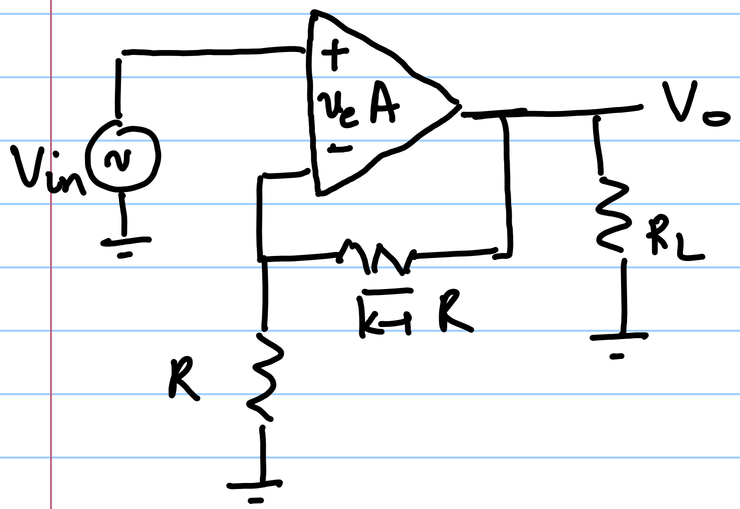
Note Title 10-02-2022

Lecture # 12



$$A = \frac{V_o(s)}{V_e(s)} = \frac{A_o}{1+s/p_1}$$

$s = -p_1$ (L.H.P)

$$\left(V_{in} - \frac{V_o}{K}\right) A(s) = V_o$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{\frac{1}{K} + \frac{1}{A(s)}}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{K}{1 + \frac{K}{A(s)}}$$

$$\frac{V_o}{V_{in}} = \frac{K}{1 + \frac{K(1+s/p_1)}{A_o}}$$

$$= \frac{K}{\left(1+\frac{K}{A_o}\right) + \frac{s}{\frac{A_o p_1}{K}}}$$

$$= \frac{K}{1+\frac{K}{A_o}} \quad \frac{1}{1 + \frac{s}{\frac{A_o p_1}{K} \times \left(1+\frac{K}{A_o}\right)}}$$

$$= \frac{K}{1+\frac{K}{A_o}} \quad \frac{1}{1 + \frac{s}{\frac{A_o p_1}{K} + p_1}}$$

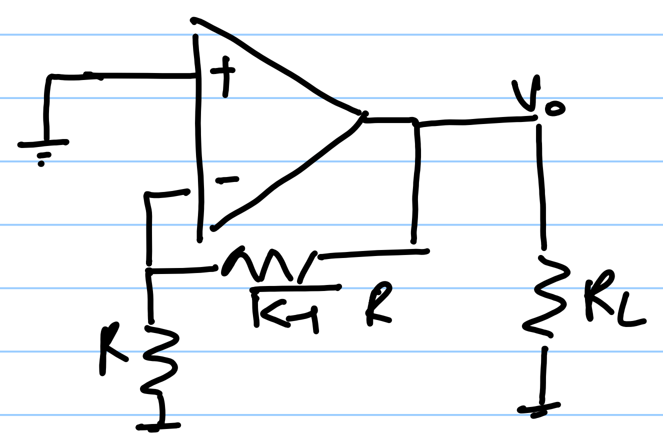
$$\frac{V_o(s)}{V_{in}(s)} = \boxed{\frac{A_{DC}}{1+s/p_1'}}$$

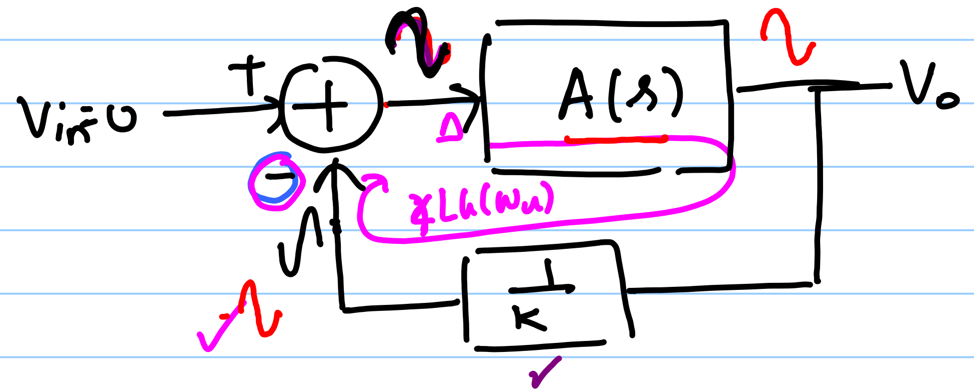
$$A_{DC} = \frac{K}{1+K/A_0}$$

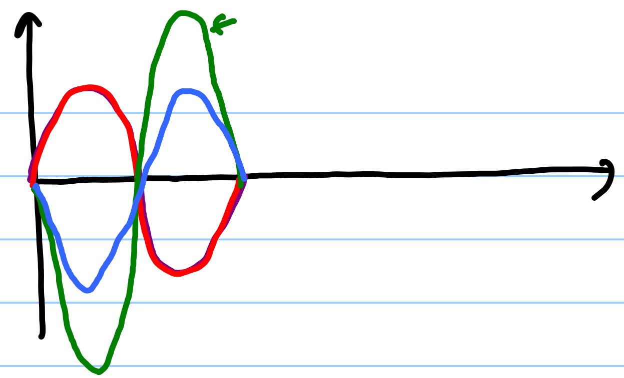
$$s = -p_1' = \frac{A_0 p_1}{K} + p_1$$

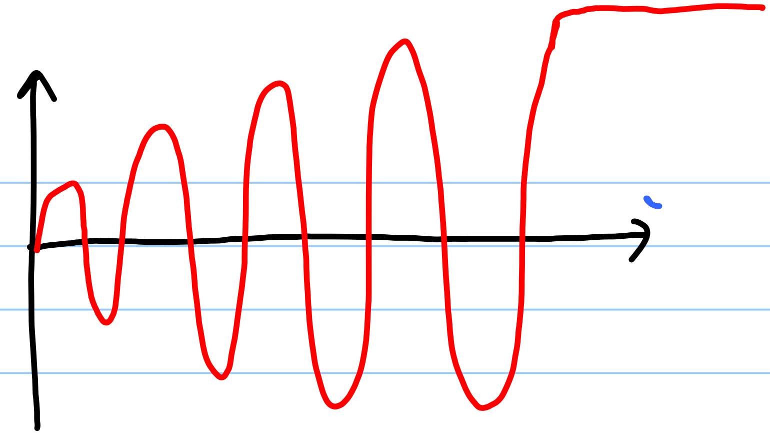
- if you have even one R.H.P pole, the system is unstable (closed loop times)
- Phase margin < 0, the system is unstable.

- Nyquist Stability Criterion.
- For all poles system, it is stable if $\phi_m > 0$









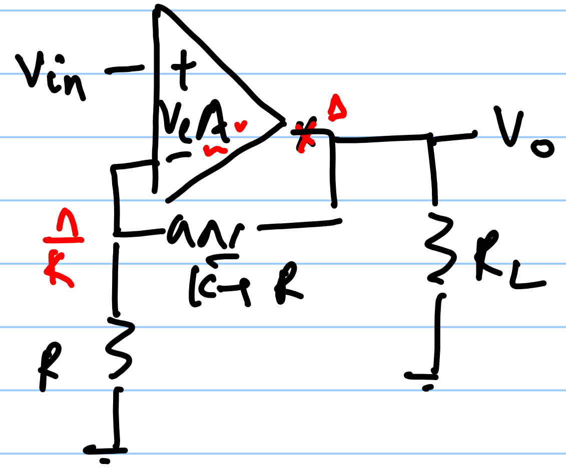
$$H(s) = \frac{1}{\left(1+\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)} \quad ; \quad p_1 > 0, \; p_2 > 0$$

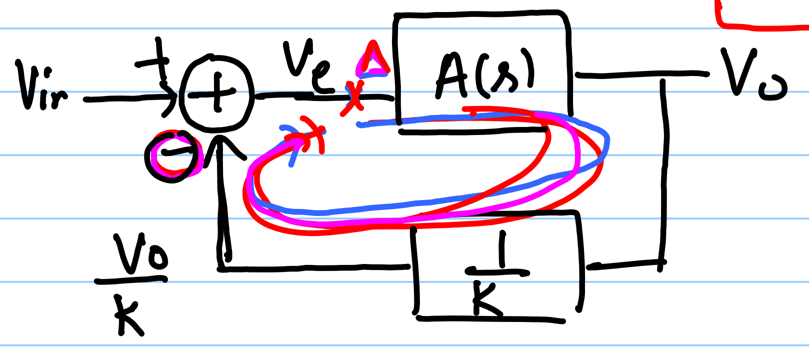
$$\underset{\sim}{X(s)} \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

$$Y(s) = X(s) \; H(s)$$

$$= \frac{1}{s} \cdot \frac{1}{\left(1+\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)} = \frac{P}{s} + \frac{Q}{s+p_1} + \frac{R}{\underline{s-p_2}} \checkmark$$

$$y(t) = \mathcal{L}^{-1}(Y(s)) = \overset{?}{P}. u(t) + Q e^{-p_1 t} + \underline{R. e^{p_2 t}}$$





$$\boxed{\frac{V_o}{V_{in}} = \frac{A(s)}{1+\frac{A(s)}{K}}}$$

$$\frac{V_o}{V_{in}} = \frac{K}{1+\frac{K}{A_0}} \frac{1}{1+s/p_1'}$$

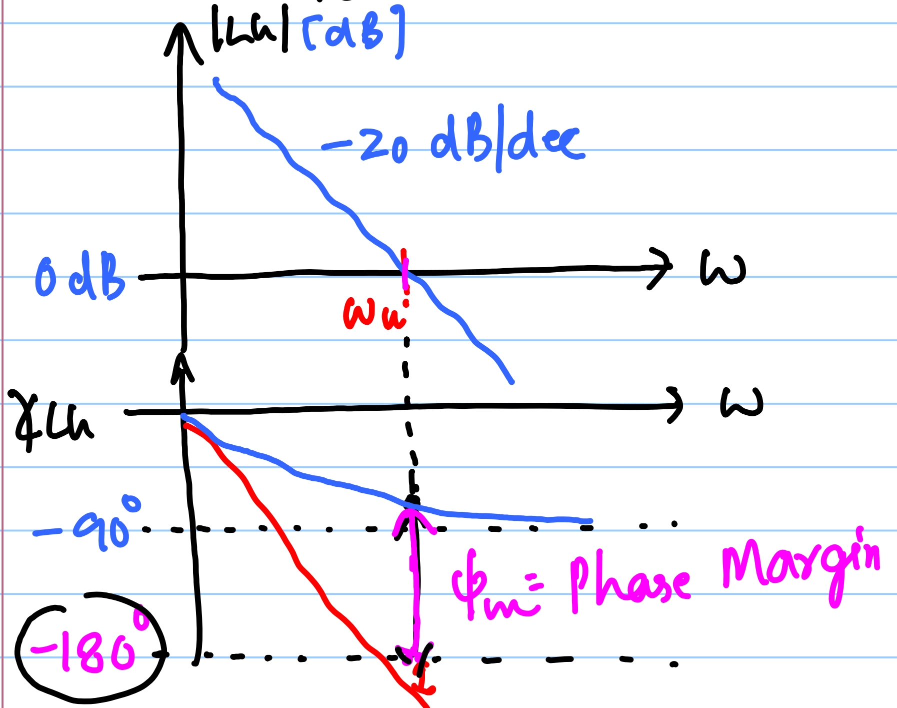
loop gain $\boxed{LG = \frac{A(s)}{K}}$

$$= \frac{A_0}{K} \frac{1}{1+s/p_1}$$

$$\angle LG = -\tan^{-1}(\omega/p_1)$$

$$|LG| \,[dB] = 20 \log_{10}(|LG|)$$

$$= 20 \log_{10}\left(\frac{A_0/K}{\sqrt{1+\frac{\omega^2}{p_1^2}}}\right)$$



$$|LG(\omega_u)| = 1$$

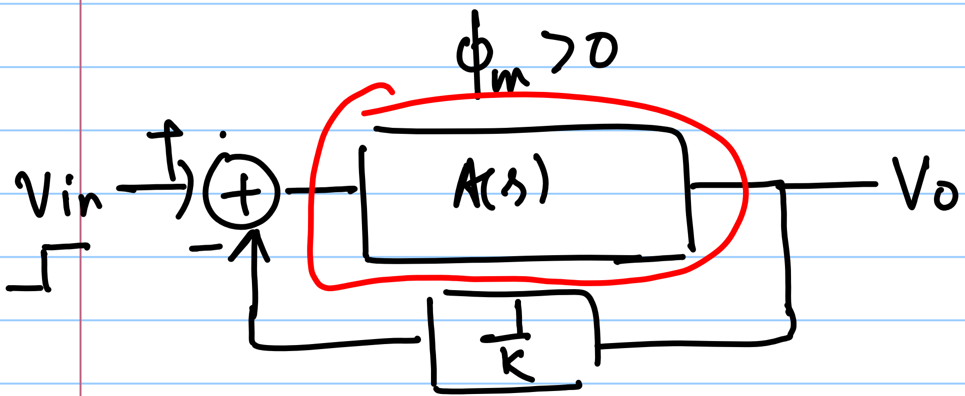
$\phi_m = \measuredangle La - (-180°)$ ✓

$\phi_m > 0$

- if for open loop La, $\phi_m < 0$ in an all pole system then closed loop system will have R.H.P poles.

$$\frac{V_o}{V_{in}} = \frac{k}{1+k/A_o} \frac{1}{1+s/p_1'}$$

loop gain, $La = \frac{A_o}{(1+s/p_1)} \times \frac{1}{k}$



Closed loop transfer function

$$V_o(s) = V_{in}(s) \frac{k}{1+k/A_o} \frac{1}{1+s/p_1'}$$

$$= \frac{k}{1+k/A_o} \frac{1}{s} \frac{1}{1+s/p_1'}$$

$$= \frac{k}{1+k/A_o} \left[\frac{1}{s} + \frac{-1}{s+p_1'} \right]$$

$$v_o(t) = \mathcal{L}^{-1}[V_o(s)]$$

$$= \frac{k}{1+k/A_o} \left[u(t) - e^{-p_1' t} u(t) \right]$$

$$v_o(t) = \frac{k}{1+k/A_0}\left[1-e^{-p_1' t}\right]u(t)$$

$$\lim_{t\to\infty} v_o(t) = \frac{k}{1+k/A_0}$$

Ideal $\lim_{\substack{t\to\infty \\ A_0\to\infty}} v_o(t) = k$

Static error, ϵ_s

$$\epsilon_s = \lim_{\substack{t\to\infty \\ A_0\to\infty}} v_o(t) - \lim_{t\to\infty} v_o(t)$$

$$= k - \frac{k}{1+k/A_0}$$

$$= \frac{k}{1+A_0/k}$$

$$\frac{P}{s} + \frac{Q}{s+p_1'} = \frac{1}{s}\cdot\frac{p_1'}{s+p_1'}$$

$$P + s\cdot\frac{Q}{s+p_1'} = \frac{p_1'}{s+p_1'}$$

$$P = \frac{p_1'}{p_1'} = 1$$

$$\frac{P(s+p_1')}{s} + Q = \frac{1}{s}\cdot p_1'$$

$$Q = \frac{p_1'}{-p_1'} = -1$$

Dynamic error, ϵ_d

$$\epsilon_d = \frac{\lim_{t\to\infty} v_o(t) - v_o(t_d)}{\lim_{t\to\infty} v_o(t)} = e^{-p_1' t_d}$$

- To reduce static error, A_0 should be increase.