IMPROVING CHANNEL ESTIMATION IN OFDM SYSTEMS FOR SPARSE MULTIPATH CHANNELS

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Abstract — **We describe an algorithm for sparse channel estimation in orthogonal frequency division multiplexing (OFDM) systems. The proposed algorithm uses Least Squares (LS) technique for channel estimation and a Generalized Akaike Information Criterion to estimate the channel length and tap positions. This effectively reduces the signal space of the LS estimator, and hence improves the estimation performance as demonstrated using computer simulations. For example, the proposed modified LS with sparse channel estimation algorithm has a 5dB lower mean square error in channel estimation when compared to the conventional LS approach [2] which translates to approximately 0.5dB improvement in SNR at receiver.**

I. Introduction

OFDM has received considerable interest in the last few years for its advantages in high bit rate transmissions over frequency selective fading channels. In OFDM systems, the high-rate data stream is divided into many low-rate streams that are transmitted in parallel, thereby increasing the symbol duration and reducing the inter-symbol interference (ISI). The ISI can be completely eliminated by introducing guard interval (Cyclic Prefix-CP) between adjacent OFDM symbols, given that the cyclic prefix length is greater than the length of channel impulse response. The cyclically extended guard interval also converts linear convolution of signal and channel into circular convolution. As a result, a traditional complex Time-domain EQualizer (TEQ) can be replaced by a simple single tap Frequency-domain EQualizer (FEQ).

The channel estimation method based on a parametric channel model is proposed in [3] which uses Minimum Description Length (MDL) to estimate the channel order and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique) to acquire mutipath time delays. Most Significant Taps (MST) approach is used in [4] to estimate the channel order. Recently, a delay subspace tracking algorithm applicable for OFDM has been proposed in [5], which cam potentially also exploit sparse multipath. In this paper, we propose an algorithm to estimate sparse channels using the Generalized Akaike Information Criterion (GAIC). This modified LS with Sparse Channel Estimation (mLS-SCE) algorithm yields a significant improvement in normalized mean squared error in channel estimation.

In this paper, bold faces denotes vectors or matrices; $(.)^T$ denotes transposition; $(.)^H$ denotes hermitian transposition and $\mathbf{I}_N$ is the $N \times N$ identity matrix.

II. OFDM System Model

The OFDM system is modeled employing the following assumptions:
1. Accurate timing and frequency synchronization are perfect at the receiver.
2. The channel impulse response length (L) is smaller than the cyclic prefix length L_{cp} of the OFDM symbol.

The channel consists of L multi-path components and has the form

$$h(k) = \sum_{m=0}^{L-1} \alpha_m \delta(k-m) \tag{1}$$

where α_m is a zero-mean complex Gaussian variable with $E[\alpha_i \alpha_j^*] = 0$ for $i \neq j$, and $E[|\alpha_m|^2] = e^{-\beta m}$ where $\beta = 4/L_{cp}$ for the exponential delay profile, and $\beta = 0$ for the uniform delay profile. In the above model, path delays are sample spaced. For non-sample spaced multipath channels, the GAIC based parameter pruning may not be useful. However, with oversampled OFDM systems, sample spaced channel models which are sparse may still be realized and the proposed estimator can be useful for such channels too.

The bits to be transmitted are first mapped onto constellation points X_i. The X_i are modulated in blocks of N, using a N point IDFT. The last L_{cp} samples of the IDFT output are appended as cyclic prefix to form one OFDM symbol of $N + L_{cp}$ samples. In the frequency domain we can write the measurement vector as

$$\mathbf{R} = \mathbf{XH} + \mathbf{n} \tag{2}$$

where, $\mathbf{R} = [R_1, R_2, ..., R_N]^T$ is the received symbol retaining only the appropriate N samples per OFDM symbol, $\mathbf{X}$ is a diagonal matrix of transmitted symbols i.e., $\mathbf{X} = \text{diag}[X_1, X_2, ..., X_N]$ and $\mathbf{H}$ is the N point DFT of the channel impulse response $\mathbf{h}$. The zero mean complex Gaussian vector $\mathbf{n}$ has an auto correlation matrix $\sigma_n^2 \mathbf{I}_N$, i.e., $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$ where σ_n^2 is the noise variance.

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III. Channel Estimation

In OFDM systems, channel estimation is done by sending known data on all N subcarriers. We can write equation (2) initially as

$$\mathbf{R} = \mathbf{X}\mathbf{F}\mathbf{h} + \mathbf{n} \tag{3}$$

where $\mathbf{F}$ is the $N \times N$ DFT matrix and $\mathbf{X}$ is known at the receiver. The LS estimator for impulse response $\mathbf{h}$ is then given by

$$\hat{\mathbf{h}}_{ls} = \mathbf{F}^{-1}\mathbf{X}^{-1}\mathbf{R} \tag{4}$$

The modified LS [2] exploits the correlation in frequency domain using the fact that channel length cannot exceed cyclic prefix length, and gives

$$\hat{\mathbf{h}}_{mls} = (\mathbf{F}_1^H\mathbf{X}^H\mathbf{X}\mathbf{F}_1)^{-1}\mathbf{F}_1^H\mathbf{X}^H\mathbf{R} \tag{5}$$

where $\mathbf{F}_1$ is a DFT matrix retaining only the first L_{cp} columns, namely

$$[\mathbf{F}_1]_{p,q} = e^{-j2\pi pq/N} \tag{6}$$
$$p = 0, 1, .., N-1, \quad q = 0, 1, .., L_{cp} - 1.$$

If alphabets X_i are chosen from an M-ary PSK constellation, then modified LS is same as truncating the LS channel estimates (4) to L_{cp}. This is not true if alphabets X_i are chosen from an M-ary QAM constellation. The channel estimation algorithm proposed in [6] is equivalent to the modified LS method.

With the linear, Gaussian measurement model in (3) all unbiased estimators lead to the Best Linear Unbiased Estimator [1] given by (5). The performance of the channel estimator can be improved by using Bayesian estimators, but this requires prior knowledge of the channel.

If the channel is known to be sparse, then the knowledge of channel length and position of channel taps will help in improving the performance of the modified LS estimator. In this paper, we have used the GAIC criterion to estimate the length of the channel. By successively canceling the estimated taps in time domain, we can also estimate the tap positions. This is discussed in the next section.

IV. Sparse Channel Estimation

A channel is said to be sparse if number of multipath components is smaller than channel length, i.e., some of the tap gains in (1) are zero. GAIC has been a popular statistical criterion for model structure selection in system identification. The GAIC cost function has the form [7]

$$\text{GAIC}(L) = V_L + \gamma \ln(\ln(N))(L+1) \tag{7}$$

where the first term reflects the modeling error and the second term is the penalty function. Here γ is a parameter which the user can choose, and we have chosen $\gamma = 2$ for our simulations.

For the system model in (3), the expression for V_L is given by

$$V_L = \frac{N}{2}\ln(\hat{\sigma}^2_{n,L}) \tag{8}$$

where, $\hat{\sigma}^2_{n,L}$ is the estimate of the noise variance for channel length L and is given by

$$\begin{aligned}\hat{\sigma}^2_{n,L} &= \frac{1}{N}(\mathbf{R} - \mathbf{X}\mathbf{F}\hat{\mathbf{h}}_{mls,L})^H(\mathbf{R} - \mathbf{X}\mathbf{F}\hat{\mathbf{h}}_{mls,L}) \\ &= \frac{1}{N}(\hat{\mathbf{h}}_{ls} - \hat{\mathbf{h}}_{mls,L})^H\mathbf{F}^H\mathbf{X}^H\mathbf{X}\mathbf{F}(\hat{\mathbf{h}}_{ls} - \hat{\mathbf{h}}_{mls,L})\end{aligned} \tag{9}$$

where $\hat{\mathbf{h}}_{mls,L}$ is the modified LS estimate (5) of the channel $\mathbf{h}$ assuming channel length L and padded with $(N-L)$ zeros.

The GAIC estimate of true channel length is obtained by minimizing (7) with respect to L.

The following steps constitute the GAIC test,

Step 1. Initially set the limit $P = L_{cp}$

Step 2. Calculate the cost function GAIC(L) for $L = 1, 2, ..., P$

Step 3. The GAIC estimate of L is then obtained as

$$\hat{L} = \arg\min_L\{\text{GAIC}(L)\}. \tag{10}$$

If the channel is sparse (which is usually the case in practice), then the channel length estimation alone will not fully exploit the sparsity. The sparsity can only be exploited if we know the tap positions. The procedure for estimating tap position is now given.

Step 4. Remove the effect of estimated tap by setting $\hat{h}_{ls}(\hat{L}) = 0$, $\hat{\mathbf{H}}_{ls} = \mathbf{F}\hat{\mathbf{h}}_{ls}$ and redefine $\mathbf{R} = \mathbf{X}\hat{\mathbf{H}}_{ls}$

Step 5. Repeat steps 1-3 with $P = \hat{L} - 1$ to estimate *next* significant tap position

Step 6. If $\hat{L} \neq 1$ go to Step 4.

The values of $\{\hat{L}\}$ give the position of channel taps. With these estimated tap positions, the improved channel estimates are obtained as

$$\hat{\mathbf{h}}_{ils} = (\mathbf{F}_2^H\mathbf{X}^H\mathbf{X}\mathbf{F}_2)^{-1}\mathbf{F}_2^H\mathbf{X}^H\mathbf{R} \tag{11}$$

where $\mathbf{F}_2$ is the modified DFT matrix retaining only those columns corresponding to estimated channel tap positions. In other words, the $(p,q)^{th}$ element of $\mathbf{F}_2$ is given by

$$[\mathbf{F}_2]_{p,q} = e^{-j2\pi pq/N} \tag{12}$$
$$p = 0, 1, .., N-1, \quad q \in \{0, 1, .., L_{cp} - 1\},$$

where q reflects the estimated channel tap positions in the range 0 to $L_{cp} - 1$.

V. Application to Spectral Shaping Systems

In spectral shaping systems, the DC subcarrier and some subcarriers at the edges of the spectrum are not used. IEEE 802.11a WLAN [9] standard is an example where the 11 subcarriers at the edges of the spectrum act as guard band.

In [8], an algorithm for channel estimation using GAIC is given. This uses only channel length information, but does not exploit sparsity. It is possible to have channel impulse response equal to the CP length, and yet the number of significant (non-zero) taps is much less. In such situations channel length estimation alone is not useful, and exploiting

the sparsity of the channel taps can give significant advantage.
Considering only useful $M \leq N$ subcarriers in (3) we have

$$\mathbf{R}' = \mathbf{X}'\mathbf{F}'\mathbf{h} + \mathbf{n}' \quad (13)$$

where $\mathbf{n}' \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I}_M)$.
The LS estimator of the frequency response is given by

$$\hat{\mathbf{H}}'_{ls} = \mathbf{X}'^{-1}\mathbf{R}' \quad (14)$$

The modified LS estimator $\hat{\mathbf{H}}_{mls}$ is given by

$$\hat{\mathbf{H}}_{mls} = \mathbf{F}_1(\mathbf{F}'^H\mathbf{X}'^H\mathbf{X}'\mathbf{F}')^{-1}\mathbf{F}'^H\mathbf{X}'^H\mathbf{R}' \quad (15)$$

where $\mathbf{F}'$ is the DFT whose rows corresponding to zero subcarriers are removed, and only the first L_{cp} columns are retained. In other words, the $(p,q)^{th}$ element of $\mathbf{F}'$ is given by

$$[\mathbf{F}']_{p,q} = e^{-j2\pi pq/N} \quad (16)$$
$$p = 1, .., r, s, .., N-1, \quad q = 0, 1, .., L_{cp} - 1.$$

{where we have assumed that the number of subcarriers in the guard band is $(s - r)$}
The modeling error in this case is given by

$$V_L = \frac{M}{2}\ln(\hat{\sigma}^2_{n,L}) \quad (17)$$

$$\hat{\sigma}^2_{n,L} = \frac{1}{M}(\hat{\mathbf{H}}'_{ls} - \hat{\mathbf{H}}'_{mls,L})^H\mathbf{X}'^H\mathbf{X}'(\hat{\mathbf{H}}'_{ls} - \hat{\mathbf{H}}'_{mls,L}) \quad (18)$$

$\hat{\mathbf{H}}'_{mls,L}$ is the modified LS estimate (considering only the useful subcarriers) for channel length L. The procedure for sparse channel estimation is same as explained in the previous section. Here the effect of estimated tap is removed in the frequency domain. After estimating the positions of channel taps $\{\hat{L}\}$, the improved channel estimates are given by

$$\hat{\mathbf{H}}_{ils} = \mathbf{F}_1(\mathbf{F}'^H_1\mathbf{X}^H\mathbf{X}\mathbf{F}'_1)^{-1}\mathbf{F}'^H_1\mathbf{X}^H\mathbf{R} \quad (19)$$

where $\mathbf{F}'_1$ is the DFT whose rows corresponding to zero subcarriers are removed and only those columns corresponding to estimated channel tap positions are retained, i.e.,

$$[\mathbf{F}'_1]_{p,q} = e^{-j2\pi pq/N} \quad (20)$$
$$p = 1, .., r, s, .., N-1, \quad q \in \{0, .., L_{cp} - 1\}.$$

VI. Simulation and Results

An OFDM systems was simulated using $N = 64$ useful subcarriers with cyclic prefix of $L_{cp} = 16$ samples. The preamble data is taken from a 16-QAM constellation. A spectral shaped OFDM system similar to IEEE 802.11a [9] was also simulated with $N = 64, M = 52, L_{cp} = 16, r = 26$ and $s = 38$.
The channel estimation algorithms are compared for the following channel models

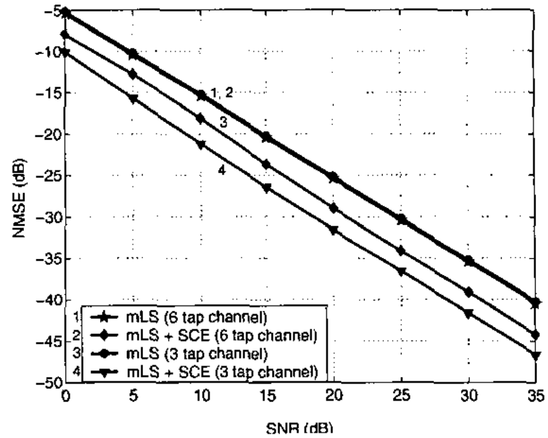

Fig. 1. NMSE comparison for exponential power delay profile

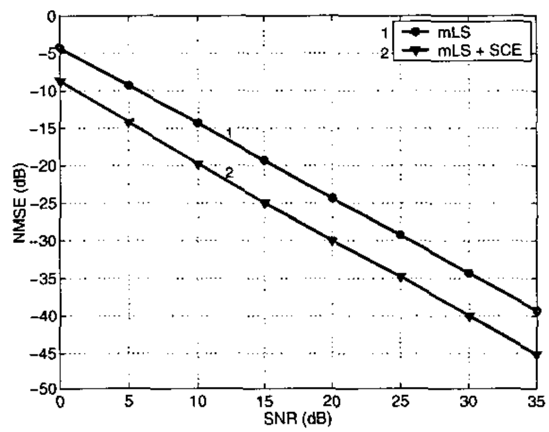

Fig. 2. NMSE for exponential power delay profile with 3 significant taps (802.11a)

1. 6-tap sparse channel with $L = 14$ for a system with 64 useful subcarriers
2. 3-tap sparse channel with $L = 14$ for a system with 64 useful subcarriers
3. 3-tap sparse channel with $L = 8$ for an 802.11a system.

The location of taps between first and last tap was randomized. The performance is averaged over the independent, quasi-static channel realizations. Channel estimation was done using both modified LS (mLS) and modified LS with sparse channel estimation (mLS-SCE) techniques. Their performances were compared in terms of Normalized Mean Square Error (NMSE) defined by

$$NMSE = \frac{E\left[\sum_k \left|H[k] - \hat{H}[k]\right|^2\right]}{E\left[\sum_k |H[k]|^2\right]}$$

(for spectral shaped systems only errors in useful subcarrier locations are considered). The NMSE plots are as shown

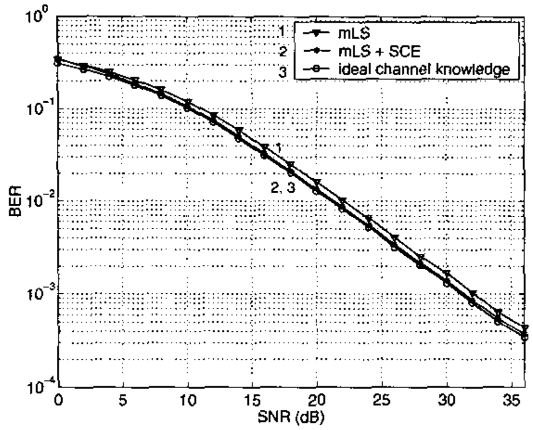

Fig. 3. BER comparison for 802.11a with 3 significant taps (exponential power delay profile)

in Figure 1 (for 64 useful subcarriers system) and 2 (for spectral shaped system). It can be seen that as the number of significant taps decreases, the mLS-SCE performs better than mLS since the number of parameters to be estimated is small.

The raw bit error rate (BER) plot for 16 QAM data is shown in Figure 3 (for spectral shaped system). This reveals an improvement of 0.5 dB in SNR at BER of 10^{-3} with mLS-SCE when compared to mLS.

VII. Conclusions

In this paper, we have presented an improved channel estimator for sparse channels. The proposed mLS-SCE algorithm uses a combination of GAIC and successive tap cancellation in time domain to estimate the tap positions. This can also be applied to track the channel length and tap positions of slowly time varying channels when used along with pilot data.

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