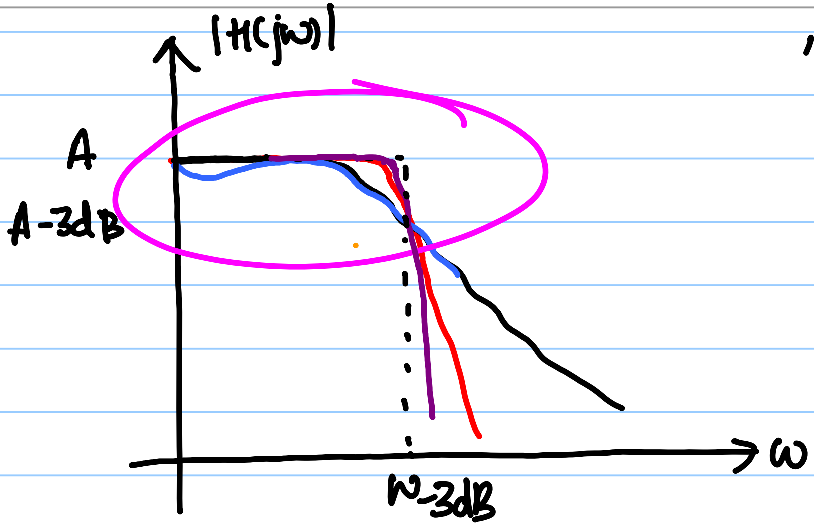
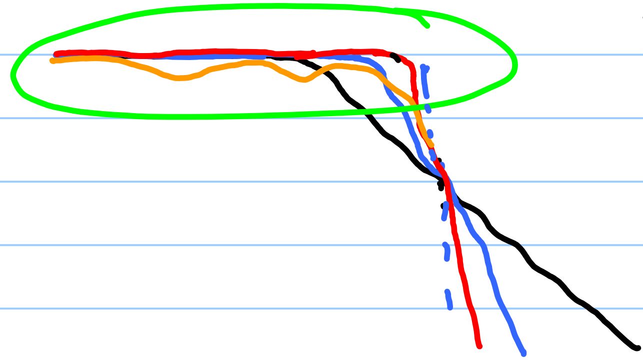
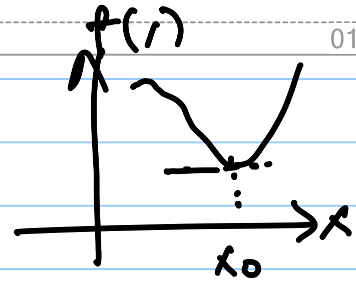
Note Title 01-04-2022

lecture # 35



"Butterworth" Filter.





f(x)

$$H(s) = \frac{1}{D(s)}$$

$$D(s) = K_0' + K_1' s^1 + K_2' s^2 + \ldots + K_n' s^n$$

$$|H(j\omega)|^2 = \frac{1}{D(j\omega)\, D^*(j\omega)} = \left| \frac{1}{\sum_p K_p s^{2p} + \sum_q K_q s^{2q+1}} \right|^2$$

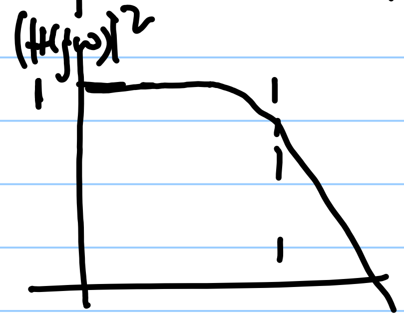
$$= \left| \frac{1}{\sum K_p (j)^{2p} \omega^{2p} + \sum K_q\, j^{2q+1} \omega^{2q+1}} \right|^2$$

$$= \frac{1}{\left(\sum K_q (-1)^p \omega^{2p}\right)^2 + \left(\sum K_q (-1)^q \omega^{2q+1}\right)^2}$$

$$= \frac{1}{\omega^0 + 0\,\omega^1 + \omega^2 + \cdots + \omega^{2n}}$$

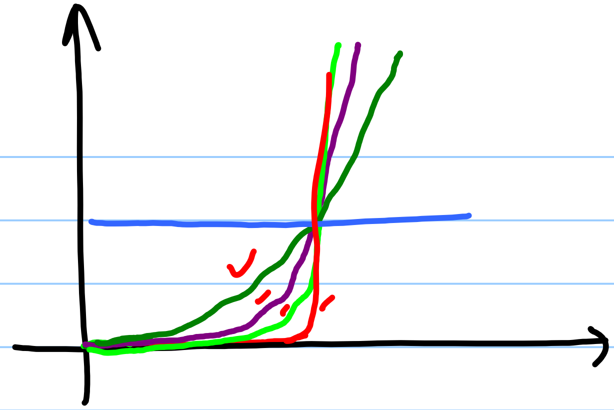
$$= \frac{1}{K_0 + K_1\omega^2 + K_2\omega^4 + \cdots + K_n\omega^{2n}}$$

$$|D(j\omega)|^2 = K_0 + K_1\omega^2 + K_2\omega^4 + \cdots + K_n\omega^{2n}$$



$$|H(j\omega)|^2 = \frac{1}{1 + K_1\omega^2 + K_2\omega^4 + \cdots + K_n\omega^{2n}}$$

$$|D(j\omega)|^2 = 1 + \underbrace{K_1\omega^2 + K_2\omega^4 + \cdots + K_n\omega^{2n}}$$



$$-\frac{d^p |D(j\omega)|^2}{d(\omega^2)^p}$$

$$\frac{d\,(|D(j\omega)|^2)}{d\omega^2} = K_1 + 2K_2\omega^2 + \cdots$$

$$\frac{d}{d\omega^2}\left(\frac{d(|D(j\omega)|^2)}{d\omega^2}\right) = 2K_2 + \cdots \omega^2 +$$

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}} = \frac{1}{1+\underbrace{\left(\frac{s}{j}\right)^{2n}}}$$

$$= (H(j\omega))\,(H(j\omega))^*$$

$$= \frac{1}{D(j\omega)}\,\frac{1}{D(-j\omega)}$$

$$= \underbrace{\frac{1}{D\left(\frac{s}{j}\right)}\,\frac{1}{D\left(\frac{-s}{j}\right)}}$$

$$H(s) = \frac{1}{D(s)}$$

$$H(j\omega) = \frac{1}{D(j\omega)}$$

$$(H(j\omega))^* = \frac{1}{D^*(j\omega)}$$

$$= \frac{1}{D(-j\omega)}$$

$$s = j\omega$$

$$H(s) = \frac{1}{D(s)} =$$

$$|H(j\omega)|^2 = \frac{1}{\underbrace{1+\omega^{2n}}} = \frac{1}{\underbrace{D(j\omega)\,D^*(j\omega)}}.$$

$$= \frac{1}{\underbrace{1+\left(\frac{s}{j}\right)^{2n}}} = \frac{1}{D(s)\;D^*(s)}$$

$D(s) = s + \omega_p$ ✓

$D(j\omega) = j\omega + \omega_p = s + \omega_p$

$\underbrace{D^*(j\omega)} = -j\omega + \omega_p$ ✓

$= -s + \omega_p$

$$\left(\frac{s}{j}\right)^{2n} + 1 = 0 \checkmark$$

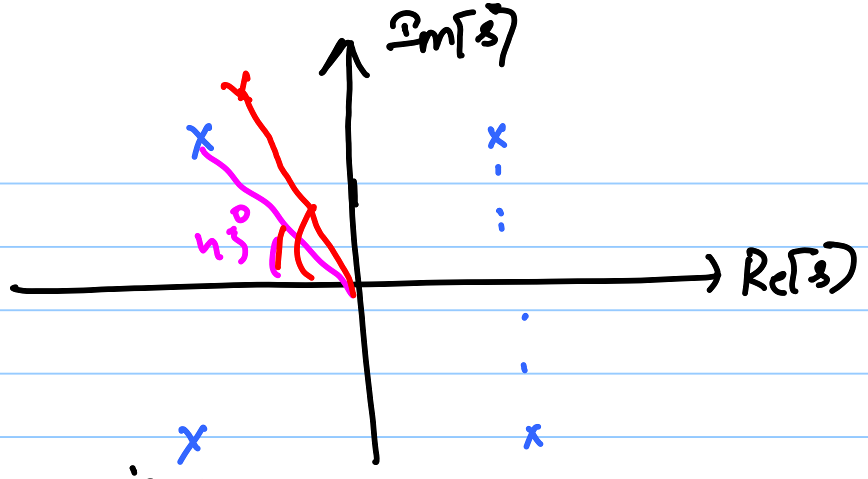
$$s^{2n} = -1 \quad j^{2n}$$

if n is even : $\quad s^{2n} = -1\,(j^2)^n = -1\cdot(-1)^n = -1 = e^{j(\overline{2m+1}\pi)}$

n is odd : $\quad s^{2n} = (-1)\cdot(j^2)^n = -1\cdot(-1)^n = 1 = e^{j\,2m\pi}$

$$\boxed{s^{2n} = e^{j(\overline{2m+1}\pi)}}$$

$$s = e^{j(\overline{2m+1}\pi)/2n}$$



Ex. $n=2$: $s = e^{j(\overline{2m+1}\pi)/4}$

$$= e^{j\pi/4},\ e^{j3\pi/4},\ e^{j5\pi/4},\ e^{j7\pi/4}$$

$$-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}},\quad -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$H(s) = \frac{1}{D(s)} = \frac{1}{\left(s+\frac{1}{\sqrt{2}}-\frac{j}{\sqrt{2}}\right)\left(s+\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}\right)}$$

$$= \frac{1}{s^2+\frac{1}{2}+\sqrt{2}s+\frac{1}{2}}$$

$$= \frac{1}{s^2+\sqrt{2}s+1}$$

$$|H(j\omega)|^2 = \frac{1}{(1-\omega^2)^2+2\omega^2}$$

$$= \frac{1}{1+\omega^4-2\omega^2+2\omega^2}$$

$$= \frac{1}{1+\omega^4}$$

$n = 3$

$$s^{2n} = e^{j2m\pi}$$

$$s = e^{jm\pi/n}$$

$$s = e^{jm\pi/3}$$

$$= 1,\ e^{j\pi/3},\ e^{j2\pi/3},\ e^{j\pi},\ e^{j4\pi/3},\ e^{j5\pi/3}$$

60, 120°, 180°, 240°

$$H(s) = \frac{1}{(s+1)\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{1}{(s+1)\left(s^2+\frac{1}{4}+s+\frac{3}{4}\right)}$$

$$= \frac{1}{(s+1)(s^2+s+1)}$$