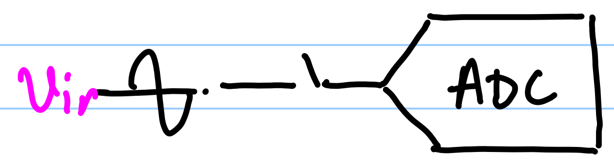
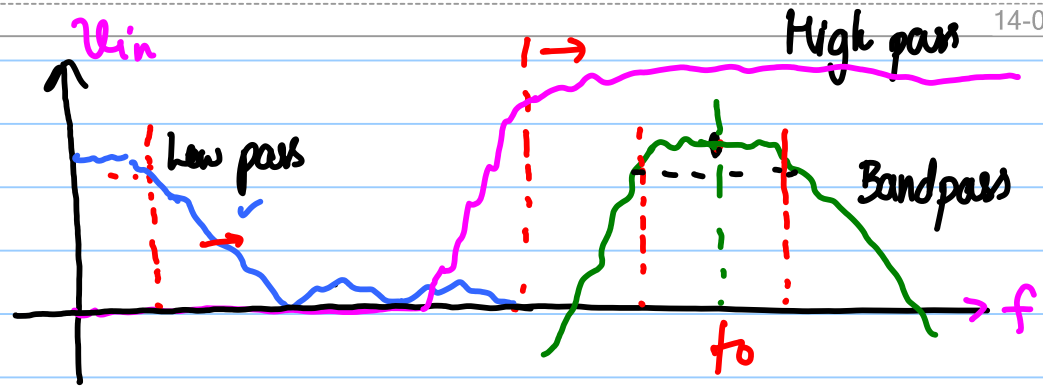
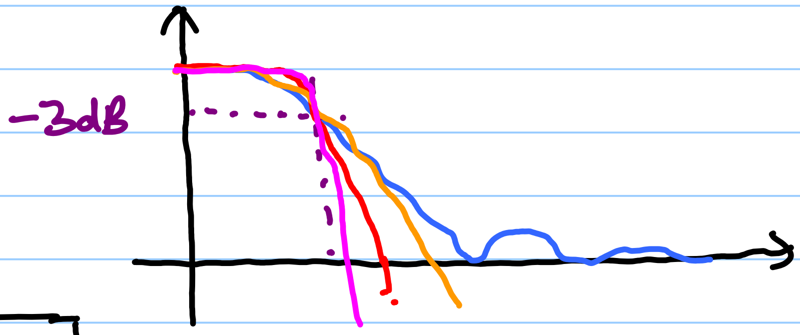
lecture #33

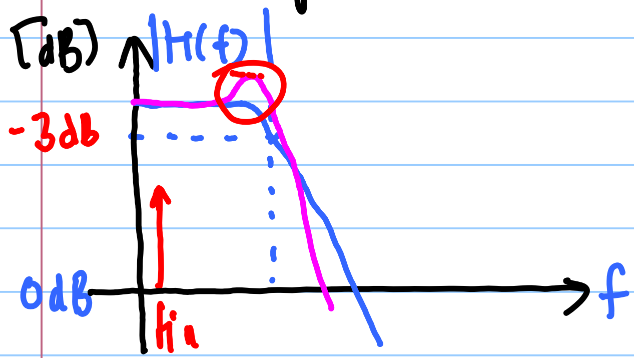
Note Title 14-04-2021

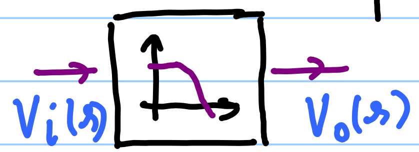






low pass filter (LPF)





$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{A_o}{1 + s/\omega_p}$$

DC gain : A_0

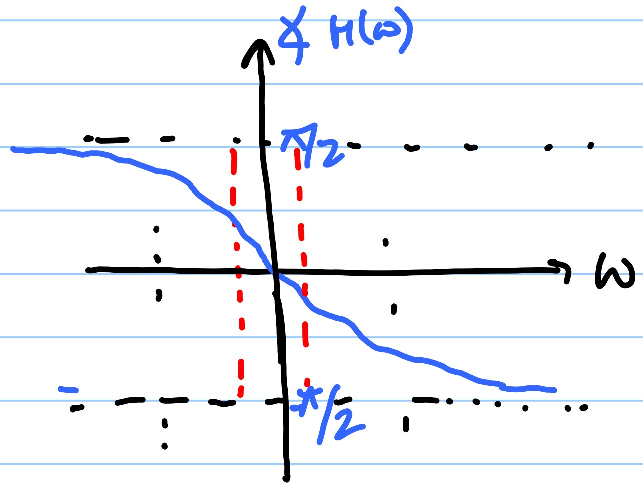
Pole @ $s = -\omega_p$

$$|H(s)| = \frac{A_0}{\sqrt{1 + \omega^2/\omega_p^2}}$$

$$\angle H = -\tan^{-1}(\omega/\omega_p)$$

$$20\log_{10}(|H(j\omega)|) = 20\log_{10}\left(\frac{A_0}{\sqrt{1+\omega^2/\omega_p^2}}\right)$$

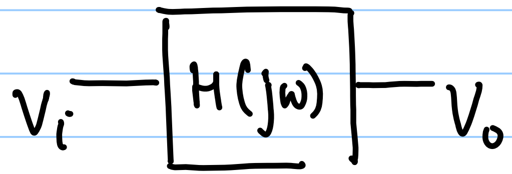
at very high freq. $|H(j\omega)|\,[dB] \approx 20\log_{10}\left(\frac{A_0}{\omega/\omega_p}\right)$



-3dB BW of filter

$$|H(j\omega_{-3dB})| = \frac{1}{\sqrt{2}}|H(0)|$$

$$\omega_{-3dB} = 2\pi f_{-3dB}$$



$$V_i = A\sin(\omega_0 t)$$

$$V_o = A \times |H(j\omega_0)| \times \sin(\omega_0 t + \measuredangle H(j\omega_0))$$

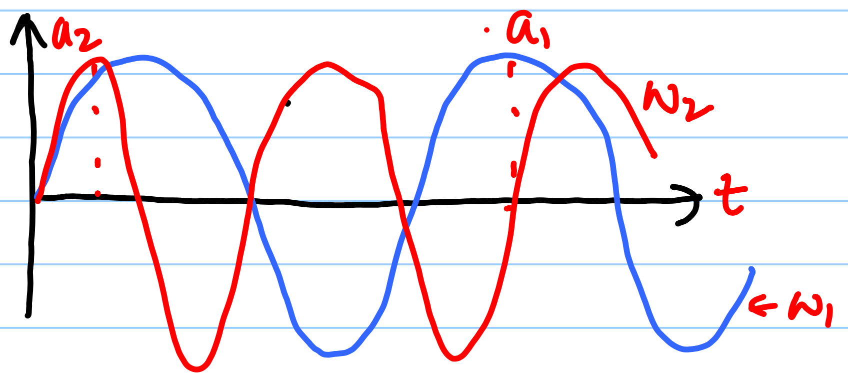
$$V_o(s) = V_i(s)\,H(s)$$

$$v_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$$

$$V_i = a_1 \sin(\omega_1 t) + a_2 \sin(\omega_2 t)$$

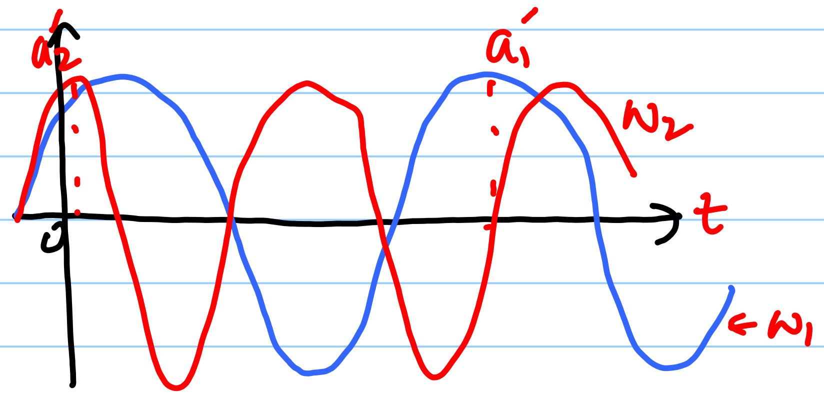
$$V_o(t) = \underbrace{a_1 |H(j\omega_1)|} \sin(\omega_1 t + \phi_1) + a_2 |H(j\omega_2)| \sin(\omega_2 t + \phi_2)$$

$$= a_1' \sin(\omega_1 t + \phi_1) + a_2' \sin(\omega_2 t + \phi_2)$$



$$= a_1' \sin\left(\omega_1 \left(t + \frac{\phi_1}{\omega_1}\right)\right)$$

$$+ a_2' \sin\left(\omega_2 \left(t + \frac{\phi_2}{\omega_2}\right)\right)$$



$\phi_1 - \phi_2 = 2\pi k.$ ✗

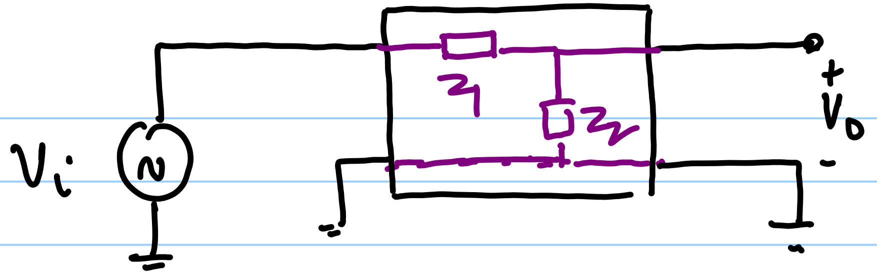
$\Delta t_1 = \frac{\phi_1}{\omega_1}$, $\Delta t_2 = \frac{\phi_2}{\omega_2}$

$\Delta t_1 = \Delta t_2 \Rightarrow \frac{\phi_1}{\omega_1} = \frac{\phi_2}{\omega_2}$

$\Rightarrow \frac{\phi}{\omega} =$ constant ✓

✓ $\phi = -\tan^{-1}(\omega/\omega_p)$

$$= -\left[\frac{\omega}{\omega_p} - \frac{(\omega/\omega_p)^3}{3} + - \cdot\right]$$



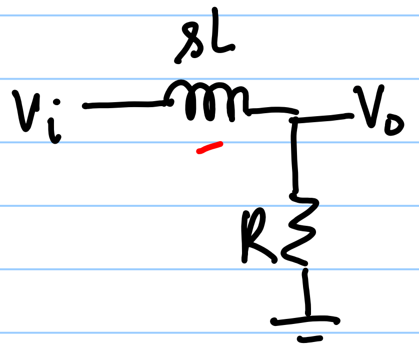
$$\frac{V_o}{V_i} = \frac{A}{1+s/\omega_p} = \frac{Z_2}{Z_2+Z_1}$$

$$= \frac{1}{1+\frac{Z_1}{Z_2}}$$

$$\boxed{\frac{Z_1}{Z_2} = \frac{s}{\omega_p}}$$

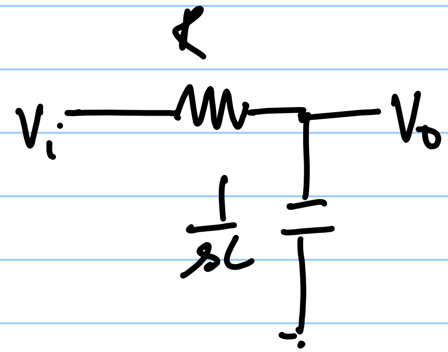
Z_1: sL , R

Z_2: R , $1/sC$



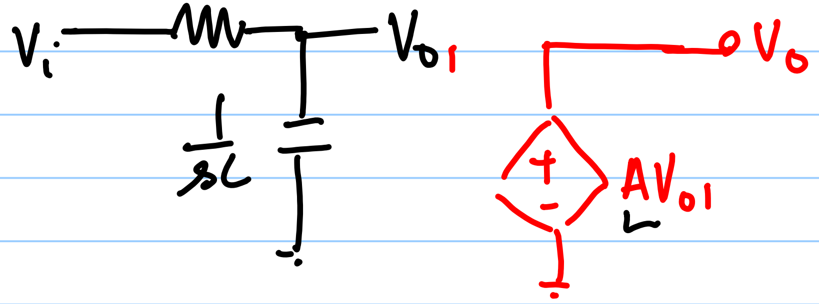
$$\omega_p = \frac{R}{L}$$

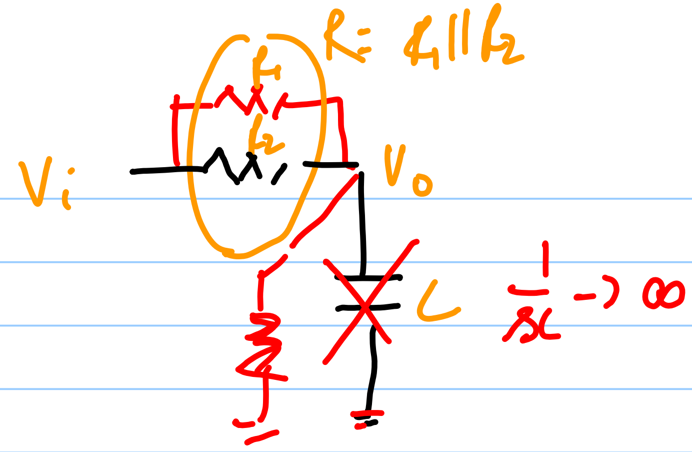
$$\frac{V_o}{V_i} = \frac{1}{1+sL/R}$$



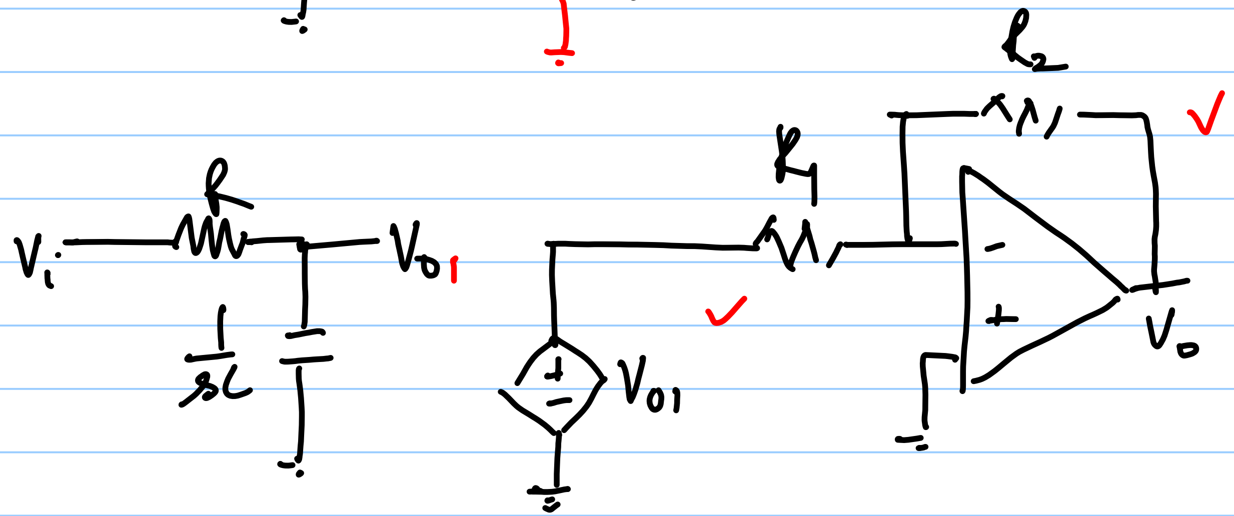
$$\omega_p = \frac{1}{RC}$$

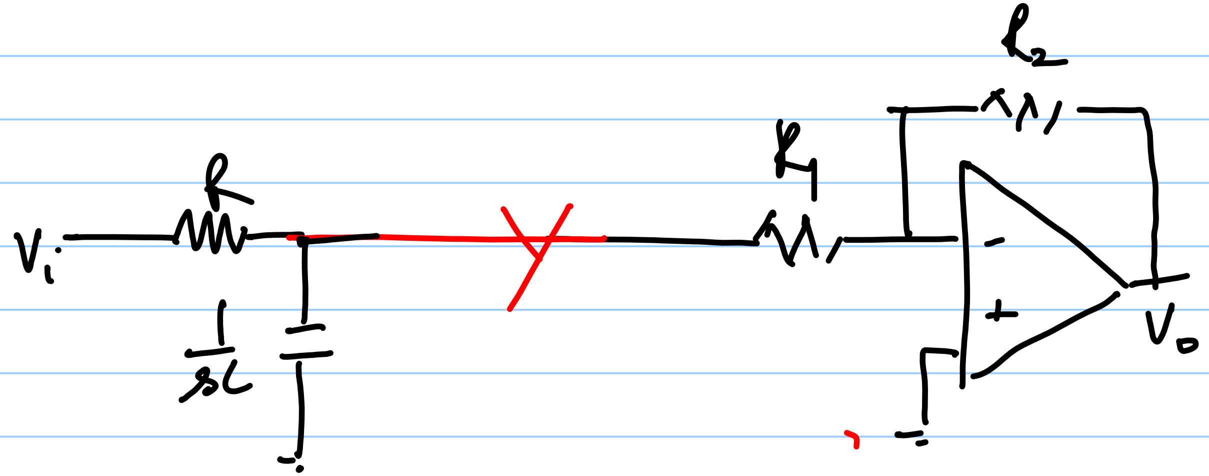
$$\frac{V_o}{V_i} = \frac{1}{1+sRC}$$

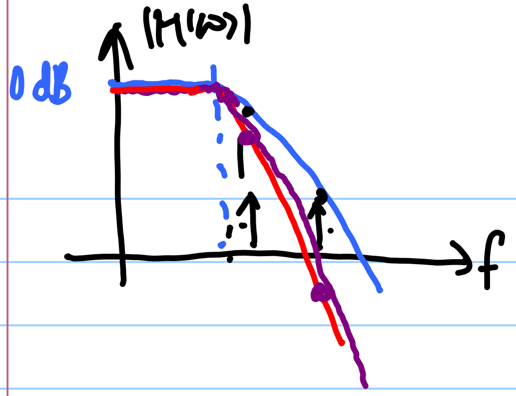




$$\frac{V_o}{V_i} = \frac{1}{1 + sRC}$$



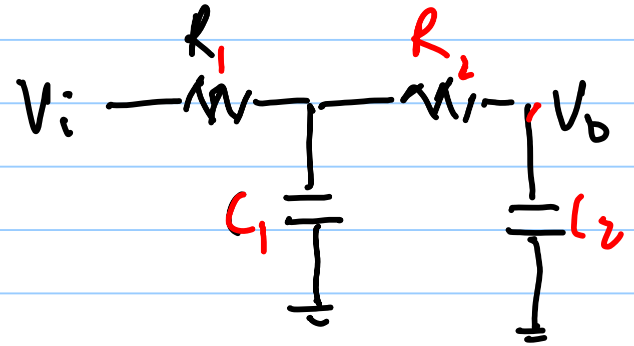




$$H(s) = \frac{1}{1+s/\omega_p}$$

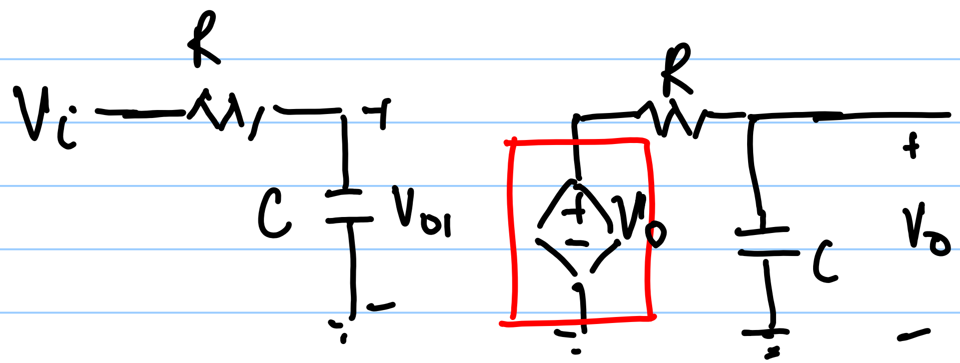
@ high freq. $\quad -20$ dB/dec

$$H(s) = \frac{1}{1+s^2/\omega_p^2 + s/\omega_p Q_p}$$



$$\frac{V_o}{V_i} = \frac{1}{1+s(C_2R_2+C_1R_1+C_2R_1) + \underbrace{s^2C_1C_2R_1R_2}} \checkmark$$

$$\approx \frac{1}{s^2C_1C_2R_1R_2}$$



$$\frac{V_o}{V_i} = \left(\frac{1}{1+sRC}\right)^2 \checkmark$$

-40 dB/dec

at higher freq. rate of change of $|H(j\omega)| = -40$ dB/dec

$$H(s) = \frac{1}{1 + s(C_2R_2 + C_1R_1 + C_2R_1) + s^2 R_1R_2C_1C_2}$$

$$= \frac{1}{1 + \frac{s}{\omega_p Q_p} + \frac{s^2}{\omega_p^2}}$$

$$\omega_p = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q_p = \frac{1}{\sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}}}$$