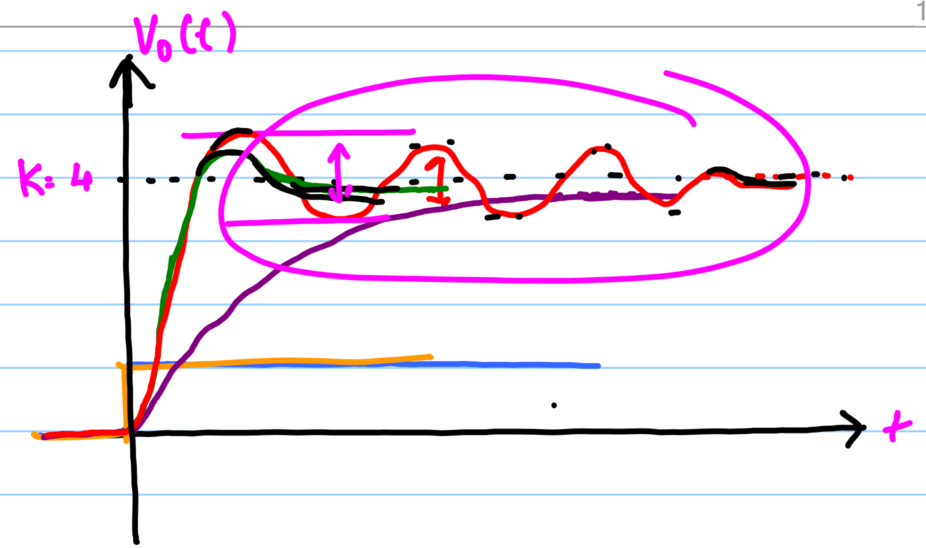
Lecture #14

Note Title 16-02-2022

Second-Order System.



$$\underset{\sim}{H}(s) = \frac{A_{DC} \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

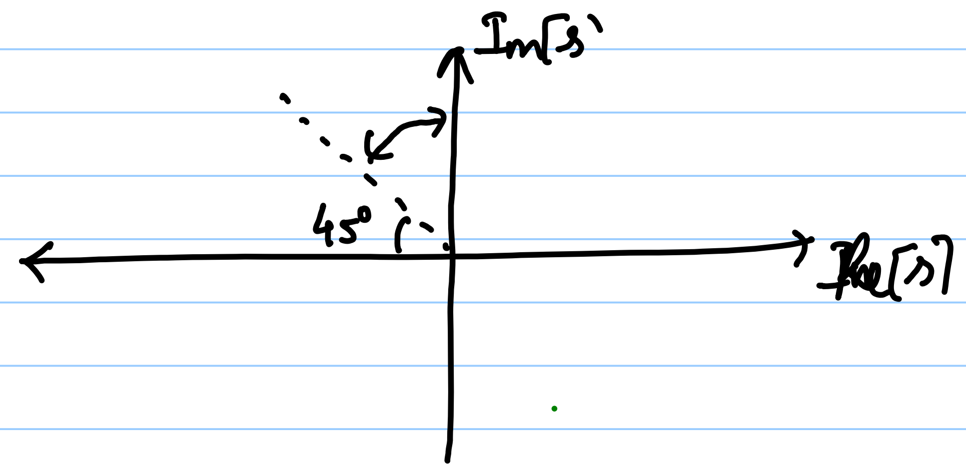
$$A_{DC} = \frac{k}{1 + k/A_0}$$

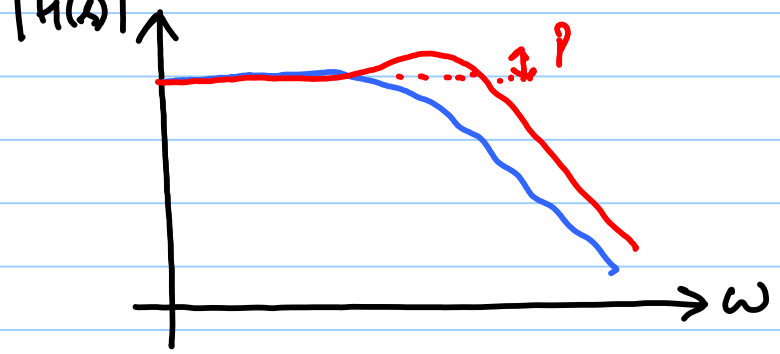
When $\zeta \gg 1$, Real poles along -ve Re axis.

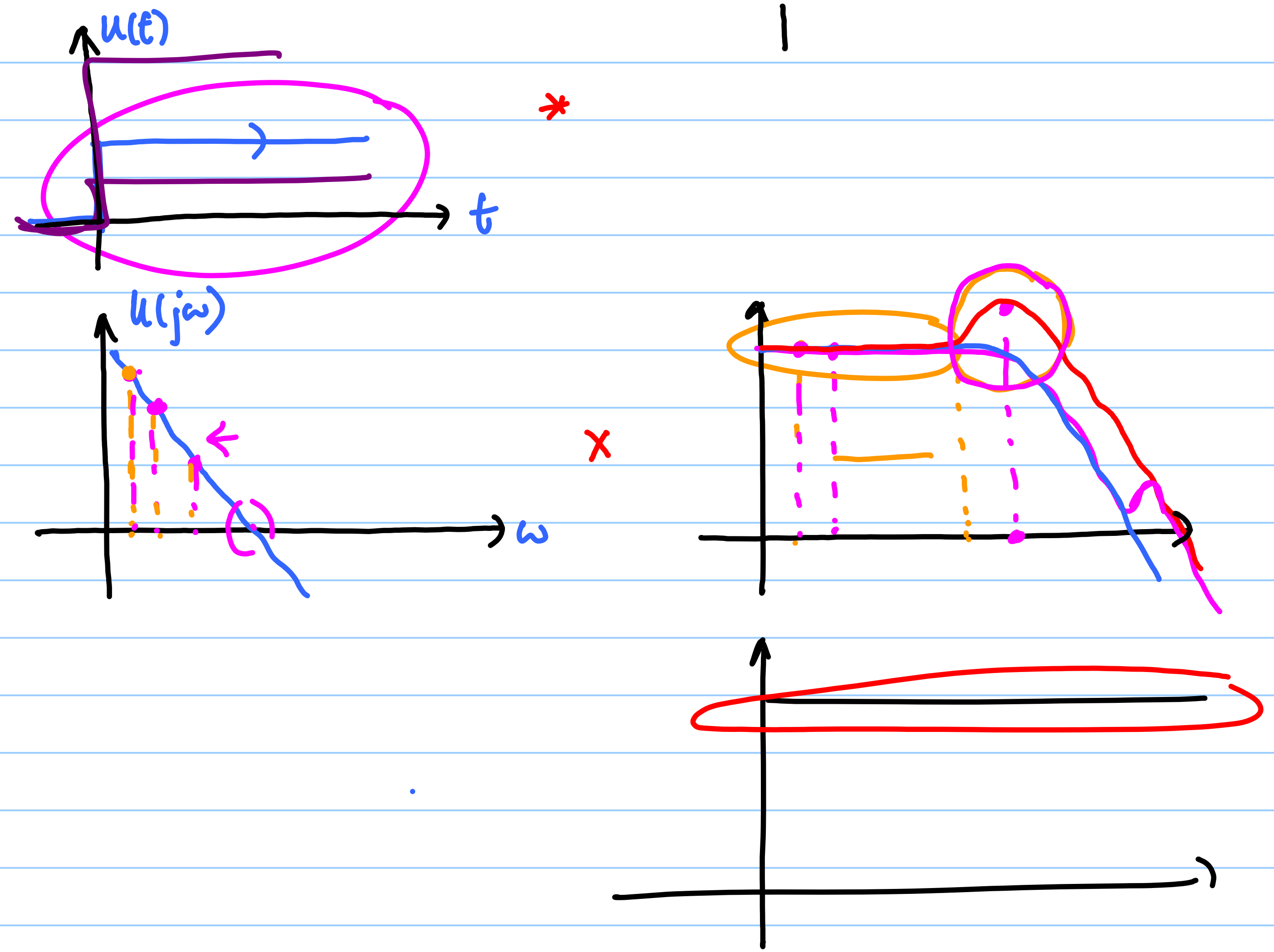
$\zeta < 1$, Complex poles.

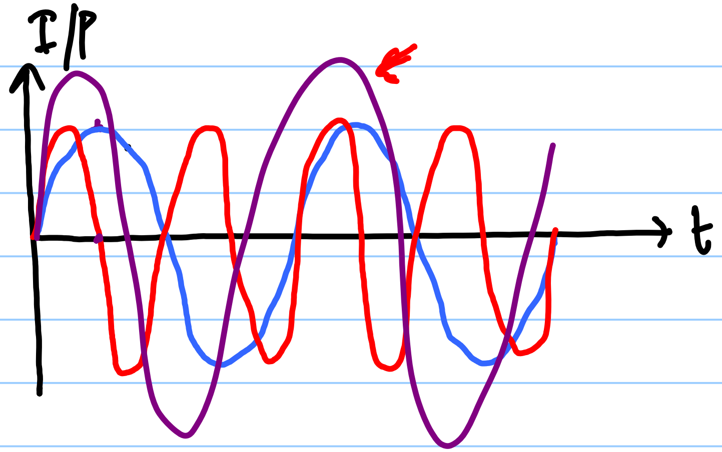
if $\zeta \ll 1$

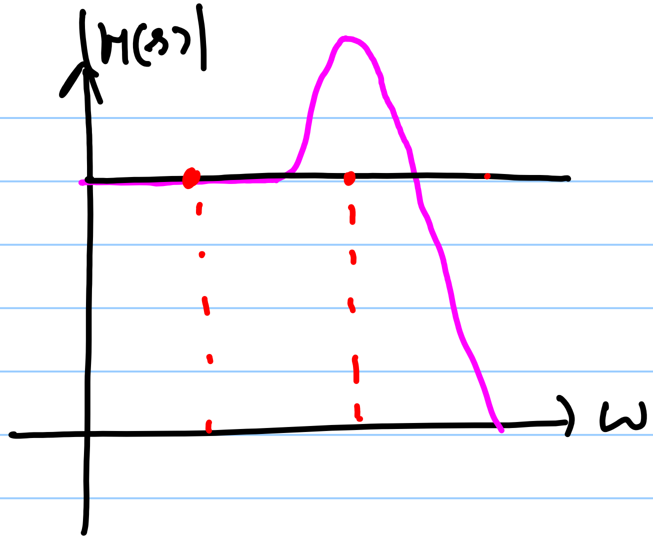
if ζ :

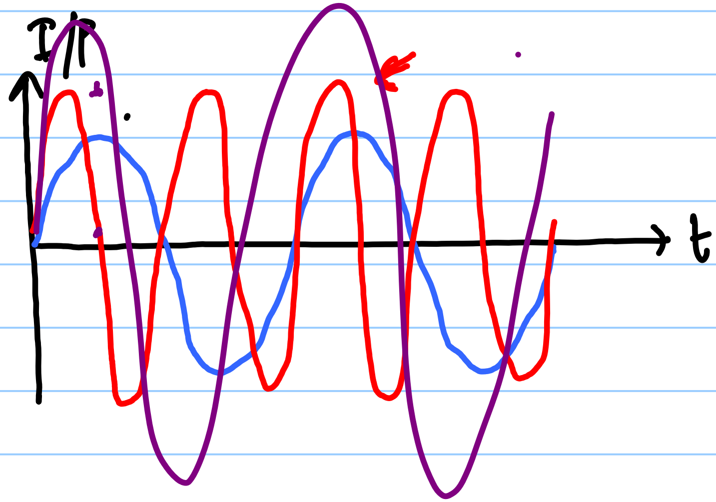





u(t)
t
u(jω)
ω







Phase- Linear

$$\frac{V_o}{V_{in}} = \frac{A_{DC} \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{p_1 p_2 \left(1 + \frac{A_0}{K}\right)}$$

$$\zeta = \frac{1}{2\sqrt{1 + \frac{A_0}{K}}} \left(\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}}\right)$$

$\zeta = 1$

$$\frac{V_o}{V_{in}} = \frac{A_{DC} \cdot \omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$= \frac{A_{DC} \cdot \omega_n^2}{(s + \omega_n)^2}$$

$$V_o(s) = V_{in}(s) \times \frac{A_{DC} \cdot \omega_n^2}{(s + \omega_n)^2}$$

if $V_{in}(t) = u(t)$

$$V_o(t) = \mathcal{L}^{-1}\left\{ \frac{1}{s} \cdot \frac{A_{DC} \cdot \omega_n^2}{(s + \omega_n)^2} \right\}$$

$$= \mathcal{L}^{-1}\left\{ A_{DC}\left(\frac{1}{s} + \frac{-1}{(s + \omega_n)} + \frac{-\omega_n}{(s + \omega_n)^2} \right) \right\}$$

$$= A_{DC}\left[1 - \underbrace{e^{-\omega_n t}(1 + \omega_n t)} \right] u(t)$$

$$\zeta = \frac{1}{2}\left[\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}}\right]\sqrt{\frac{1}{1+A_0/K}}$$

$$\zeta = 1 \Rightarrow \left(\frac{1}{2}\sqrt{\frac{1}{1+A_0/K}}\right)^2 \left[\qquad\right]^2 = 1$$

$$\underbrace{\frac{p_1}{p_2} + \frac{p_2}{p_1}} + 2 = 4\left(1 + \frac{A_0}{K}\right)$$

$$\frac{p_2}{p_1} + 2 \approx 4\frac{A_0}{K} + 4 \Rightarrow \boxed{\frac{p_2}{p_1} \approx 4\frac{A_0}{K} + 2}$$

$$\frac{p_2}{p_1} \approx \frac{4A_0}{K}$$

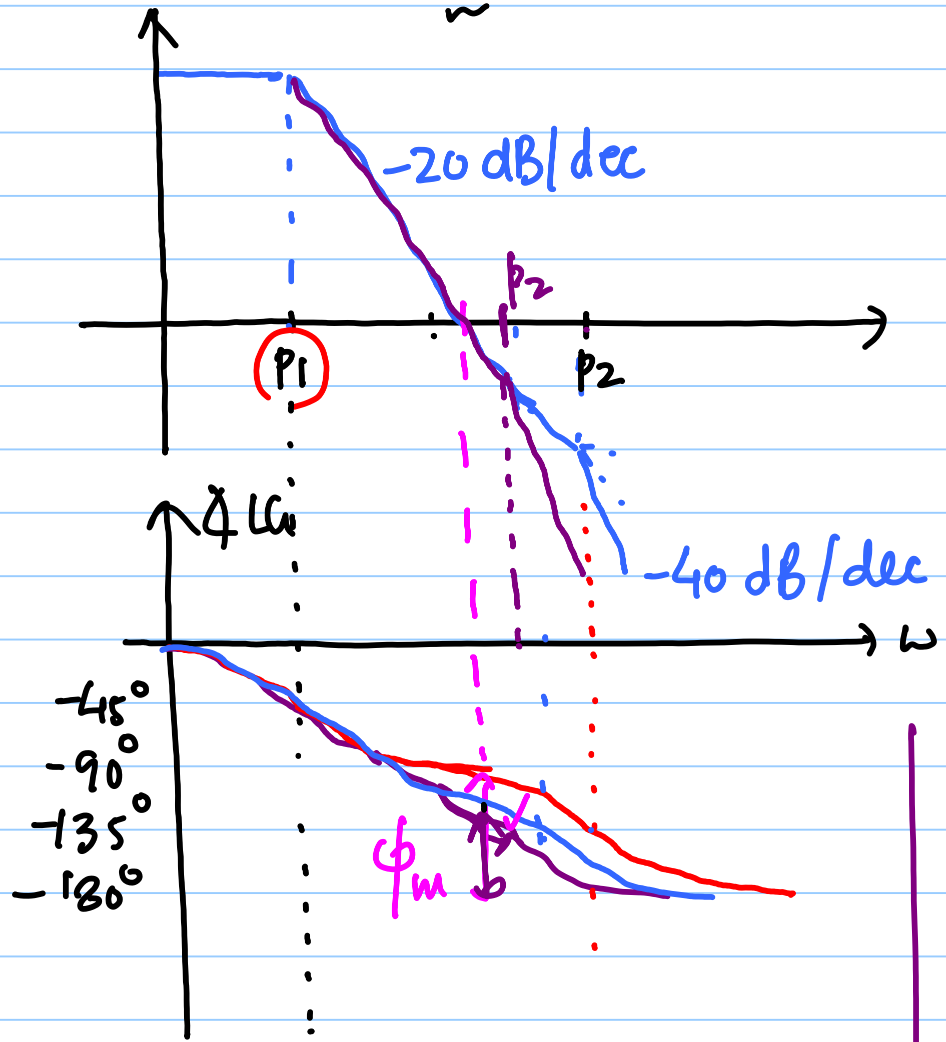
$$p_2 = \frac{4A_0 p_1}{K}$$

if $p_1 \ll p_2 \Rightarrow \frac{p_1}{p_2} \ll \frac{p_2}{p_1}$

$$\frac{p_2}{p_1} \approx 4\left(1 + \frac{A_0}{K}\right)\zeta^2 - 2$$

$$\approx \left(\frac{4A_0}{K}\zeta^2 + 2\right)$$

$$LG = \frac{A_0}{K} \frac{1}{(1+s/p_1)} \frac{1}{(1+s/p_2)}$$



if $\frac{p_2}{p_1} \gg 1$

$|LG(j\omega_u)| = 1$

$$\frac{A_0}{K} \frac{1}{\sqrt{1+\frac{\omega_u^2}{p_1^2}}} \times \frac{1}{\sqrt{1+\frac{\omega_u^2}{p_2^2}}} = 1$$

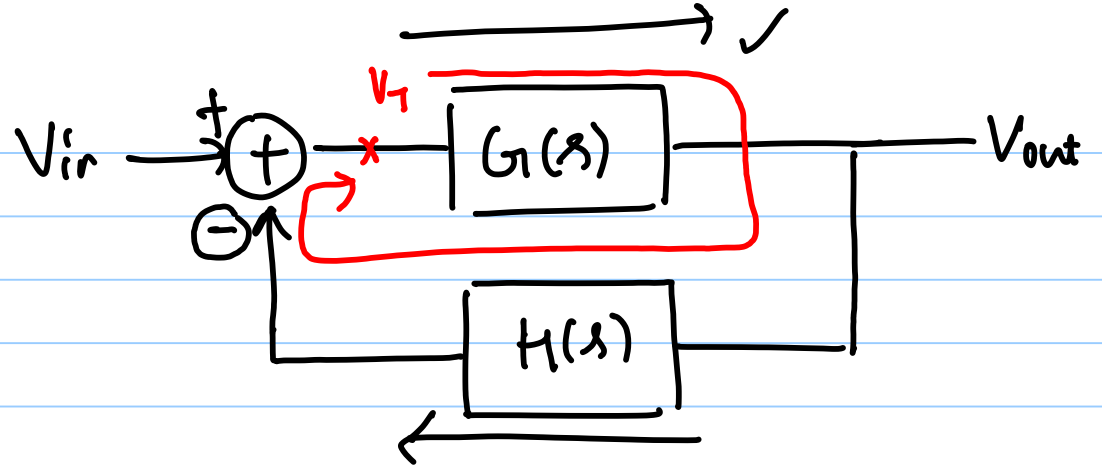
$$\Rightarrow \frac{A_0}{K} \frac{1}{\frac{\omega_u}{p_1}} \times \frac{1}{1} = 1$$

$$\omega_u = \frac{A_0 p_1}{K} \checkmark$$

$$\measuredangle LG = -\tan^{-1}\left(\frac{\omega}{p_1}\right) - \tan^{-1}\left(\frac{\omega}{p_2}\right)$$

$$\phi_m = \measuredangle LG(\omega_u) - (-180^\circ)$$

$$= 180^\circ - \tan^{-1}\left(\frac{\omega_u}{p_1}\right) - \tan^{-1}\left(\frac{\omega_u}{p_2}\right)$$



$(V_{in} - H(s)V_{out})\,G(s) = V_{out}(s)$

$LG = G(s)H(s)$

$$\frac{V_o}{V_{in}} = \frac{G(s)}{1+G(s)H(s)} = \frac{G(s)\cdot H(s)}{1+G(s)H(s)} \times \frac{1}{H(s)}$$

$$\frac{V_o}{V_{in}} = \frac{1}{H(s)} \quad \frac{LG}{1+LG}$$

$$\phi_m = 180^\circ - \tan^{-1}\left(\frac{\omega_u}{p_1}\right) - \tan^{-1}\left(\frac{\omega_u}{p_2}\right)$$

$$\omega_u = \frac{A_0 p_1}{K}, \quad p_2 = 4\frac{A_0}{K}p_1 = 4\omega_u$$

$$\phi_m = 180^\circ - \tan^{-1}\left(\frac{A_0}{K}\right) - \tan^{-1}\left(\frac{1}{4}\right)$$

$$= 180^\circ - 90^\circ - 14^\circ = 76^\circ$$