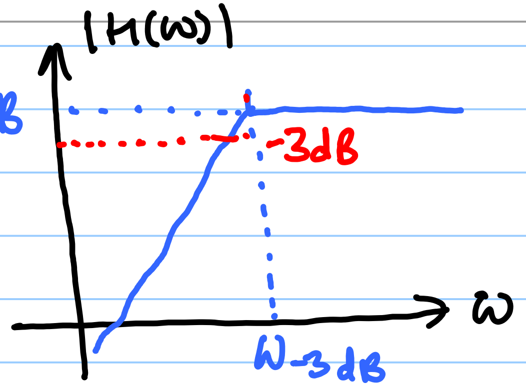
Note Title — 15-04-2021

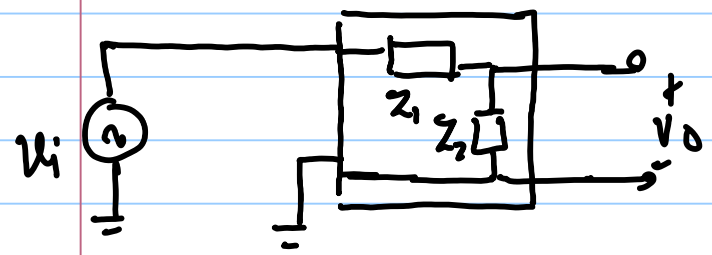
Lecture # 34

High-pass filters (HPF)

$$H(s) = \frac{s/\omega_p}{1 + s/\omega_p} = \frac{s}{s+\omega_p}$$



for $\frac{s}{\omega_p} \gg 1$, $H(s) = \frac{s/\omega_p}{s/\omega_p} = 1$

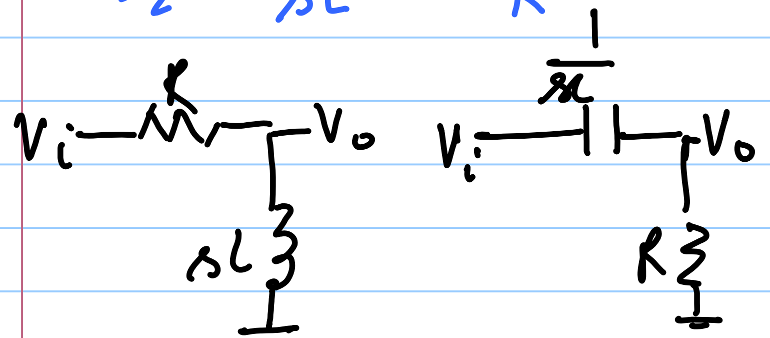


$$\frac{V_o}{V_i} = \frac{Z_2}{Z_2+Z_1} = \frac{1}{1+\frac{Z_1}{Z_2}} = \frac{s}{s+\omega_p} = \frac{1}{1+\frac{\omega_p}{s}}$$

$$\boxed{\frac{Z_1}{Z_2}} = \frac{\omega_p}{s} = \frac{1}{(s/\omega_p)} = \boxed{\frac{R}{sL}} = \frac{1/sC}{R} = \boxed{\frac{1}{sRC}}$$

Z_1 : R $1/sC$

Z_2 : sL R



$$\angle H(j\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

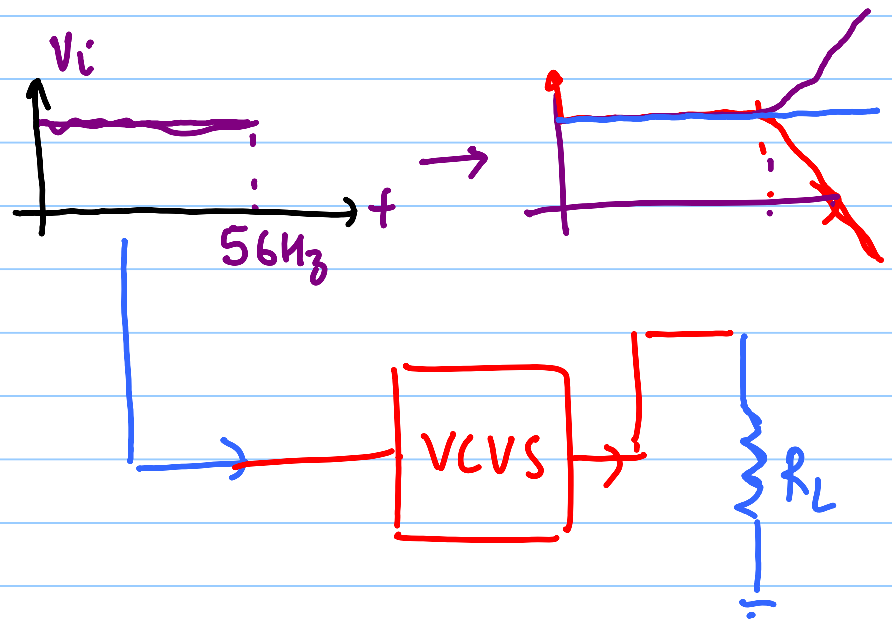
All-Pass filters

V_i — $\boxed{H(s)}$ — V_o

$$H(s) = \frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

for all pass $\quad \omega_p = \omega_z$ ✓

$$\measuredangle H(j\omega) = \tan^{-1}\left(\frac{\omega}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$



$|H(s)| = 1$ for all frequencies.

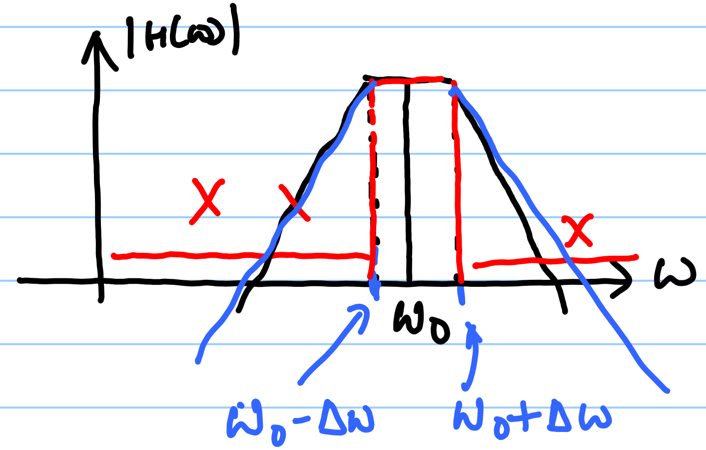
$$H(s) = \frac{1+s/\omega_z'}{1+s/\omega_p'} = \frac{1-s/\omega_p'}{1+s/\omega_p'}$$

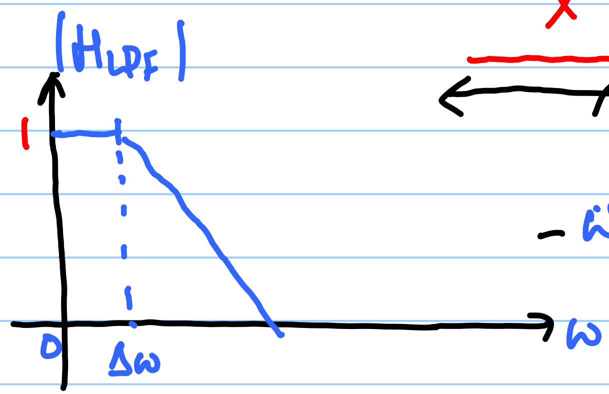
but $\omega_z' \neq \omega_p'$

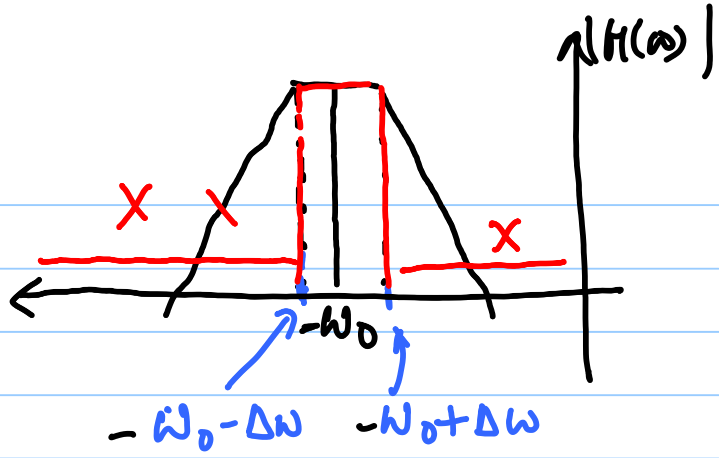
if $\omega_z' = -\omega_p'$

$$\measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_p'}\right) - \tan^{-1}\left(\frac{\omega}{\omega_z'}\right)$$

Band Pass filters (BPF)







$$H(s) = \frac{1}{1 + s/\omega_p} \longrightarrow H(s) = \frac{1}{1 + \frac{f(s)}{\omega_p}}$$

#1 @ $s = j\omega_0, -j\omega_0$

@ $s = 0$ ✓

#2 @ $s = 0$, $j\omega$ as $\omega \to \infty$

@ $s = j\omega$ as $\omega \to \infty$ ✓

$+j\omega$ as $\omega \to -\infty$

$$H_{BPF} = \frac{1}{1 + \frac{s^2 + \omega_0^2}{s\,\omega_p}}$$

$s = 0$ (LPF) $\longrightarrow$ $s = \pm j\omega_0$ (BPF)

$$= \frac{1}{1 + \frac{s + \omega_0^2/s}{\omega_p}}$$

$$s \longrightarrow \frac{s+\omega_0^2}{s} = Q\left(\boxed{\frac{s}{\omega_0}} + \boxed{\frac{\omega_0}{s}}\right), \qquad Q\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)$$

$$[Q] = [\omega]$$

$$H_{LPF}(s) = \frac{1}{1+s/\omega_p} \xrightarrow{s \to \frac{s}{\omega_0}+\frac{\omega_0}{s}} H_{BPF}(s) = \frac{1}{1+\frac{Q}{\omega_p}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)}$$

$|Q| = 1$ $\qquad [Q] = [\omega]$

$$= \frac{1}{1+\frac{Q}{\omega_p\omega_0 s}(s^2+\omega_0^2)}$$

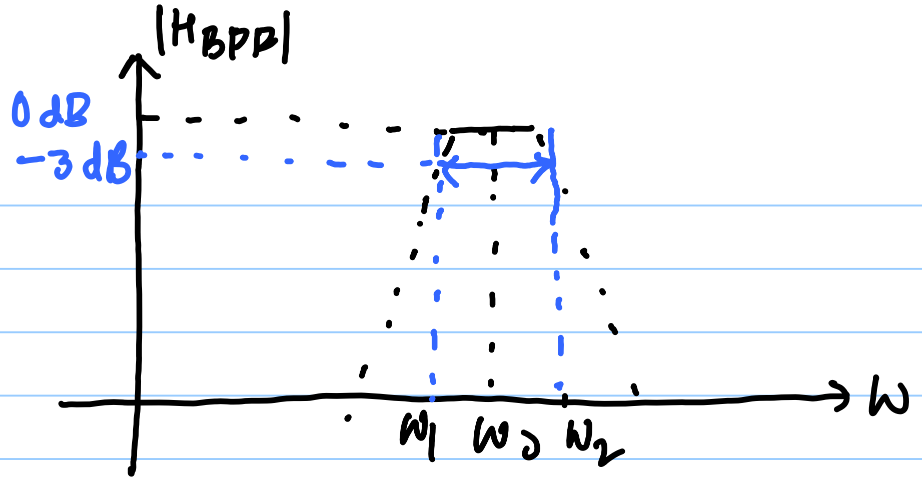
$$H_{BPF} = \frac{s.\omega_p\omega_0/Q}{s^2+\frac{\omega_p\omega_0}{Q}s+\omega_0^2},$$

For very low freq: $H_{BPF} \approx \frac{s.\omega_p\omega_0/Q}{\omega_0^2} = \frac{s.\omega_p/Q}{\omega_0}$ $\qquad$ +20dB/dec

For very high. freq: $H_{BPF} \approx \frac{s.\omega_p\omega_0}{s^2} = \frac{\omega_p\omega_0/Q}{s}$ $\qquad$ −20 dB/dec

$$H_{BPF}(s) = \frac{s\,\omega_p\omega_0/Q}{\left(s^2+\frac{\omega_p\omega_0}{Q}s+\omega_0^2\right)}$$

−40



$$D(s) = s^2 + \frac{\omega_p \omega_0}{Q} s + \omega_0^2$$

$$= \omega_0^2 \left[\underbrace{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0} \frac{\omega_p}{Q} + 1} \right]$$

$$\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0} \frac{\omega_p}{Q} + 1 = 0$$

$$\frac{s}{\omega_0} = \frac{-\frac{\omega_p}{Q} \pm \sqrt{\left(\frac{\omega_p}{Q}\right)^2 - 4}}{2} \checkmark$$

$$= -\frac{\omega_p}{2Q} \pm j\sqrt{1 - \left(\frac{\omega_p}{2Q}\right)^2} \checkmark$$

$$|H_{BPF}(\omega_0)| = 1$$

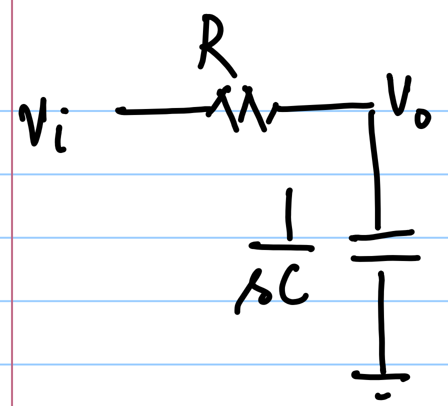
$$|H_{BPF}(\omega)| = \frac{1}{\sqrt{2}} \quad |H_{BPF}(j\omega_0)| = \frac{1}{\sqrt{2}} \checkmark$$

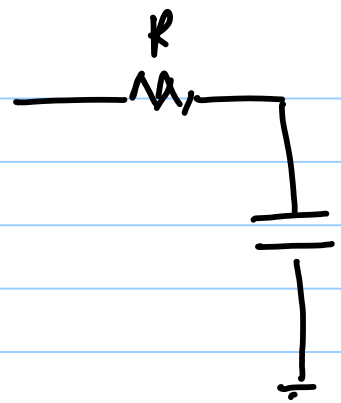
$$\Delta\omega = \omega_2 - \omega_1$$

$$s = -\omega_0 \left(\frac{\omega_p}{2Q} \pm j\sqrt{1 - \left(\frac{\omega_p}{2Q}\right)^2} \right)$$

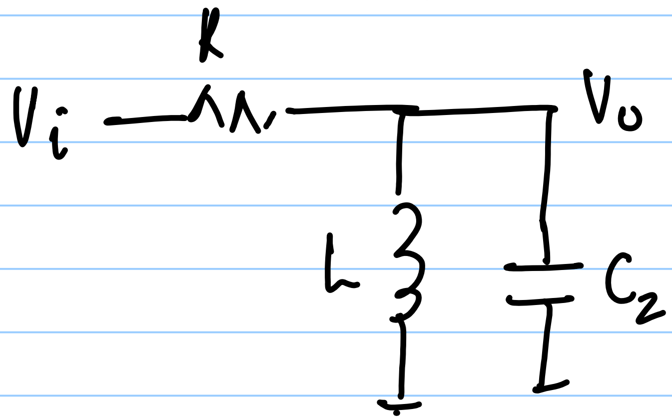
$$s = -\omega_0 \left(\frac{\omega_p}{2Q} \pm \underbrace{\sqrt{\left(\frac{\omega_p}{2Q}\right)^2 - 1}} \right)$$

$$\boxed{\omega_2 - \omega_1 \approx \frac{\omega_0}{Q_p} = \frac{\omega_0 \omega_p}{Q}}$$





$$\frac{1}{sC} \to \frac{1}{Q\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)C} = \frac{1}{\frac{QC}{\omega_0}s + \frac{Q\omega_0 C}{s}}$$



$$\frac{QC}{\omega_0}s + Q\omega_0 C\frac{1}{s} \checkmark$$

$$C_2 s + \frac{1}{sL}$$

$$C_2 = \frac{QC}{\omega_0}, \quad L = \frac{1}{Q\omega_0 C}$$