Multistage Relaying Using Interference Networks

Bama Muthuramalingam, Srikrishna Bhashyam and Andrew Thangaraj
Department of Electrical Engineering
Indian Institute of Technology Madras, Chennai, India 600036
{bama, skrishna, andrew}@iitm.ac.in

I. Introduction

One of the key technologies in next generation systems for achieving high throughput and providing better coverage is *relaying*. Relaying has attracted a high level of recent research interest with several papers focusing on various aspects of communicating using relays with different constraints and assumptions. In this work, we are concerned with the capacity of multistage relaying from one source to one destination through an arbitrary network of half duplex relays.

An example network that we consider in detail for ease of explanation and clarity is the two stage relay network shown in Fig. 1. In this 6-node network, the source node $S = 1$ intends to communicate with the sink node $D = 6$ through 4 relay nodes $\{R_1 = 2, R_2 = 3, R_3 = 4, R_4 = 5\}$ connected as shown. The channel gains (α, β, γ) are shown next to the corresponding edges. For simplicity, some of the gains are assumed to be identical. For a multistage half-duplex relay network such as the one in Fig. 1, we study coding methods and protocols needed to achieve the best possible rate from source to destination for different ranges of the channel gains.

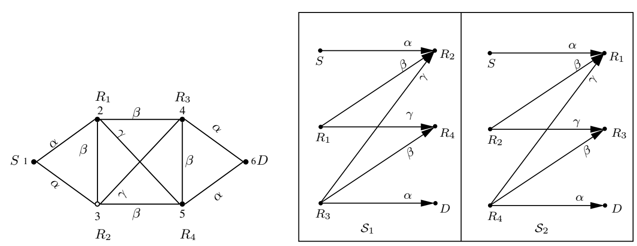

Fig. 1. Two stage relay network and interference states

There are two different aspects to multistage relaying when the relays are connected in an arbitrary fashion: (1) scheduling transmissions by nodes, and (2) coding methods employed by nodes during transmissions. One strategy for scheduling is to avoid interference altogether. However, the maximum data rate under Interference Avoidance (IA) is limited, because the source is transmitting only for a fraction of the total time. To improve upon IA, more states of the network with the source node in transmit mode need to be considered. For the network in Fig. 1, assuming three states per node (transmit, receive, idle), there are $2^2 \cdot 3^4 = 324$ possible states (the destination need not transmit and the source need not receive).

The scheduling task is to determine those states that are crucial for obtaining higher rates. When multiple nodes transmit, interference network states are created in the network based on the connectivity. Two important interference network states are shown in Fig. 1 for the network of Fig. 1. In one state, S, R_1, and R_3 are transmitters and, in the other state, S, R_2, and R_4 are transmitters. In both the states shown in Fig. 1, the source node is a transmitter and the destination node is a receiver. This property improves the flow of information, and is useful for improving the transmission rate from the source.

In each interference network state, three different coding strategies of increasing complexity are considered for transmitters - Common broadcast (CB), Superposition coding (SC) and Dirty paper coding (DPC) for the source node alone. The receiving nodes in the interference network employ multiple access (MAC) receivers that work by successive interference cancellation. For different combinations of coding strategies, suitable rate regions are determined for each state (or interference network). The overall rate achievable from the source to the destination is computed using an optimization over the time-sharing of the rate regions for each state, subject to additional flow constraints that ensure compatibility of the rate vectors used for individual states.

To place our work better, we review a sample of the relevant prior literature. The relay channel is a classic setting, introduced in [1], and studied extensively [2]–[4]. One result of particular interest is the cut-set bound for half-duplex relay networks operating by time-sharing over a finite number of states [5]. This "cheap relay" bound has been used by several authors as an outer bound for achievable rates.

Recently, the half-duplex diamond network with two relays has been studied in [6]–[9]. The *multi-hopping decode and forward* (MDF) protocol, proposed in [6] and extended in [7], achieves rates close to the cheap relay cut-set bound. Wang *et al* [8] consider a modified diamond network with an additional link between the relays and propose a coding strategy using Dirty Paper Coding (DPC), which is shown to approach the cut-set bound. More protocols for general half-duplex wireless relay networks have been studied in [10], [11].

For an arbitrary number of relays in a general topology, capacity approximations have been established in [12] under the full-duplex and full-cooperation assumptions. The optimal DMT for arbitrary relay networks with full-duplex and half-duplex nodes have been determined in [13] and [14], respectively.

In relation to the above, in our work, we propose and study multi-hopping decode and forward (MDF) protocols for a general relay network with half-duplex nodes in the following setting: (1) *No cooperation* is assumed for encoding and decoding (except in one protocol for the source node alone), (2) Achievable rates are compared against the cheap relay cut-set bound at *finite SNRs*, (3) The protocols and methods apply for a *general topology* of relays. The results are illustrated by evaluation on two different networks, where we show that the cut-set bound is approached for some values of channel gains.

II. Numerical Results

We evaluate and compare the rate achieved by the Multihopping Decode-and-Forward (MDF) relaying schemes with (1) Common Broadcast (CB), (2) Superposition Coding (SC), and (3) Dirty Paper Coding (DPC-CB) for two different network topologies and several channel realizations. The cheap relay cut-set upper bound for half-duplex relay networks [5] and the rate achieved by the IA scheme are also evaluated. The rate achieved by each scheme is obtained by solving an optimization problem with appropriate rate region constraints.

We first consider the two-stage relay network shown in Fig. 1. For evaluating the cut-set bound, all the $2^2 \cdot 3^4 = 324$ states were considered. The states that avoid interference (called IA states) are the states with a single transmitting node. For the proposed MDF protocols, interference network states with two transmitters ($\binom{5}{2} = 10$ states) and some states with three transmitters (5 out of $\binom{5}{3} = 10$ states) are used along with the IA states. Two of the states with three transmitters are shown in Fig. 1 for illustration. In Fig. 2, the cut-set bound, determined by the source cut, is at 1 for all β. For large

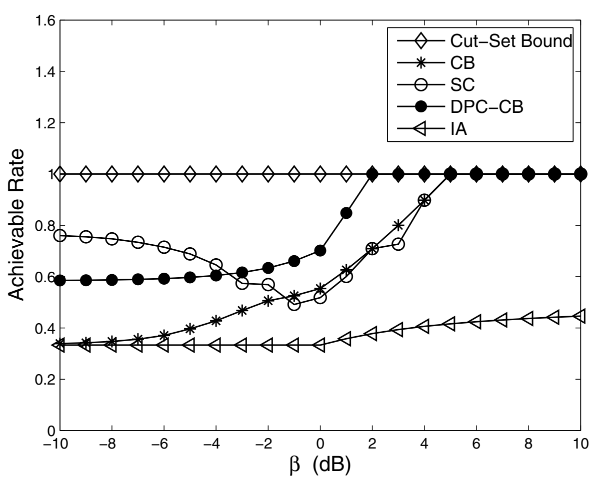

Fig. 2. $\alpha = 1, \gamma = 1$, vary β.

β, the states used are $S_3 = (\{S, R_2, R_3\}, \{R_1, R_4, D\})$ and $S_4 = (\{S, R_1, R_4\}, \{R_2, R_3, D\})$. The receivers in both these states see strong interference, which can be canceled at the receiver. For instance, in state S_3, the receiver R_1 can decode the source's message in the presence of strong interference from R_2 and R_3. Because of this, all three MDF schemes achieve capacity of 1 by equal time-sharing of states S_3 and S_4. For small β, common broadcast at the relays is limited by a weak receiver with close-to-zero capacity. Superposition coding, which enables different rates to receivers, proves to be better at low values of β. For SC, states S_1 and S_2 (shown in fig. 1) are chosen, and the rate is limited by the interference at relays R_1 and R_2. DPC is marginally weaker, since the relays continue to do common broadcast when the source does DPC. However, when $\beta = 1$ (0 dB), DPC is better as SC becomes identical to CB for identical channel gains.

Consider the 4×3 rectangular grid network shown in Fig. 3. Since the number of possible states is prohibitively large, we first select three non-overlapping paths from the source node $S = 2$ to the destination node $D = 11$. We know from the two-stage relay network example that multiple flow paths used appropriately with interference processing can be effective. The paths chosen are $S \to 4 \to 7 \to D$, $S \to 5 \to 8 \to D$ and $S \to 6 \to 9 \to D$. Using the nodes on these paths, the three states chosen for scheduling are $(\{S, 6, 8\}, \{4, 9, D\})$, $(\{S, 4, 9\}, \{5, 7, D\})$, and $(\{S, 5, 7\}, \{6, 8, D\})$. Note that the source node is a transmitter and the destination node is a receiver in all three chosen states. Also, the other two transmitters are chosen to be at different distances from the source. With this choice of states, we have a two-stage relay network with six relay nodes $\{4, 5, 6, 7, 8, 9\}$ aiding communications from the source to the destination.

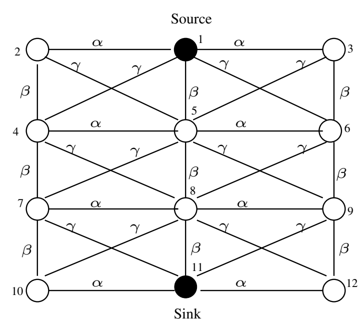

Fig. 3. 4×3 Grid Network.

In Fig. 4, the gains β and γ are set to 1, and the gain α is varied. We notice that the DPC-CB scheme approaches the capacity for a large range of values of $\alpha > 1$. The CB and SC schemes are limited by the interference at relays 4 and 5 even for large α. For small α, the DPC-CB and CB schemes are limited by the common broadcast constraint at the relays. While SC scheme can perform better, it is still limited by interference at relays 4 and 5 compared to the cut-set bound.

III. Conclusions

Based on this work, two interesting comparisons are possible for multistage half-duplex relay networks based on the cut-set bound. For the network in Fig. 1, the cut-set bound evaluates to $C_{pp} = \log(1+\alpha^2 P/\sigma^2)$, which can be interpreted as the capacity of a point-to-point link with power constraint P and channel gain α. Using the protocols in this work, we have shown that rates up to C_{pp} are achievable by multistage half-duplex relaying in the network of Fig. 1 for certain ranges of the channel gains α, β and γ. A necessary condition for achieving the point-to-point capacity under the half-duplex

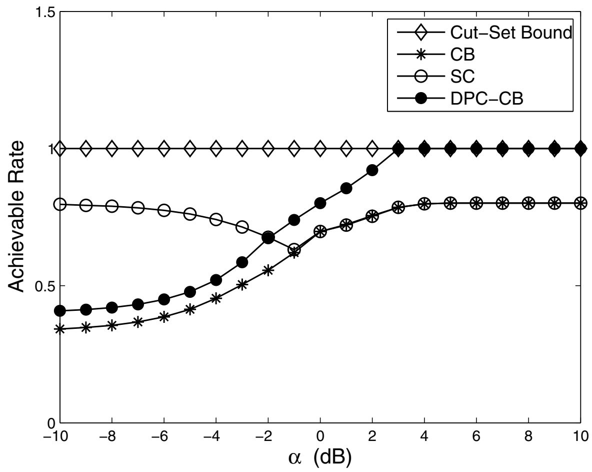

Fig. 4. Performance in Grid Network, $\beta = 1, \gamma = 1$, vary α.

constraint is that the source needs to be in transmit mode at all times. It appears that continuous transmission by the source and information transfer through the half-duplex relays is possible as long as there are two or more non-overlapping paths from the source to the destination (which is true in Figs. 1 and 3). Further, coding in interference networks created by multiple transmitters and receivers of the relay network is crucial for enabling the information flow.

The second comparison is with full-duplex relays. The achievable rate even with full duplex relays is bounded by the sum rate across the source-broadcast cut, which is equal to C_{pp}, for the network in Fig. 1. Once again, we observe that two non-overlapping paths through the relays and interference-network coding enable a half-duplex relay network to achieve the full-duplex cut-set bound for certain ranges of channel gains.

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