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Parametric Channel Estimation in Reuse-1 OFDM Systems

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Abstract— **We propose an improved channel estimator for reuse-1 orthogonal frequency division multiplexing (OFDM) cellular systems[1]. The proposed channel estimation technique exploits delay subspace structure in reducing the interference on channel estimation. The proposed pilot-based channel estimation technique initially estimates the multipath delay locations of both the desired and interference channels. In estimating multipath delays, we assume that the time-of-flight difference between the desired and interfering signals ensures that the multipath delay locations of the corresponding channels are distinct. This information is used to suppress interference in the multipath-delay domain, and define a channel interpolator with a lower normalized mean squared error (NMSE) when compared to the conventional modified least-squares technique (mLS). We also derive the analytical expression for the bit-error-rate of a zero-forcing (ZF) receiver based on the proposed channel estimator. In particular, we show that for uncoded OFDM, the match between the estimated BER and analytical BER is very good and the proposed estimator can outperform mLS by more than a order of magnitude in BER if the interference on the data subcarriers is significantly lower than the interference seen on the pilot subcarriers. Simulation results are also presented with turbo-coded OFDM which further demonstrates the efficacy of the proposed algorithm.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) based future cellular networks, are expected to be universal frequency reuse (reuse-1). Nearly half of the users in a reuse-1 cellular network will typically see signal to interference ratio (SIR) below 0dB on the broadcast pilot subcarriers. Good channel estimation and tracking become very difficult for these interference limited users. By co-operative scheduling of subcarriers across cells or sectors, power control, and precoding, and the SIR seen on the data subcarriers (subchannels) are likely to be higher than the corresponding ratios seen on the pilot subcarriers.

A number of interference rejection techniques for data subcarriers using multiple/single antenna system have been proposed in literature. For example, the interference suppression techniques in [4], [3], [7] assume full channel knowledge at the receiver. Antenna array based interference suppression techniques along with channel estimation have been proposed in [1], [2], [5]. These techniques linearly combine different antenna outputs in order to estimate the transmitted information, and channel estimation plays an important role in defining the combiner. While [1] uses a minimum mean squared error (MMSE) based channel estimator (designed for fading channel without interference as in [18]), a pilot based channel estimation method capable of mitigating synchronous interference has been presented in [2]. This [2] assumes apriori knowledge of the multipath delays of desired and interferer signals in defining a channel estimator. Another pilot based MMSE (minimum mean square error) estimation method for time-varying channels with strong co-channel interference is presented in [5] for DVB-T (digital video broadcasting-terrestrial) systems. The estimator exploits the pseudo-random properties of the pilot sequences on interfering transmitters in mitigating co-channel interference. Since it models the interference components as white noise, it suffers from an irreducible error floor at high signal to noise ratios (SNR). Furthermore, the performance of the channel estimator degrades significantly for non-sample spaced channel models. A co-channel interference suppression technique with a single receiver for coded OFDM system has been discussed [6]. The preamble based channel estimation method used here assumes the channel to be time invariant over the burst and also the knowledge of frequency correlation of the desired channel at the receiver.

A number of parametric channel estimation techniques for OFDM/OFDMA systems have been proposed for sample-spaced and non-sample-spaced channel models [9], [10], [11], [12]. The parametric channel estimation techniques specular channel model where each path is characterized by a delay and a complex gain. In this paper, we propose a channel estimation technique with co-channel interference for a single receiver antenna system. We assume that the multipath delay locations of the desired and interferer channels do not exactly overlap. In other words, even if the power delay profiles are nearly the same, we assume that the time-of-flight difference between the different signals ensures that the multipath locations are distinct. The proposed method estimates the multipath delays of both the desired and interfering channels at the receiver. Exploiting this non-overlapping nature of the multipath locations, we suppress the interferer channel components in multipath-delay domain and also define an improved channel interpolator. This new channel interpolator removes the error floor typically associated with discrete Fourier transform (DFT) based interpolation.

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A. Basic Notation

Bold face letters denote vectors or matrices; $(.)^T, (.)^*, (.)^H$ denote transpose, complex conjugate, Hermitian respectively; $E[.]$ denotes the expectation operator; $\mathcal{CN}(\mathbf{x}, \mathbf{C})$ represent complex Gaussian vector with mean $\mathbf{x}$ and variance $\mathbf{C}$; $\mathbf{I}_K$ denotes the $K \times K$ identity matrix; $\mathbf{0}_{p\times q}$ denotes the matrix of size $p \times q$ with zero entries; $\text{diag}(\mathbf{x})$ is the diagonal matrix with elements of the vector $\mathbf{x}$ on its main diagonal.

II. OFDM SYSTEM MODEL

Consider an OFDM system operating with a bandwidth of $B = \frac{1}{T}$Hz (T is the sampling period). The system consists of K subcarriers of which K_u are useful subcarriers (excluding guard bands and DC subcarrier) with the set $\mathcal{I}$ indicating useful subcarrier positions.

The baseband equivalent of the time-varying channel impulse response is modeled as a wide-sense stationary uncorrelated scatterer (WSSUS) zero mean complex Gaussian process. The channel impulse response has L multipath components where each path is characterized by a complex gain factor h_l and a delay τ_l [9], and has the form $h(\tau, t) = \sum_{l=1}^{L} h_t(l)\delta(\tau - \tau_l)$, where $h_t(m)$ is a zero-mean complex Gaussian random variable with $E[h_t(m)h_t^*(m)] = \sigma_l^2$ and $E[h_t(k)h_t^*(m)] = 0$ for $k \neq m$. The paths fade independently according to the time-correlation function $E\left[h_t(l)h_{t'}(l)\right] = \sigma_l^2 J_0\left(2\pi f_\text{d}(t-t')\right)$ where $J_0(.)$ is the zeroth order Bessel function of the first kind and f_d is Doppler frequency in Hz. The sampled channel in frequency domain is given as $\mathbf{H}_n = \mathbf{F}\mathbf{h}_n$ where $\mathbf{h}_n = [h_n(1), h_n(2), ..., h_n(L)]^T$ and the $(l,k)^{th}$ element of the Fourier basis matrix $\mathbf{F}$ is

$$[\mathbf{F}]_{l,k} = \exp\left(\frac{-j2\pi k\tau_l}{KT}\right) \quad k = 0, 1, ..., K-1, \quad l = 1, 2, ..., L. \tag{1}$$

If the variation in the multipath delay locations over N OFDM symbols is smaller than the resolution of the system ($\frac{1}{T}$), then we can assume delay locations to be constant over N symbols[2](for details refer section 3, [11]). Assuming accurate frequency synchronization and neglecting inter-carrier-interference (ICI), the received signal vector in frequency domain at time n is,

$$\mathbf{Y}_n = \mathbf{X}_n\mathbf{H}_n + \mathbf{V}_n \tag{2}$$

where $\mathbf{X}_n$ is the diagonal matrix with data symbols $[X_{0,n}, X_{1,n}...., X_{K-1,n}]$ and $\mathbf{H}_n$ is the sampled frequency response of the channel at n^{th} OFDM symbol. The zero mean complex Gaussian noise vector $\mathbf{V}_n$ has distribution $\mathbf{V} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_K)$.

Consider a downlink scenario with users at cell edge or sector edge. We restrict our discussion to a single interferer case, even though this method can be extended for multiple interferers as well. The user receives signals from both desired and interfering base stations (BS's). It is assumed that the receiver is synchronized to the desired BS and that all the base stations are frequency and frame synchronized. Let $\{\tau_\text{d}\}, \mathbf{h}_{\text{d},n}, \mathbf{H}_{\text{d},n}$ and $\{\tau_\text{i}\}, \mathbf{h}_{\text{i},n}, \mathbf{H}_{\text{i},n}$ denote the multipath delay locations (with $\tau_{\text{d},1} = 0$), the channel impulse responses and the sampled channel frequency responses of desired and interferer, respectively. Let $L = L_\text{d} + L_\text{i}$ denote the total number of multipath components. We assume that the maximum delay spread of each of the channel (with respect to $\tau_{\text{d},1} = 0$) is smaller than the duration of cyclic prefix. Without loss of generality, we assume the average signal and channel variance is unity i.e., $E\left[|H|^2\right] = 1$ and $E\left[|X|^2\right] = 1$ for both desired and interferer signals. Let K_d represent the number of subcarriers allocated to the desired user, with the set $\mathcal{I}_\text{d}$ indicating their subcarrier indices. The received signal vector in frequency domain on the data subcarriers of the desired user at n^th OFDM symbol is

$$\mathbf{Y}_n = \mathbf{X}_{\text{d},n}\mathbf{H}_{\text{d},n} + \frac{1}{\sqrt{\gamma_\text{d}}}\mathbf{X}_{\text{i},n}\mathbf{H}_{\text{i},n} + \mathbf{V}_n \tag{3}$$

where γ_d represent signal to interference ratio (SIR) on data subcarriers. The signal to noise ratio (SNR) per subcarrier is defined (in dB) as

$$\lambda \triangleq \frac{E\left[|H_\text{d}|^2\right] E\left[|X_\text{d}|^2\right]}{E\left[|V|^2\right]} = \frac{1}{\sigma^2}. \tag{4}$$

The OFDM frame structure is similar to the IEEE 802.16d/e WMAN standard [21] as captured follows. At the start of every OFDM frame, a preamble symbol is transmitted from all base-stations (BS). Assuming BS with a single transmit antenna[3], the three BS's (one per sector) are allocated orthogonal, equi-spaced subcarriers (i.e., preamble symbol is based on 1/3 reuse design). Let K_pre represent the number of subcarriers allocated with the set $\mathcal{I}_\text{pre}$ indicating subcarrier positions for the desired transmitter. The following OFDM symbols contain pilot subcarriers in reuse-1 mode "shared" by all BS's for channel tracking. Let K_pil represents the number of pilots with the set $\mathcal{I}_\text{pil}$ indicating their subcarrier indices. For a fixed number of pilot subcarriers in an OFDM symbol, it has been shown that the mean square error in the channel estimation is minimum if the pilots are *equi-spaced* and *equi-powered* on the frequency grid [13], [14].

III. CHANNEL ESTIMATION

The channel "sounding" is carried out by inserting pilot subcarriers in the time-frequency grid. In conventional channel estimation, the estimates on the pilot subcarriers are then interpolated over data subcarriers. In the presence of interference, since the pilot subcarrier positions of the desired and interferer BS's overlap, the channel measured at the edge of the cell be severely corrupted and cannot be used for data detection. In order to combat interference on pilot carriers, the use of pseudo-random (PR) pilot sequences from the desired and interferer BS's is proposed in [21], [22]. While this gives some

[2]For the system parameters considered in the simulation, we can assume $N \leq 600$

[3]Method is extendable to multiple transmit antennas too, but we do not discuss here for brevity

co-channel interference (CCI) mitigation[4], such schemes fail when interference is strong. In further discussions we refer the above channel estimation technique as PR-mLS.

In this paper, we propose to transmit same pilot sequences from desired and interferer BS's so as to measure the statistics of the combined channel. From the estimated statistics, the proposed method computes the multipath delays of both the desired and interfering channels at the receiver. Exploiting this information, the interferer is suppressed in multipath-delay domain and the improved channel interpolator is defined. The proposed channel estimation method involves :
(1) estimating multipath delays
(2) detecting the desired user multipath delays
(3) interference rejection and channel interpolation.

Considering the pilot subcarrier positions in (3) we have,

$$\mathbf{Y}'_n = \mathbf{X}'_{\mathrm{d},n}\mathbf{H}'_{\mathrm{d},n} + \frac{1}{\sqrt{\gamma_{\mathrm{p}}}}\mathbf{X}'_{\mathrm{i},n}\mathbf{H}'_{\mathrm{i},n} + \mathbf{V}'_n \tag{5}$$

where γ_{p} denotes the SIR on pilot subcarriers.

In order to estimate channel multipath delay locations of desired and interfering channels, the same pilot symbols are sent from both the transmitters, i.e., $\mathbf{X}'_{\mathrm{d},n} = \mathbf{X}'_{\mathrm{i},n}$ with the pilot symbol energy $|X'_{\mathrm{d},n}|^2 = 1$. The least squares (LS) channel estimates on pilot subcarriers [8] are given by

$$\widehat{\mathbf{H}}'_n = \mathbf{X}'^{-1}_{\mathrm{d},n}\mathbf{Y}'_n = \mathbf{H}'_{\mathrm{d},n} + (1/\sqrt{\gamma_{\mathrm{p}}})\mathbf{H}'_{\mathrm{i},n} + \mathbf{X}'^{-1}_{\mathrm{d},n}\mathbf{V}'_n \tag{6}$$

$$= \mathbf{F}'_{\mathrm{d}}\mathbf{h}_{\mathrm{d},n} + (1/\sqrt{\gamma_{\mathrm{p}}})\mathbf{F}'_{\mathrm{i}}\mathbf{h}_{\mathrm{i},n} + \underbrace{\mathbf{X}'^{-1}_{\mathrm{d},n}\mathbf{V}'_n}_{\mathbf{V}''_n \sim \mathcal{CN}(\mathbf{0},\sigma^2\mathbf{I}_{K_{\mathrm{pil}}})} \tag{7}$$

where $\mathbf{F}'_{\mathrm{d}}, \mathbf{F}'_{\mathrm{i}}$ are sub-matrices derived by retaining rows corresponding to pilot subcarrier locations in $\mathbf{F}_{\mathrm{d}}, \mathbf{F}_{\mathrm{i}}$. The matrices $\mathbf{F}_{\mathrm{d}}$ and $\mathbf{F}_{\mathrm{i}}$ are constructed as in (1) with multipath delays $\{\tau_{\mathrm{d}}\}$ and $\{\tau_{\mathrm{i}}\}$, respectively. Since we have assumed the multipath delays of desired and interferer channels do not overlap, the columns of the matrix $[\mathbf{F}_{\mathrm{d}}\ \ \mathbf{F}_{\mathrm{i}}]$ are different.

The first part of the algorithm is estimating multipath delay locations of the channels involved in (6). This equation can be viewed as the output of a sensor array with L far-field narrowband sources. The problem of multipath delay estimation in (6) is equivalent to the direction of arrival (DOA) estimation of different narrow band sources. If the observed signal-space exhibits the required shift invariance property, then the computationally efficient ESPRIT (estimation of signal parameters using rotational invariance method) algorithm can be used for DOA estimation [16]. Indeed, the signal-space spanned by the pilot channel estimates exhibits this shift-invariance property enabling ESPRIT for multipath delay estimation [9][5]. The autocorrelation matrix $\widehat{\mathbf{R}}_{\mathrm{H}}$ of channel estimates is estimated as

$$\widehat{\mathbf{R}}_{\mathrm{H}} = \sum_{n=1}^{M} \widehat{\mathbf{H}}'_n \widehat{\mathbf{H}}'^H_n \tag{8}$$

where M is the number of OFDM symbols considered for averaging. The rate of updation of the auto-correlation matrix can be improved further by exploiting the signal-space structure of pilot channel estimates [17].

The estimated autocorrelation matrix $\widehat{\mathbf{R}}_{\mathrm{H}}$ can be decomposed using eigenvalue decomposition (EVD) as $\widehat{\mathbf{R}}_{\mathrm{H}} = \sum_{m=1}^{K_{\mathrm{pil}}} \widehat{\lambda}_m \widehat{\mathbf{u}}_m \widehat{\mathbf{u}}_m^H$ where $\widehat{\lambda}_1, \widehat{\lambda}_2, ..., \widehat{\lambda}_{K_{\mathrm{pil}}}$ are the eigenvalues arranged such that $\widehat{\lambda}_1 \geq \widehat{\lambda}_2 \geq ... \geq \widehat{\lambda}_{K_{\mathrm{pil}}}$ and $\left[\widehat{\mathbf{u}}_1, \widehat{\mathbf{u}}_2, ..., \widehat{\mathbf{u}}_{K_{\mathrm{pil}}}\right]$ are the corresponding eigenvectors.

A. Estimation of multipath delays

The number of paths are estimated as the number of dominant eigenvalues $(\widehat{L})$ of $\widehat{\mathbf{R}}_{\mathrm{H}}$ as in [15]. The eigenvectors corresponding to $\widehat{L}$ dominant eigenvalues forms the delay-subspace basis $\widehat{\mathbf{U}}_{\mathrm{s}} = \left[\widehat{\mathbf{u}}_1\ \widehat{\mathbf{u}}_2\ ...\ \widehat{\mathbf{u}}_{\widehat{L}}\right]$. Let $\widehat{\mathbf{U}}_1$ and $\widehat{\mathbf{U}}_2$ represent the shift invariant subspaces derived from $\widehat{\mathbf{U}}_{\mathrm{s}}$. The placement of zero-tones (guard carriers) decides the choice of $\widehat{\mathbf{U}}_1$ and $\widehat{\mathbf{U}}_2$ from $\widehat{\mathbf{U}}_{\mathrm{s}}$. Exploiting the shift invariance structure of the subspaces $\widehat{\mathbf{U}}_1$ and $\widehat{\mathbf{U}}_2$, the multipath delays are estimated as [16] $\widehat{\tau}_l = \arg(\phi_l^*)KT/(2\pi P)$ for $l = 1, 2, ..., \widehat{L}$ where P denote the shift (in subcarriers) between the subspaces $\widehat{\mathbf{U}}_1$ and $\widehat{\mathbf{U}}_2$, $\arg(\phi_l^*)$ denotes the phase angle (in the range $[0, 2\pi)$) of ϕ_l^* and $\{\phi_l\}_{l=1}^{l=\widehat{L}}$ are the eigenvalues of the matrix $\psi = (\widehat{\mathbf{U}}_1^H\widehat{\mathbf{U}}_1)^{-1}\widehat{\mathbf{U}}_1^H\widehat{\mathbf{U}}_2$.

The next stage of the algorithm is separating multipath delays corresponding to the desired and interfering channels. The preamble being $1/3$ reuse, it is possible to estimate channel frequency response without interference on a set of carriers with indices $\mathcal{I}_{\mathrm{pre}}$. The "interference free" channel estimates derived from the preamble corresponding to the desired BS are projected onto the combined basis derived from the estimated multipath delays. The contribution along different basis vectors are then used to separate the multipath delays corresponding to the desired channel.

Let $\widehat{\mathbf{H}}^{\mathrm{p}}$, a $K_{\mathrm{pre}} \times 1$ vector, represent the "interference free" channel estimates corresponding to desired transmitter derived from the preamble symbol. The interference free channel estimates $\widehat{\mathbf{H}}^{\mathrm{p}}$ are projected onto the combined Fourier basis derived from the estimated multipath delays as,

$$\widehat{\mathbf{h}} = (\widetilde{\mathbf{F}}_{\mathrm{p}}^H\widetilde{\mathbf{F}}_{\mathrm{p}})^{-1}\widetilde{\mathbf{F}}_{\mathrm{p}}^H\widehat{\mathbf{H}}^{\mathrm{p}} \tag{9}$$

where $\widetilde{\mathbf{F}}_{\mathrm{p}}$ is $K_{\mathrm{pre}} \times \widehat{L}$ matrix with $[\widetilde{\mathbf{F}}_{\mathrm{p}}]_{k,l} = \exp\left(-\frac{j2\pi\widehat{\tau}_l\mathcal{I}_{\mathrm{pre}}(k)}{KT}\right)$ for $k = 1, 2, ..., K_{\mathrm{pre}}$ and $l = 1, 2, ..., \widehat{L}$. The vector $\widehat{\mathbf{h}}$ represents the components of $\widehat{\mathbf{H}}^{\mathrm{p}}$ along different basis-vectors associated with multipath delays. The projections are then compared with scaled noise variance $\alpha\sigma^2$ and the components that exceed the scaled noise variance are counted as dominant. Let $\mathcal{T}_{\mathrm{d}}$ (a $\widehat{L}_{\mathrm{d}} \times 1$ vector) denote the indices of multipath delays corresponding to dominant projections.

B. Interference rejection and channel interpolation

Once the multipath delay locations of the desired channel are known, the delay-domain channel gains are estimated by

[4]CCI mitigation on channel estimates is possible provided the CCI is weak and/or the number of pilots is much larger than channel impulse response length

[5]The presence of zero-tones should be accounted in deriving shift invariant subspaces from the signal-space of the channel estimates

projecting the pilot channel estimates onto the Fourier basis as follows:

$$\widehat{\mathbf{h}}_{\mathrm{c},n} = (\widetilde{\mathbf{F}}_{\mathrm{c}}'^H \widetilde{\mathbf{F}}_{\mathrm{c}}')^{-1} \widetilde{\mathbf{F}}_{\mathrm{c}}'^H \widehat{\mathbf{H}}_n' \tag{10}$$

where $\widetilde{\mathbf{F}}_{\mathrm{c}}'$ is $K_{\mathrm{pil}} \times \widehat{L}$ matrix with $[\widetilde{\mathbf{F}}_{\mathrm{c}}']_{k,l} = \exp\left(-\frac{j2\pi\widehat{\tau}_l \mathcal{I}_{\mathrm{pil}}(k)}{KT}\right)$ for $k = 1, 2, ..., K_{\mathrm{pil}}$ and $l = 1, 2, ..., \widehat{L}$. The desired channel impulse response estimates are the components of $\widehat{\mathbf{h}}_{\mathrm{c},n}$ with indices $\mathcal{T}_{\mathrm{d}}$ i.e., $\widehat{\mathbf{h}}_{\mathrm{d},n} = \widehat{\mathbf{h}}_{\mathrm{c},n}(\mathcal{T}_{\mathrm{d}})$. Finally, we derive frequency domain channel estimates on the subcarriers of the desired user as

$$\widehat{\mathbf{H}}_{\mathrm{d},n} = \widetilde{\mathbf{F}}_{\mathrm{d}} \widehat{\mathbf{h}}_{\mathrm{d},n} \tag{11}$$

where $\widetilde{\mathbf{F}}_{\mathrm{d}}$ is a $K_{\mathrm{d}} \times \widehat{L}_{\mathrm{d}}$ matrix with $[\widetilde{\mathbf{F}}_{\mathrm{d}}]_{k,l} = \exp\left(-\frac{j2\pi\widehat{\tau}_l \mathcal{I}_{\mathrm{d}}(k)}{KT}\right)$ for $k = 1, 2, ..., K_{\mathrm{d}}$ and $l \in \mathcal{T}_{\mathrm{d}}$. The channel estimates $\widehat{\mathbf{H}}_{\mathrm{d},n}$ can be further improved by exploiting the time correlation of temporal channel estimates $\widehat{\mathbf{h}}_{\mathrm{d},n}$ due to Doppler using a FIR (finite impulse response) filter. The co-efficients of the FIR filter are derived using Weiner filter theory [18]. However, in this paper we have not exploited channel time correlation information but have focused only on the interference mitigation aspect.

IV. BER ANALYSIS

In this section, we measure the performance of the proposed estimator by evaluating bit error rate (BER) with Gray coding. The BER is analyzed for the case of strong co-channel interference on pilot subcarriers and zero interference on data subcarriers ($\gamma_{\mathrm{d}} = \infty$).

The effect of channel estimation error on the BER performance of OFDM systems in Rayleigh fading channels has been studied in [19]. The analysis yields a closed form expression for BER for constellations with Gray coding. The BER is derived for a general case where the channel estimates on data subcarriers is a linear function of pilot channel estimates. The channel estimate on a k^{th} data subcarrier is derived as,

$$\widehat{H}_k = \mathbf{a}_k^H \widehat{\mathbf{H}}' \tag{12}$$

where $\mathbf{a}_k^H$ is the linear combiner and $\widehat{\mathbf{H}}'$ is the channel estimates on pilot subcarriers. The expression for BER involves statistics of the channel estimate $\widehat{H}_k$ and the true channel response H_k on k^{th} data subcarrier. Define

$$\sigma_{\mathrm{H},k}^2 \triangleq \frac{1}{2} E[|H_k|^2], \quad \sigma_{\widehat{\mathrm{H}},k}^2 \triangleq \frac{1}{2} E[|\widehat{H}_k|^2], \tag{13}$$

where $E[|H_k|^2]$ and $E[|\widehat{H}_k|^2]$ represent the variance of the actual and estimated channel responses on k^{th} subcarrier, respectively. The cross-correlation between actual and estimated channel and the corresponding correlation coefficients are defined as

$$\mu_{1,k} + j\mu_{2,k} \triangleq \frac{1}{2} E[\widehat{H}_k H_k^*], \tag{14}$$

$$\kappa_{1,k} \triangleq \frac{\mu_{1,k}}{\sigma_{\mathrm{H},k}\sigma_{\widehat{\mathrm{H}},k}}, \quad \kappa_{2,k} \triangleq \frac{\mu_{2,k}}{\sigma_{\mathrm{H},k}\sigma_{\widehat{\mathrm{H}},k}}. \tag{15}$$

The closed form expression for BER (with QPSK modulation) on k^{th} subcarrier is given by

$$P_{\mathrm{b}}(k) = \frac{1}{2}\left[1 - \frac{C_1}{2} - \frac{C_2}{2}\right] \tag{16}$$

where

$$C_1 = \frac{\frac{(\kappa_{1,k}+\kappa_{2,k})}{\sqrt{2}}}{\sqrt{1 + \frac{1}{\lambda} - \frac{(\kappa_{1,k}-\kappa_{2,k})^2}{2}}}; \quad C_2 = \frac{\frac{(\kappa_{1,k}-\kappa_{2,k})}{\sqrt{2}}}{\sqrt{1 + \frac{1}{\lambda} - \frac{(\kappa_{1,k}+\kappa_{2,k})^2}{2}}}.$$

Finally the overall BER is given by

$$P_{\mathrm{b}} = \frac{1}{K_{\mathrm{d}}} \sum_{k \in \mathcal{I}_{\mathrm{d}}} P_{\mathrm{b}}(k) \tag{17}$$

A. BER for the proposed subspace based method

Assuming the multipath delays are estimated and tagged without any error, the linear combiner for k^{th} subcarrier is given by

$$\mathbf{a}_k^H = \mathbf{e}_k^T \mathbf{F}_{\mathrm{d}} \mathbf{P} (\mathbf{F}_{\mathrm{c}}'^H \mathbf{F}_{\mathrm{c}}')^{-1} \mathbf{F}_{\mathrm{c}}'^H \tag{18}$$

where $\mathbf{F}_{\mathrm{c}}' = [\mathbf{F}_{\mathrm{d}}' \; \mathbf{F}_{\mathrm{i}}']$ and $\mathbf{e}_k$ denotes the column vector with K elements whose k^{th} entry is 1 and other entries are 0. The selection matrix $\mathbf{P} = [\mathbf{I}_{L_{\mathrm{d}}} \; \mathbf{0}_{L_{\mathrm{d}} \times L_{\mathrm{i}}}]$ selects the CIR components corresponding to desired channel. For the given linear combiner, the channel statistics are given by

$$\sigma_{\mathrm{H},k}^2 = \frac{1}{2} \sum_{l=0}^{L_{\mathrm{d}}-1} \sigma_l^2, \quad \sigma_{\widehat{\mathrm{H}},k}^2 = \frac{1}{2}\left(\sum_{l=0}^{L_{\mathrm{d}}-1} \sigma_l^2 + \sigma^2 \mathbf{a}_k^H \mathbf{a}_k\right) \tag{19}$$

$$\mu_{1,k} = \frac{1}{2} \sum_{l=0}^{L_{\mathrm{d}}-1} \sigma_l^2 \quad \text{and} \quad \mu_{2,k} = 0 \tag{20}$$

Substituting (19) and (20) into (15) we evaluate the BER as in (17). Let $P_{\mathrm{b,ss}}$ denote the BER evaluated for the proposed subspace based method. Observe that $P_{\mathrm{b,ss}}$ is only a function of power delay profile of the desired channel.

B. BER for PR-mLS based method

In the case of PR-mLS based method the LS channel estimates on pilots are given by

$$\widehat{\mathbf{H}}' = \mathbf{H}_{\mathrm{d}}' + (1/\sqrt{\gamma_{\mathrm{p}}}) \mathbf{X}_{\mathrm{d}}'^{-1} \mathbf{X}_{\mathrm{i}}' \mathbf{H}_{\mathrm{i}}' + \mathbf{X}_{\mathrm{d}}'^{-1} \mathbf{V}'. \tag{21}$$

The linear combiner for the PR-mLS method is given as

$$\mathbf{a}_k^H = \mathbf{e}_k^T \mathbf{F}_{\mathrm{cp}} (\mathbf{F}_{\mathrm{cp}}'^H \mathbf{F}_{\mathrm{cp}}')^{-1} \mathbf{F}_{\mathrm{cp}}'^H \tag{22}$$

where $\mathbf{F}_{\mathrm{cp}}$ is the modified matrix obtained from the K-point DFT matrix by selecting first L_{cp} columns and $\mathbf{F}_{\mathrm{cp}}'$ is the modified $\mathbf{F}_{\mathrm{cp}}$ matrix by selecting the rows corresponding to pilot indices $\mathcal{I}_{\mathrm{pil}}$. For the given linear combiner the channel statistics are given by

$$\sigma_{\mathrm{H},k}^2 = \frac{1}{2} \sum_{l=0}^{L_{\mathrm{d}}-1} \sigma_l^2 \tag{23}$$

$$\sigma_{\widehat{\mathrm{H}},k}^2 = \frac{1}{2} \mathbf{a}_k^H \left(\mathbf{R}_{\mathrm{d}} + (1/\gamma_{\mathrm{p}}) \mathbf{X}_{\mathrm{d}}'^{-1} \mathbf{X}_{\mathrm{i}}' \mathbf{R}_{\mathrm{i}} \mathbf{X}_{\mathrm{i}}'^H \mathbf{X}_{\mathrm{d}}'^{-H} + \sigma^2 \mathbf{I}_{K_{\mathrm{pil}}}\right) \mathbf{a}_k \tag{24}$$

and the cross-correlation term is given as

$$\mu_{1,k} + j\mu_{2,k} = \frac{1}{2}\left(\mathbf{a}_k^H \mathbf{r}_k\right) \quad (25)$$

where $\mathbf{R}_\mathrm{d} = \mathbf{F}'_\mathrm{d}\left(E\left[\mathbf{h}_\mathrm{d}\mathbf{h}_\mathrm{d}^H\right]\right)\mathbf{F}'^H_\mathrm{d}, \mathbf{R}_\mathrm{i} = \mathbf{F}'_\mathrm{i}\left(E\left[\mathbf{h}_\mathrm{i}\mathbf{h}_\mathrm{i}^H\right]\right)\mathbf{F}'^H_\mathrm{i}$ and $\mathbf{r}_k$ is the k^th column of $\mathbf{F}'_\mathrm{d}\left(E\left[\mathbf{h}_\mathrm{d}\mathbf{h}_\mathrm{d}^H\right]\right)\mathbf{F}^H_\mathrm{d}$. Substituting (23), (24) and (25) into (15) we evaluate the BER as in (17). Let $P_\mathrm{b,mls}$ denote the BER evaluated for the PR-mLS method. Observe that the $P_\mathrm{b,mls}$ is a function of power delay profile of both desired and interferer channels.

V. Analytical and Simulated NMSE, BER Results

We demonstrate the performance of the proposed estimator by computer simulations. An OFDM system is simulated with following parameters [21]: center frequency $f_\mathrm{c} = 2.2$GHz, bandwidth $B = \frac{1}{T} = 5$MHz, total number of subcarriers used $K = 512$, number of useful subcarriers $K_\mathrm{u} = 420$, length of cyclic prefix $L_\mathrm{cp} = 32$, OFDM symbol duration $T_\mathrm{s} = (K + L_\mathrm{cp})T$. Each transmitter is allocated $K_\mathrm{pre} = \frac{420}{3} = 140$ non-overlapping equi-spaced subcarriers over the preamble symbol and the subsequent data symbols are allocated $K_\mathrm{pil} = 26$ pilots (with reuse-1). The scalar α is set to 10. A OFDM burst consists of 5 frames and each frame consists of 100 OFDM symbols. A preamble symbol is transmitted at the start of every frame.

We have considered a 4 tap channel model for the desired and interferer signals. The multipath delays are generated with a minimum spacing of $0.5T$ between any two paths. The maximum delay difference between the desired signal and the interferer signal is within the duration of cyclic prefix. Since the auto-correlation matrix is estimated with finite averaging the ability of the ESPRIT algorithm to resolve closely spaced paths is limited. This is the only reason why we limit the minimum spacing between any two paths to $0.5T$. The power delay profile (pdp) of the channel is assumed to be uniform. The channel paths fade independently according to Jakes' power spectrum [20]. The normalized fade rate is $f_\mathrm{d}T_\mathrm{s} = 0.004$ (vehicular speed = 10m/s).

The performance of the algorithm is evaluated using normalized mean square error (NMSE) and bit error rate (BER) by averaging over channels with different multipath delay locations. In evaluating BER results, we consider both raw and coded OFDM system. For the coded OFDM system, the input bits are encoded with rate 1/2 parallel concatenated convolutional codes (turbo-codes) and then modulated using QPSK scheme. The synchronous interferer signal is also assumed to be QPSK modulated. The NMSE is defined as (for some n) NMSE $= \frac{E[||\mathbf{H}_{\mathrm{d},n} - \widehat{\mathbf{H}}_{\mathrm{d},n}||^2]}{E[||\mathbf{H}_{\mathrm{d},n}||^2]}$.

Fig. 1 compares the NMSE performance of proposed subspace (SS) method and pseudo-random (PR-mLS) pilot data based channel estimation methods with co-channel interference with pilot SIR (γ_p) = 0dB, 6dB. The NMSE is measured at the end of $n = 60$ OFDM symbol. The performance of PR-mLS based method is evaluated based on channel interpolation method as explained in section IV-B. It is observed that the PR-mLS based channel estimation method fails for severe

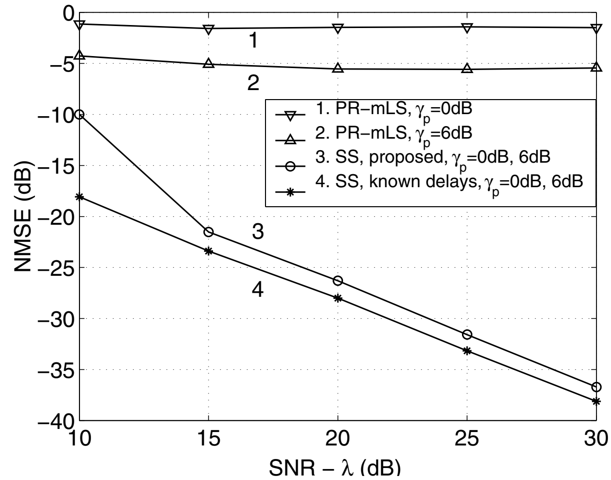

Fig. 1. NMSE convergence plot comparing proposed subspace method (SS) with PR-mLS based method for different γ_p at $n = 60$

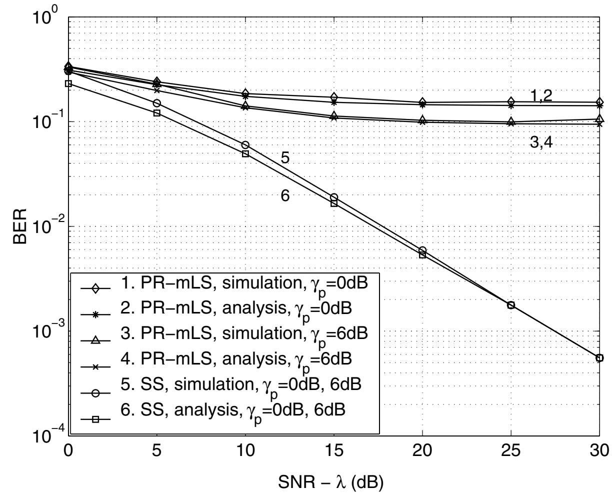

Fig. 2. Raw BER performance comparing proposed subspace method (SS) with PR-mLS based method for different γ_p

CCI and suffers from an irreducible error floor. The plot shows a significant improvement in the performance of the subspace-based channel estimation technique. The subspace based estimates the multipath delay locations of the desired and interferer channel and projects the pilot channel estimates onto the basis derived from the multipath delay information. Since the multipath delay locations of the desired and the interferer channel are assumed to be different, high quality channel estimation is possible. We have also plotted the NMSE performance of subspace based estimator with ideal multipath delay information as a reference for the proposed channel estimator.

The fig. 2 shows the BER performance of an uncoded OFDM system for different pilot SIRs ($\gamma_\mathrm{p} = 0$dB, 6dB). The data is assumed to be interference free. As expected, the BER for the proposed method decreases with increase in SNR where as the BER saturates for PR-mLS. Also observe that

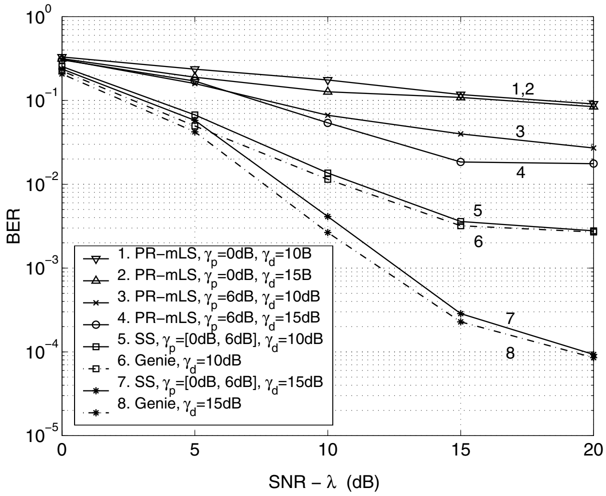

Fig. 3. Coded BER performance comparing proposed subspace method (SS) with PR-mLS based method and the Genie receiver (full channel knowledge) for different $\gamma_{\rm p}$ and $\gamma_{\rm d}$

the performance of the PR-mLS based estimator improves with the increase in the pilot SIR ($\gamma_{\rm p}$). Note that the BER performance of the proposed subspace based method is not a function of pilot SIR, $\gamma_{\rm p}$ (as discussed in BER analysis). The analytically derived BER ($P_{\rm b,ss}, P_{\rm b,mls}$) are also plotted for reference. Observe the close match between the simulated results and analysis.

Fig. 3 compare the performance of channel estimation techniques for a coded system for different data and pilot SIRs. It is clear from the plot that the proposed subspace based method (SS) outperforms the PR-mLS based method at all SNRs. We have also plotted the BER performance with ideal channel information (Genie receiver), which indicates that the proposed method is very close to the ideal channel knowledge.

VI. Conclusion

We presented a subspace based channel estimation method for cellular reuse-1 OFDM systems with strong co-channel interference. In such reuse-1 systems, the pilot subcarrier positions of the desired and interferer BS overlap which severely corrupts the pilot channel estimates especially for the users at the sector or cell edge. The use of pseudo-random pilot sequences for interference suppression [5], [21] results in an irreducible channel estimation error.

The proposed subspace based interference rejection method suppress the interferer in the multipath-delay domain. This method assumes that the multipath delay locations of the desired and interferer channels do not exactly overlap. In the paper the multipath delays are generated with a minimum spacing of $0.5T$ since the accuracy of the delay estimation algorithm (ESPRIT) is limited by extent of auto-correlation averaging. Analytical and simulated BER results indicate a significant improvement in the performance of the channel estimator over the conventional channel estimators for single receiver OFDM system. The proposed algorithm can be extended to OFDM systems operating with multiple transmit and receiver antennas.

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