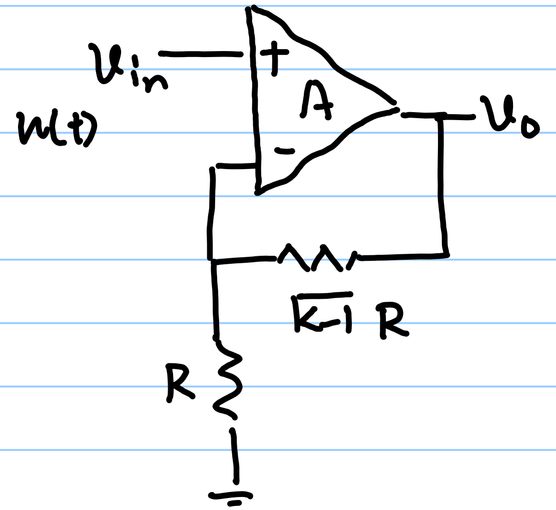
Lecture # 16



$$\frac{v_o}{v_e} = A(s) = \frac{A_0}{1+s/p_1}$$

$$v_o(t) = \mathcal{L}^{-1}\{V_{in}(s)\cdot H(s)\}$$

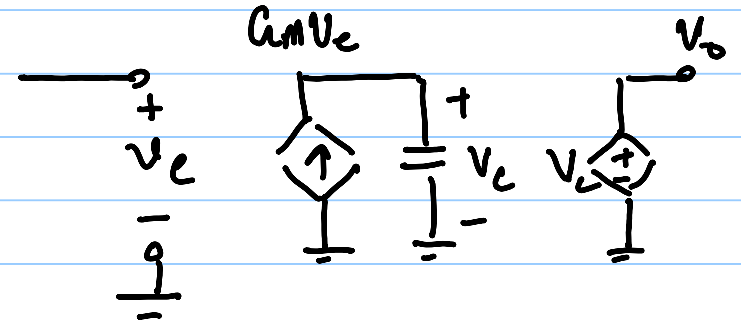
$$H(s) = \frac{K}{1+\frac{K}{A(s)}}$$

$$\epsilon_s = \frac{v_o^{ideal}(t\to\infty) - v_o(t\to\infty)}{v_o^{ideal}(t\to\infty)}$$

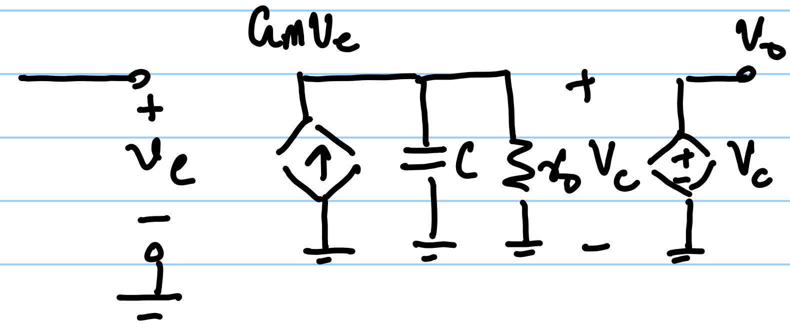
$$= \frac{K - K/(1+K/A_0)}{K} = 1 - \frac{1}{1+K/A_0}$$

$$= \frac{K/A_0}{1+K/A_0} = \frac{1}{1+A_0/K}$$

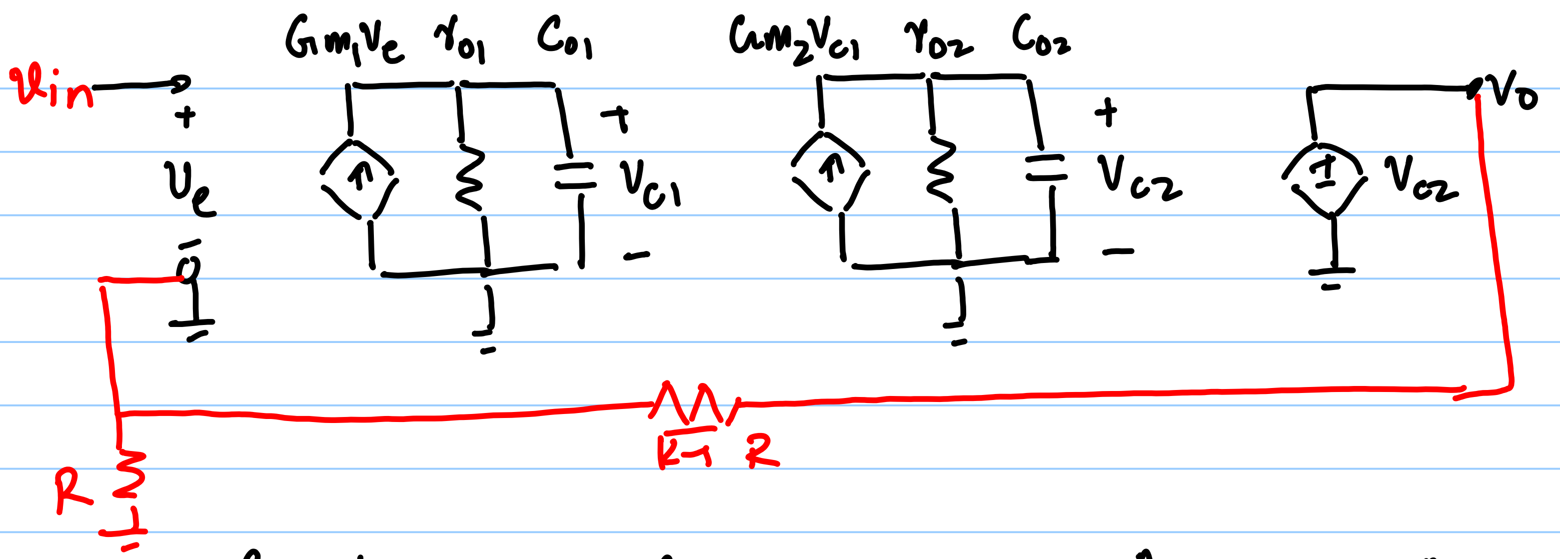
$$\epsilon_s \propto \frac{1}{A_0}$$



$$\frac{v_o}{v_e} = \frac{G_m}{sC}$$



$$\frac{v_o}{v_e} = \frac{A_0}{1+s/p_1}$$



$$A(s) = \frac{V_o}{V_e} = \frac{G_{m1} r_{o1}}{1 + s r_{o1} C_{o1}} \times \frac{G_{m2} r_{o2}}{1 + s r_{o2} C_{o2}} = \frac{A_{DC1}}{1+s/p_1} \times \frac{A_{DC2}}{1+s/p_2} = \frac{A_0}{(1+s/p_1)(1+\frac{s}{p_2})}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{K}{1+\frac{K}{A(s)}} = \frac{K}{1+\frac{K(1+s/p_1)(1+s/p_2)}{A_0}}$$

$$= \frac{K}{1+\frac{K}{A_0}\left(1+\frac{s}{p_1}+\frac{s}{p_2}+\frac{s^2}{p_1 p_2}\right)}$$

$$= \frac{K}{1+\frac{K}{A_0}+\frac{K}{A_0}\left(\frac{s}{p_1}+\frac{s}{p_2}\right)+\frac{K}{A_0} s^2/p_1 p_2}$$

$$= \frac{K/(1+K/A_0)}{\underbrace{1 + \frac{K/A_0}{1+K/A_0}\left(\frac{1}{p_1}+\frac{1}{p_2}\right)s + \frac{K/A_0}{1+K/A_0}\frac{1}{p_1p_2}s^2}_{D(s)}} \checkmark$$

$$D(s) = 1 + \frac{1}{1+A_0/K}\left(\frac{1}{p_1}+\frac{1}{p_2}\right)s + \underbrace{\frac{1}{1+A_0/K}\frac{1}{p_1p_2}}s^2$$

$$= 1 + \frac{s}{\omega_p Q_p} + \frac{s^2}{\omega_p^2} \checkmark$$

Q_p: Quality factor

ζ: damping coefficient.

$$= \frac{1}{\omega_n^2}\left(\underbrace{\omega_n^2 + 2\zeta\omega_n s} + s^2\right) \checkmark$$

Control

$\omega_n = \omega_p$

$Q_p = \frac{1}{2\zeta}$

$$\omega_n = \sqrt{p_1p_2(1+A_0/K)}$$

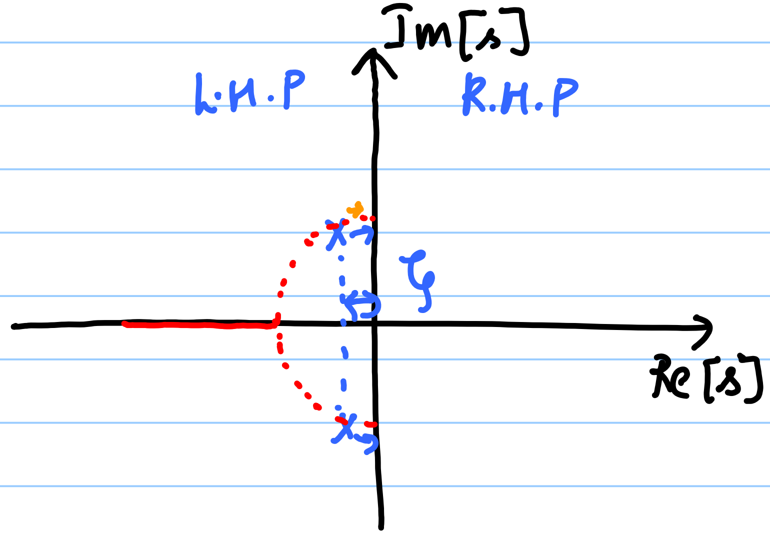
$$\zeta = \frac{1}{2}\frac{\sqrt{\frac{p_1}{p_2}}+\sqrt{\frac{p_2}{p_1}}}{\sqrt{1+A_0/K}}$$

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$s_1, s_2 = \Big[\underbrace{-\zeta \pm \underbrace{\sqrt{\zeta^2-1}}_{<\zeta}}\Big]\omega_n \checkmark$$

$\zeta > 1$: $\zeta^2 - 1 > 0 \Rightarrow s_1, s_2 < 0$

$0 \leq \zeta < 1$: $s_1, s_2 = \left(-\zeta \pm j\sqrt{1-\zeta^2}\right)\omega_n$



$\zeta > 1$: Closed loop poles are real

$$\frac{V_o(s)}{V_{in}(s)} = \frac{K}{1+K/A_0} \frac{1}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

$$V_o(s) = \frac{K}{1+K/A_0} \frac{1}{s} \frac{1}{\left(1+\frac{s}{\omega_{p1}}\right)(1+s/\omega_{p2})}$$

$$v_o(t) = K\,u(t)\left[1 - \frac{e^{-\zeta\omega_n t}}{2\sqrt{1-\frac{1}{\zeta^2}}}\left\{\left(1+\sqrt{1-\frac{1}{\zeta^2}}\right)e^{\zeta\omega_n\sqrt{1-\frac{1}{\zeta^2}}\,t} - \left(1-\sqrt{1-\frac{1}{\zeta^2}}\right)e^{-\zeta\omega_n\sqrt{1-\frac{1}{\zeta^2}}\,t}\right\}\right]$$

for $\zeta < 1$, complex poles

$$V_o(t) = K\,u(t)\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left\{\zeta\sin(\omega_n\sqrt{1-\zeta^2}\,t) + \sqrt{1-\zeta^2}\cos(\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_n\sqrt{1-\zeta^2}}\,t)\right\}\right]$$

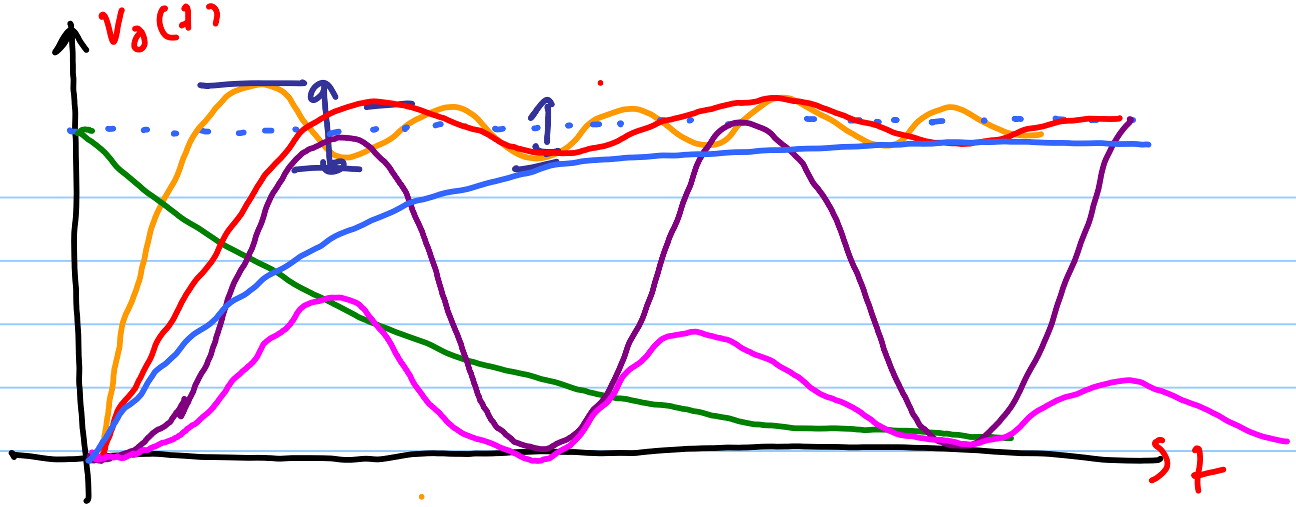
x

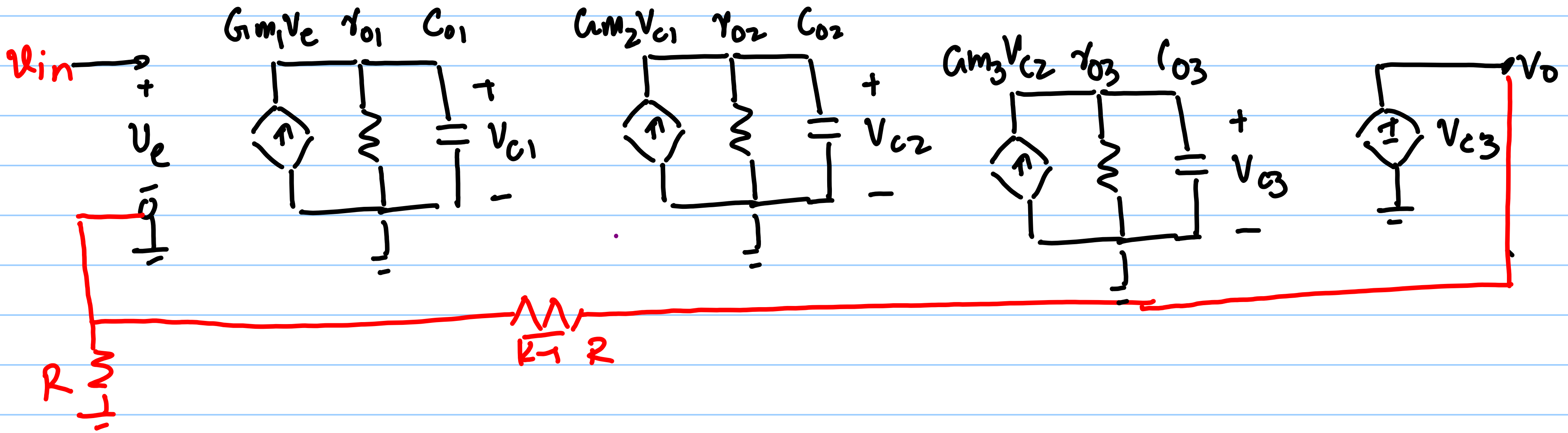
$\zeta \ll 1$

$$v_o(t) \simeq K\,u(t)\left[1 - \frac{e^{-\zeta\omega_n t}}{1}\cos(\omega_n\sqrt{1-\zeta^2}\,t)\right]$$

$$= \underbrace{K\,u(t)\left[1 - e^{-\zeta\omega_n t}\right]}_{\text{1st term}} + \underbrace{K\,u(t)\left[1-\cos(\omega_n\sqrt{1-\zeta^2}\,t)\right]e^{-\zeta\omega_n t}}_{\text{IInd term}}$$

$\to 0$





$$A(s) = \frac{V_o}{V_e} = \frac{(G_{m1}r_{o1})(G_{m2}r_{o2})(G_{m3}r_{o3})}{\left(1+\frac{s}{p_1}\right)\left(1+s/p_2\right)\left(1+s/p_3\right)} = \frac{A_0}{\quad}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{K}{1+\frac{K}{A(s)}}$$

$$A(s) = \frac{A_0}{(1+s/p_1)^3}$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{K}{1+\frac{K}{A_0}\left(1+\frac{s}{p_1}\right)^3}$$

$$D(s) = 1 + \frac{K}{A_0}\left(1+\frac{s}{p_1}\right)^3 = 0$$

$$\left(1+\frac{s}{p_1}\right)^3 = (-1)\left(\frac{A_0}{K}\right)$$

$$\frac{s}{p_1} = (-1)^{1/3}\left(\frac{A_0}{K}\right)^{1/3} - 1$$

$$-1 = e^{j\pi} = \cos(\pi) + j\sin(\pi)$$

$$= -1 + \left(\frac{A_0}{K}\right)^3 (-1)^{1/3}$$

$$= -1 + \left(\frac{A_0}{K}\right)^3 e^{j(2k\pi+\pi)/3}$$

$$= -1 + \left(\frac{A_0}{K}\right)^3 e^{j\pi/3} \cdot e^{j2\cancel{k}\pi/3}$$

$$s = p_1\left[-1 - \left(\frac{A_0}{K}\right)^{1/3}\right], \quad p_1\left[\underbrace{-1 + \left(\frac{A_0}{K}\right)^{1/3}\frac{1}{2}} \pm j\frac{\sqrt{3}}{2}\left(\frac{A_0}{K}\right)^{1/3}\right]$$

$$< 0$$

first order: Unconditionally stable

2nd order: Stable but with ripples at o/p

3rd order: can be unstable.

$$-1 + \left(\frac{A_0}{K}\right)^{1/3}\frac{1}{2} > 0$$

$$\left(\frac{A_0}{K}\right)^{1/3} > 2$$

$$\left(\frac{A_0}{K}\right) > 8$$