^2}{\omega_p^2} + \frac{s}{\omega_p Q_p} + 1}$$

$$= \frac{1}{\frac{s^2}{\omega_p^2} + \frac{\sqrt{2}\, s}{\omega_p} + 1}$$

$$|H(j\omega)|^2 = \frac{1}{\left(1 - \frac{\omega^2}{\omega_p^2}\right)^2 + 2\frac{\omega^2}{\omega_p^2}} = \frac{1}{1 + \left(\frac{\omega^2}{\omega_p^2}\right)^2}$$

$$|H(j\omega)| = \frac{1}{1+\omega^{2n}} = \frac{1}{K_0 + K_1\omega^2 + K_2\omega^4 + K_n\omega^{2n}}$$




## Analog-to-Digital Converter.