Rate and Power Adaptation in OFDM with Quantized Feedback

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Abstract

Adaptive modulation can be used in conjunction with Orthogonal Frequency Division Multiplexing (OFDM) to obtain high spectral efficiencies. In this paper, we analyze two types of adaptive OFDM systems with quantized feedback. In the first type, rate and power allocation algorithm is performed at the transmitter to maximize throughput using quantized channel-to-noise ratio (CNR) feedback from the receiver. In the second type, power allocation is implicit such that the receiver decides the constellation to be used in each subcarrier and sends back only the rate allocation vector to the transmitter. We determine the spectral efficiency of these adaptive OFDM systems and quantify the number of bits of feedback needed in each case. Results show that even though type 2 systems have lower spectral efficiency than type 1 systems, they achieve significant spectral efficiency gain compared to no-feedback schemes and have lower feedback requirements. In both types of systems, correlation among subcarriers can be utilised to further reduce the feedback requirements while retaining the benefits of adaptive modulation. Various approaches to use subcarrier correlation in the presence of quantized feedback are compared.

1. Introduction

In wireless systems, Orthogonal Frequency Division Multiplexing (OFDM) is being increasingly used because of its ability to mitigate multipath effects and fading. OFDM converts a frequency selective channel into parallel flat fading channels [1]. Adaptive modulation has been discussed in [2] as a technique to enhance spectral efficiency. In conjunction with OFDM, adaptive modulation is highly promising for high data rate transmission. With perfect channel knowledge at the transmitter and receiver, [3] describes the optimal strategy to allocate rate and power across subcarriers to maximize the data rates under a total power constraint and an equal BER constraint on all subcarriers. Other suboptimal algorithms of lower complexity are proposed in [4, 5].

In this paper, we provide a comprehensive comparison of adaptive OFDM algorithms in terms of feedback requirements and spectral efficiency. First, we categorize the various adaptive OFDM systems into two types based on which side of the communication link the allocation algorithms are being performed. Subsequently, we determine the feedback requirement for both types of algorithms in terms of number of feedback bits per subcarrier. Finally, we also propose and compare different techniques to further reduce the feedback requirement by utilizing the correlation among subcarriers.

The paper is organized as follows. In the next section, we describe the system model and the rate maximization problem. In Sections 3 and 4, we describe the two classes of algorithms and a CNR quantization strategy that can be used to analyze the feedback requirements and spectral efficiency performance. In section 5, we describe different techniques that utilize subcarrier correlation to reduce feedback. The simulation setup is explained in Section 6. Finally, we summarize our results in Section 7.

2. System Model

We assume a block fading channel which remains constant within a block of OFDM symbols, and varies independently from block to block. The receiver is assumed to have perfect channel estimates of all subcarriers, and a finite-rate feedback channel exists between receiver to transmitter. We assume a point-point communication link in this paper. However, the results can be extended to other communication scenarios as well.

The transmission model is $Y_k = H_k X_k + n_k$ where H_k is the channel coefficient in frequency domain of kth subcarrier ($k = 1, 2, \cdots, K$). X_k is the symbol to be transmitter and is drawn from one of the available constellations as decided by the rate allocation algorithm. The power on the symbol X_k is P_k. n_k is the complex AWGN noise of variance N_0. The SNR on each subcarrier is $\frac{P_k |H_k|^2 T_s}{N_0}$. T_s is the OFDM symbol duration. Without loss of generality, we assume $T_s = 1$ in our work. We define channel to noise ratio (CNR) as $C_k = \frac{|H_k|^2}{N_0}$. $\{\Psi(m)\}_{m=1}^{M}$ are set of constellations available for modulation and transmission and $\{r(m)\}_{m=1}^{M}$

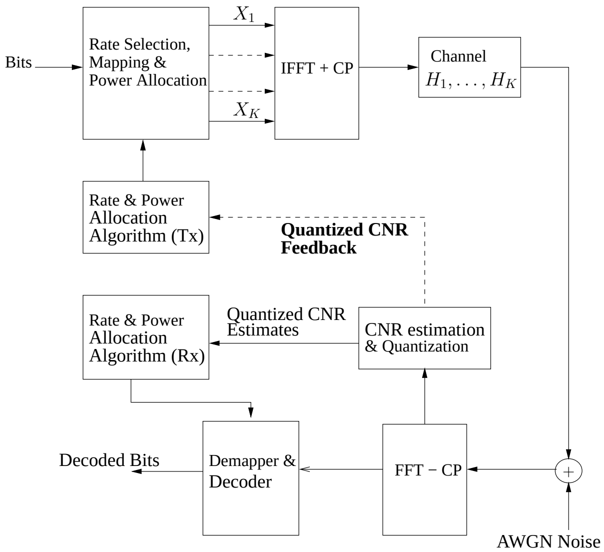

Figure 1: Same allocation algorithms performed at transmitter and receiver, and quantized CNR values fed back from receiver to transmitter.

are the set of corresponding rates. P_e is the target BER. $\{\Gamma(m)\}_{m=1}^{M}$ are the required SNRs for these constellations at the target BER P_e. The rate maximization problem with total power constraint (P_T) and target BER on each subcarrier is given below.

$$\max_{\{r(m_1),\ldots,r(m_K)\},\{P_1,\ldots,P_K\}} \frac{1}{K}\sum_{k=1}^{K} r(m_k)$$

subject to:

$$\sum_{k=1}^{K} P_k \leq P_T$$

$$P_{e,k} \leq P_e$$

$$P_k C_k \geq \Gamma(m_k) \quad m_k \in \{1,\ldots,M\}$$

The solution of the maximization problem is the well known discrete rate water-filling solution. Several algorithms to implement the solution have been proposed in [7, 9, 10]. We use the iterative algorithm proposed in [3], since it is known to be faster and is applicable for all discrete rate maximization problems with concave rate-SNR curves.

3. System 1: Algorithm performed at transmitter based on quantized CNR feedback

To perform the rate and power allocation at the transmitter, transmitter needs the CNR of all subcarriers (Figure 1). We assume that a feedback channel is available to transmit the CNR information from receiver to the transmitter. To evaluate the requirements of the feedback channel, we formulate a quantization method for CNR and then proceed to analyze how the rate and power allocation algorithm performs as the number of quantization levels change.

We divide the entire SNR range into Q quantization levels. The range depends on the actual constellations being used and the target BER. The SNR on each subcarrier is evaluated with regard to a reference power and compared against the quantization levels. The largest level lower than the SNR value is used as the quantized level for that SNR. Channel to Noise Ratio (CNR) is found by scaling this quantized level using the reference power.

$$\tilde{C}_k = \Upsilon(q)\frac{K}{P_T} \quad \text{if } \Upsilon(q) \leq \frac{P_T}{K}C_k < \Upsilon(q{+}1) \quad l \in \{q,\ldots,Q\},$$

where $\{\Upsilon(q)\}_{q=1}^{Q}$ are the set of quantization levels, $\tilde{C}_k$ is the quantized value of C_k, and P_T/K is the reference power. The $Q+1^{th}$ level is assumed to be infinity. This quantization method ensures that $\tilde{C}_k \leq C_k$, and hence, $\tilde{C}_k$ can be used to in rate and power allocation algorithm without violating the target BER constraints. For each subcarrier, the index of the level of $\tilde{C}_k$ is fed back. As the number of quantization levels are higher, the rate and power allocation algorithm performs closer to the perfect feedback case. The results are presented in Section 6. The constraints in this case become:

$$P_k\tilde{C}_k \geq \Gamma(m_k) \text{ and } \sum_{k=1}^{K} P_k \leq P_T.$$

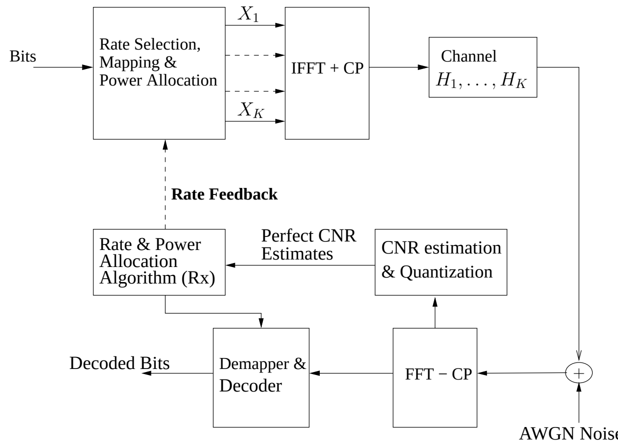

Figure 2: Allocation algorithm performed only at receiver, and rate allocation fed back to transmitter.

4. System 2: Algorithm performed at receiver and rate allocation vector fed back

In the previous section, we discussed systems where allocation is performed at the transmitter. Alternately, we can perform rate and power allocation at the receiver and feedback the rate and power allocation vectors to the transmitter as in Figure 2. However, power vector feedback would need as much bits as CNR vector feedback, and hence, overall feedback required will be higher than CNR vector feedback. Therefore, we make some implicit power allocation assumptions that would eliminate the need to feed back the power allocation vector to the transmitter. We consider the following two algorithms.

4.1. Equal Power Allocation (EPA)

In this method, each subcarrier is allocated equal amount of power (equal to P_T/K). The rate on each subcarrier is determined independently based on the subcarrier SNR and the target BER constraint.

4.2. Dual Mode Power Allocation (DMPA)

In [4], an approximate water-filling algorithm in which power is equally allocated only on subcarriers whose channel gains are above a predetermined cutoff is proposed. Utilizing the idea, we choose to allocate power only to the 'useful' subcarriers which can carry non-trivial rates on them. This algorithm works by first sorting the subcarriers in the descending order of channel gains. Subsequently, the 'useful' set of subcarriers is determined as follows.

Let $\{C_k\}_{k=1}^{K}$ be the subchannel gains sorted in descending order and $\Gamma(2)$ be the required SNR of the first nonzero rate option.

1. Initialize $j = K$.
2. If $\frac{P_T}{l} C_j < \Gamma(2)$, then set $j = j - 1$ and repeat Step 2.
   Else $K^* = j$.

Equal power allocation with SNR based rate allocation is performed on the K^* 'useful' subcarriers. The power allocation (in terms of the index k obtained after sorting the subchannel gains) is

$$P_k = \begin{cases} 0 & K^* < k \leq K \\ \frac{P_T}{K^*} & 1 \leq k \leq K^* \end{cases}$$

The rate for each subcarrier that is allocated power is determined to be the largest rate such that $\frac{P_T}{K^*} C_k \geq \Gamma(m_k)$. The rate allocation vector is then fed back to the transmitter.

It must be noted that, in both the above algorithms power allocation is discrete. In EPA, only one level of power is possible, while in DMPA two levels of allocated power is possible. In both the schemes, the amount of feedback is $\lceil log_2(M) \rceil$ bits per subcarrier. Since the rate allocation is performed at the receiver, these algorithms are more useful in point-to-point communication links.

5. Correlated subcarriers

In OFDM, subcarrier bandwidth is usually much smaller than the coherence bandwidth of the multipath channel. Therefore, in the frequency domain, channel gains of adjacent subcarriers are correlated. This correlation can be utilised to reduce the feedback requirements. Three such methods are discussed here.

5.1. FFT based interpolation and allocation

In [8], it has been shown that, the channel gains of $2L+1$ subcarriers are sufficient to fully identify the entire channel profile having $L+1$ nonzero channel taps. In OFDM systems, the cyclic prefix length P is usually designed to be larger than L the channel spread. Therefore, it is sufficient to have $2P + 1$ samples or more of the CNR vector at equal intervals. The other CNR values can be obtained at the transmitter using interpolation as long as the quantization error for the sampled values are negligibly small, i.e., there are sufficiently large number of quantizer levels to ensure good quantization. The index of quantized values of CNR samples is fed back to the transmitter. The transmitter uses an FFT-based interpolator to retrieve the entire channel profile. The three steps are IFFT, truncation to length $2P + 1$ and zero-padding, and FFT.

5.2. Grouped CNR feedback allocation

A group is a set of adjacent subcarriers. Assuming that all the subcarriers in a group are highly correlated, all the subcarriers in a group can be allocated the same rate and power. Such group-wise allocation has been addressed in [5]. $C_{\text{min},g}$ represents the minimum of the CNR values in a group. $\tilde{C}_{\text{min},g}$ represents its quantized value using the method referred above. This is fed back from receiver to the transmitter. There are G groups each having K/G subcarriers. Rate and power is allocated to a group based on $\tilde{C}_{\text{min},g}$. Since the allocation is based on minimum of CNR in a group, the BER constraint is not violated. The maximization problem is:

$$\max_{\{r(m_1),\ldots,r(m_G)\},\{P_1,\ldots,P_G\}} \frac{1}{G} \sum_{g=1}^{G} r(m_g)$$

subject to $\sum_{g=1}^{G} P_g \leq P_T \frac{G}{K}$ and $P_g \tilde{C}_{\text{min},g} \geq \Gamma(m_g)$.

5.3. Grouped EPA

Power is allocated equally to all subcarriers. The rate that can be supported on the subcarrier with minimum CNR in a group is chosen as the rate for the entire group. In the grouped allocation schemes, amount of feedback depends on the number of groups.

6. Simulation Setup and Results

The SUI-4 channel model ([11]) is used in the simulations. The three taps in the SUI-4 model fade independently with a Rayleigh distribution. The system bandwidth is assumed to be 2.5MHz with 256 subcarriers. QPSK, 16QAM and 64QAM have been used as the constellations for rate adaptation in all cases except one where all rates from 1 to 6 bits/symbol are assumed. The target BER was set at 10^{-5}. The BER-SNR relations derived in [6] were used to calculate the target SNRs for each of the rates at the target BER. For the quantization methods, thresholds $\Psi(q)$ were first fixed at these target SNRs. Subsequent levels are fixed equidistant between these levels and a few levels are fixed above and below this range as well. Fig 3 shows the spectral efficiency (in

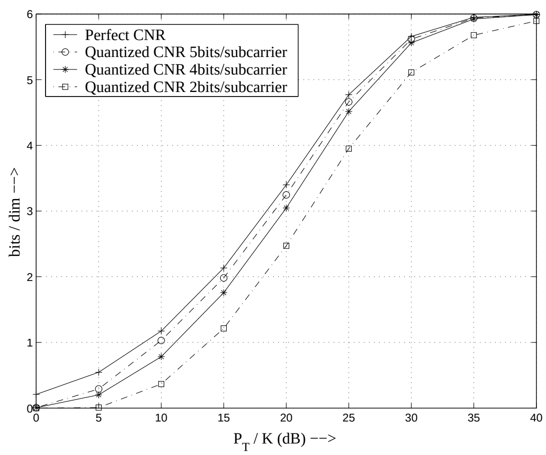

Figure 3: Performance of System 1 for various number of quantization levels. Target BER $= 10^{-5}$. Constellations used QPSK, 16QAM, 64QAM.

bits/subcarrier) vs. SNR for different number of quantization levels. As expected, spectral efficiencies increase as the number of quantization levels increase. At 5 bits quantization, spectral efficiency of the quantized feedback based allocation is within $\sim$ 0.5dB to the perfect feedback based allocation. If the number of quantization bits are further increased, the performance gains become increasingly insignificant.

Fig 4 shows the performance comparison in terms of spectral efficiency of various adaptive strategies. A performance gap of $\sim$3 dB is seen between CNR feedback based allocation and EPA. However, it must also be noted that, while CNR feedback scheme needs 5 bits feedback per subcarrier and a much more complex algorithm to achieve this performance, EPA needs 2 bits as feedback per subcarrier and uses a very simple allocation algorithm. It can be noticed that the performance gap becomes smaller as the number of rate options is increased, as shown in Fig 5. In this case, the rate can be any integer from 1 to 6 bits/symbol. However, increasing the rate options will also increase the feedback requirement of EPA to 3 bits per subcarrier. The gap is wider at lower SNRs than at high SNRs. This can be attributed to the concave nature of rate-SNR curves and the discrete nature of rate options as explained in [4]. Interestingly, DMPA

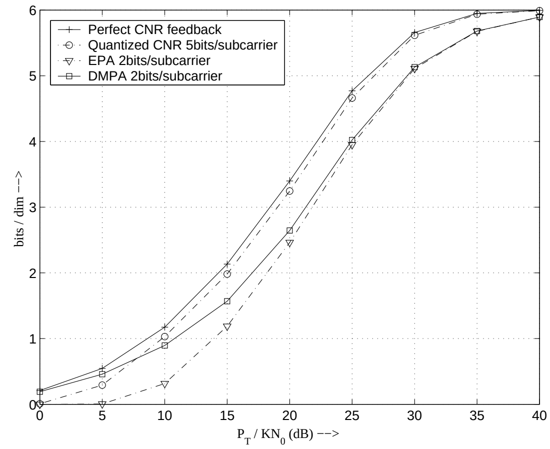

Figure 4: Performance comparison of System 1 and System 2. Constellations used QPSK, 16QAM, 64QAM.

performs better than EPA at low SNRs and DMPA has the same feedback requirement as EPA. This is expected because, DMPA allocates power only to those subcarriers which can support the minimum nonzero rate option, thus utilizing the available power more efficiently. It must be noted that, in multiple user scenarios, allocation has to be performed by the transmitter. So in such cases, CNR feedback based algorithms may be preferred.

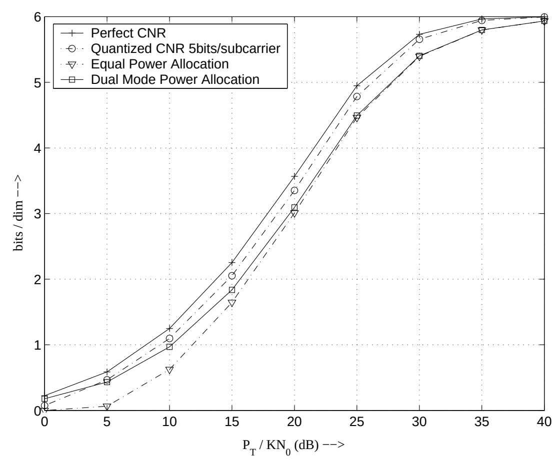

Figure 5: Performance comparison of System 1 and System 2. Constellations used with rates varying from 1 to 6. Notice the decreased performance gap between the different schemes.

FFT-based interpolation and allocation method is simulated for a group size of 4. With a delay spread $4\mu s$, 256 subcarriers, and bandwidth of 2.5 MHz, a cyclic prefix of length 32 is sufficient. Group size of 4 would make number of pilots more than 32 which is sufficient. From the Fig 6, it can be observed that, the FFT based method gives spectral efficiencies as good as the perfect feedback case. This is not surprising since the interpolation would

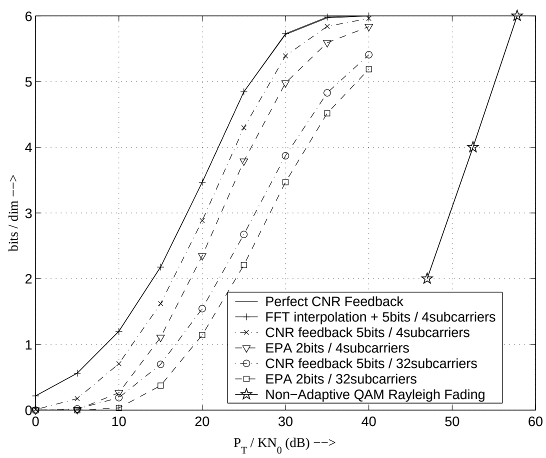

Figure 6: Spectral efficiency comparison for different grouping methods for group sizes 4, 32.

estimate the entire channel accurately, provided the quantization error is negligible and the number of pilots are sufficient. The grouped allocation schemes have spectral efficiencies falling off as the group size is increased. However, it is seen that, even with a group size of 32, the grouped allocation schemes have spectral efficiencies far better than non-adaptive QAM in Rayleigh fading.

7. Conclusion

Adaptive modulation together with OFDM is considered one of the strategies to enhance the data rates in wireless multipath fading channels. We analyzed two different class of systems, wherein, the first system uses a CNR feedback based algorithm at the transmitter to allocate rate and power across subcarriers, while in the second system, allocation is performed at the receiver and the rate allocation vector is fed back to the transmitter. We determined a quantization strategy for the CNR feedback based systems. CNR feedback based systems are seen to perform better by ∼3dB better compared to constant power allocation systems. Also, the performance gap is found to narrow down as the number of quantization levels are decreased or the number or rate options for adaptation is increased. Correlation in adjacent subcarriers can be utilised using interpolation based allocation algorithms or grouped allocation strategies. We find that, even though performance drops down as the group size increases, there is still significant spectral efficiency benefits compared to non-adaptive modulation systems.

8. References

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