Partially Blind Multiuser Detection

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Abstract— **Multiple access interference (MAI) is a significant limiting factor in the performance of direct sequence code division multiple access (DS-CDMA) systems. Various multiuser detection techniques have been developed to combat the effects of MAI. These detection techniques either assume the knowledge of all the users in the system (conventional) or assume the knowledge of the user of interest only (blind). Due to the limitations of the blind algorithms in the presence of a large number of interferers, there is a significant performance gap between these two classes of detectors. Additionally, in practice, the receiver could have only partial knowledge of the interference. In this paper, we develop a new class of detectors, partially blind multiuser detectors, that use information about a subset of interferers and bridge the performance gap between the blind and the conventional multiuser detectors.**

I. Introduction

Direct sequence code-division multiple-access (DS-CDMA) has emerged as the leading multiple access scheme for wireless communication. The idea of CDMA is to multiplex users using distinct codes, rather than the traditional methods of using distinct frequency or time intervals. CDMA systems have been shown to provide better channel utilization which can lead to support of larger number of users using limited resources. The most significant limiting factor of user capacity for the conventional DS-CDMA system is multiple access interference (MAI). This interference results from the unavoidable non-zero cross-correlations that exist between active DS-CDMA users. Conventional single-user detection ignores this interference and thus suffers from performance degradation. Multiuser detection schemes have been developed to mitigate the interference and thereby design a more efficient wireless communication system [8], [5].

Modern wireless communication systems have been designed to be inherently cellular. Each cell has an associated base-station and each user is assigned to the nearest base station. Unfortunately the actual transmitted signal of a particular user is not directional and can reach the neighboring base-stations, even though each user's signal is meant only for its associated base-station. Thus the base station receives signals not only from the users for which it is responsible but also users from those in the neighboring cells. In other words the interference to a particular user's signal consists of in-cell interference and out-of-cell interference. Actual field measurements [9] show that almost 40% of the interference come from out-of-cell users and cannot be ignored.

Most of the multiuser detection schemes assume the knowledge of the spreading codes and channel parameters of the users that contribute to the received signal and these receivers exploit this knowledge to combat MAI. However, it is fair to assume that a particular base-station will only be aware of the codes of those users for which it is responsible which constitute a fraction of all the interfering users. A second form of detectors, known as the blind detectors [1], [10], have also been designed which work only with the knowledge of the spreading codes and channel parameters of the user of concern. These detectors try to estimate background interference and recover from it. Though this restriction may very well suit a hand-set it is pessimistic for a base-station design, where we are not utilizing the entire information at hand.

Thus, there is a need to design a detector which does not assume the knowledge of all interfering users, but knows information only about a subset of the interfering users. In this paper, we will show how we can successfully exploit the partial knowledge of the interfering users and design what we call partially blind detectors.[1] We will also quantitatively evaluate the performance loss in a blind detector as compared to the traditional multiuser detectors.

II. System model

We will use a synchronous baseband DS-CDMA system model which uses short codes of spreading gain N_c. The spreading sequence of the k^{th} user is given by s_k ($\|s_k\|^2 = 1$) and it extends over the sym-

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[1]Detectors with partial knowledge of the interfering users have also been independently studied in [2], [3].

bol period [0,T). We will assume that each user sends antipodal information bits of block length N and that the received amplitude of the k^{th} user is given by A_k. Thus the contribution of the k^{th} user due to the n^{th} bit $b_k(n)$ is given by

$$A_k s_k(t - nT) b_k(n).$$

In our system we model three type of users: c the user of concern, the set K of in-cell interfering users and the set U of out-of-cell interfering users, i.e., there are $N(=| [\{c\} \cup K \cup U] |)$ users in the system. Therefore, the discretized chip matched filtered version of the received signal for the n^{th} bit interval can be expressed as

$$r(n) = S_c A_c b_c(n) + S_K A_K b_K(n) + S_U A_U b_U(n) + \eta(n), \quad (1)$$

where η is the zero-mean additive white Gaussian noise with variance σ^2. The following notational conventions are used in this paper. The subscripts of the matrices A and S denote the subset of users whose amplitudes and spreading codes are arranged in matrix format. The absence of a subscript would denote that the matrices are composed of the amplitudes and spreading codes of all the users in the system. The matrix I will denote the identity matrix and its rank would be given by its subscript.

In the subsequent sections we will base our discussions on this synchronous system model and describe how the various algorithms try to extract the information bits $b_c(n)$ from this received signal. We will start with a discussion of the minimum mean squared error (MMSE) multiuser detectors.

III. MMSE and blind MMSE detectors

A linear detector for user c is characterized by the linear transformation $f_c \in L_2[0,T)$, such that the decision on b_c is based on the statistics $\hat{b}_c = \mathrm{sgn}(\langle f_c, r \rangle)$. The linear MMSE detector is characterized by the filter

$$f_c = \min E[(A_c b_c - \langle f_c, r \rangle)^2].$$

It has been shown [11], [6] that this filter is described by the c^{th} row of the matrix

$$F_{MMSE} = [R + \sigma^2 A^{-2}]^{-1} S^\top, \quad (2)$$

where S is the matrix composed of the signature waveforms of all the users in the system, $R = S^\top S$ is the code correlation matrix and A is a diagonal matrix with nonzero elements of the received amplitudes of all the users. It is evident from the above expression that the linear MMSE receiver requires the knowledge of the received amplitudes and signature waveforms of *all* the users that contribute to the received signal r. However, in a practical system it is fair to assume that the base-station receiver would be aware of the signature waveforms of the in-cell users (c and the set of users K) only. Thus a full scale linear MMSE receiver may not be feasible.

Blind MMSE detectors are designed for those systems where the signature waveforms of the interfering users may not be known. In these receivers it is assumed that the signature waveform of the user of concern c is available. It should be noted that the conventional matched filter receiver $F_{MF} = S_c^\top$ is the simplest form of blind receiver. In this receiver the contribution of the interfering users $K \cup U$ are assumed to be additive white Gaussian. The blind MMSE receivers try to estimate the colored structure of the interfering users and exploit this knowledge. Even though the original blind MMSE receiver was proposed as the minimum output energy receiver [1], we will discuss the blind receiver proposed in [10] in this paper.

For the sake of completeness we will give a brief exposition to the blind MMSE receiver. For our analysis we will make the simplifying assumption that all users are of equal power. The blind MMSE receiver for the user c is given by

$$F_{blind} = S_c^\top [SS^\top + \sigma^2 I_{N_c}]^{-1}. \quad (3)$$

We will first give a brief proof to show that this blind MMSE detector is exactly equivalent to the original MMSE receiver and as has been argued by the authors in [1], [10], this receiver can be designed asymptotically.

A. *Proof of equivalence of the two MMSE expressions*

From equation (2), we get

$$\begin{aligned} [S^\top S + \sigma^2 I_N]^{-1} S^\top r &= b_{MMSE} \\ S^\top r &= [S^\top S + \sigma^2 I_N] b_{MMSE}. \end{aligned}$$

We can write the blind MMSE detector (from equation (3)) for all the N users as

$$S^\top \underbrace{[SS^\top + \sigma^2 I_{N_c}]^{-1} r}_{x} = b_{blind}.$$

From this equation we get that $b_{blind} = S^\top x$, where $[SS^\top + \sigma^2 I_{N_c}]x = r$. Now,

$$\begin{aligned} [S^\top S + \sigma^2 I_N] b_{blind} &= [S^\top S + \sigma^2 I_N] S^\top x \\ &= S^\top S S^\top x + S^\top \sigma^2 x \\ &= S^\top [SS^\top + \sigma^2 I_{N_c}] x \\ &= S^\top r \\ &= [S^\top S + \sigma^2 I_N] b_{MMSE}. \end{aligned}$$

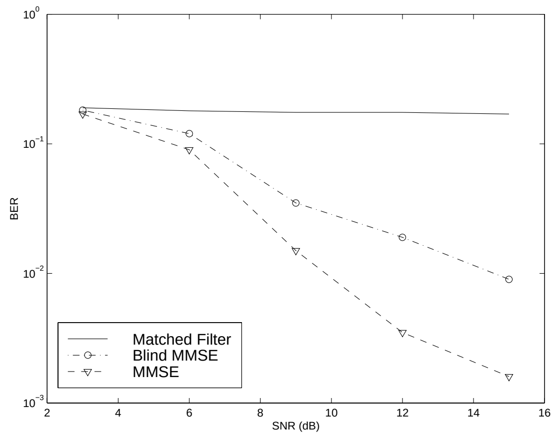

Fig. 1. Performance comparison of blind MMSE and conventional MMSE receivers – number of users = 24, all users have equal power

Therefore, b_{MMSE} and b_{blind} are equivalent. The blind MMSE receiver estimates the covariance, C, given by $[SS^\top + \sigma^2 I_{N_c}]$. Although the estimate converges to the actual C asymptotically, it is never achieved in practice. One simple way to estimate the noise covariance is to calculate $\mathrm{E}(rr^\top)$ over a long period of time. Simple algebraic manipulation (along with the assumption that all the user bits are independent of each other as well as the noise) shows that asymptotically (with infinite observations of r) $\mathrm{E}(rr^\top)$ actually estimates the covariance. Thus, asymptotically, the blind MMSE receiver should perform as well as the conventional MMSE receiver. Unfortunately, in a practical system, where the system parameters vary with time and the estimation needs to be done in finite time, the performance of the blind MMSE never reaches that of the conventional MMSE receiver. In fact, the performance loss in terms of bit error rate (BER), as shown in figure 1, is significant. Our paper aims to design receivers which can narrow this performance gap.

IV. Partially Blind Multiuser Detection

In the previous section, we described two type of detectors: the conventional MMSE detector, which requires the knowledge of the signature waveforms of all the users, and the blind MMSE detector, which works under the assumption that only the signature waveform of the user of concern is known. Unfortunately, a practical system lies in between these two extremes. The first detector is too optimistic as we cannot expect to know the signature waveforms of all the users, while, the latter, under-utilizes our knowledge of the system. In this section, we will develop partially blind multiuser detectors that bridge the performance gap between the conventional MMSE and blind MMSE detectors.

A. Error analysis of the MMSE detectors

A simple reduction of the conventional MMSE algorithm to suit our case (of partial knowledge) would be to use the knowledge of all in-cell users $\{c\} \cup K$ while treating all the out of cell users U as noise. For our notational convenience we will call this detector, which uses the information of in-cell users and treats out of cell users as noise, the *crippled MMSE* detector. As we mentioned earlier, the signature waveform matrix S is composed of the signature waveforms S_{cK} of the in-cell users and S_U, the signature waveforms of out of cell users. The crippled MMSE detector estimates the information bits of the in-cell users as

$$b_{cripp} = [R_{cK} + \sigma^2 I_{cK}]^{-1} S_{cK}^\top r, \tag{4}$$

where $R_{cK} = S_{cK}^\top S_{cK}$. Now, we can rewrite the ideal MMSE estimate as

$$[S^\top S + \sigma^2 I_N] b_{MMSE} = S^\top r.$$

Separating the contribution of in-cell users and out of cell users, we can rewrite the above equation as

$$\begin{bmatrix} R_{cK} & \rho \\ \rho^\top & R_U \end{bmatrix} + \begin{bmatrix} I_{cK} & 0 \\ 0 & I_U \end{bmatrix} \begin{bmatrix} b_{cK} \\ b_U \end{bmatrix} = \begin{bmatrix} S_{cK}^\top \\ S_U^\top \end{bmatrix} r$$

It can be seen that the correct equation

$$[R_{cK} + \sigma^2 I_{cK}] b_{cK} = S_{cK}^\top r - \rho b_U$$

to solve for the bits of the in-cell users, is incorrectly modeled by equation (4) by the crippled MMSE detector. Thus, the error $err_{cripp} = b_{cripp} - b_{cK}$ is given by

$$[R_{cK} + \sigma^2 I_{cK}] err_{cripp} = \rho b_U.$$

Thus, it is evident the more the number of out-of-cell users the more is the error in the estimate.

On the other hand for a blind MMSE system we are estimating the noise covariance matrix $C = [SS^\top + \sigma^2 I_{N_c}]$ by $\hat{C} = \mathrm{E}(rr^\top)$. The error can be given by

$$err_{blind} = S_{cK}^\top [\hat{C}^{-1} - C^{-1}] r$$

Unlike the crippled MMSE receiver, a closed form analytic expression of this error in terms of number of out-of-cell users is not possible. So we resort to simulation studies to compare the two errors.

Our simulation results in figure 2 show that for low out-of-cell interference the crippled MMSE performs better than the blind MMSE, while the situation is reversed for high out-of-cell interference.

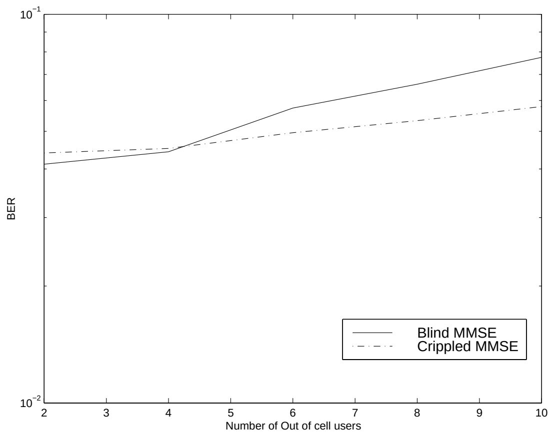

Fig. 2. Performance of Blind MMSE and Crippled MMSE receivers – number of in-cell users = 10, all users have equal power

B. *Sensitivity study of blind MMSE with interference*

Our previous result not only shows the tradeoff between the crippled MMSE and the blind MMSE receivers but also suggests that the performance of the blind MMSE receiver varies with the level of out-of-cell interference or, in general, the background interference. We performed some simulations to study the sensitivity of the blind MMSE receiver with the number of interfering users. We simulated a system with varying number of interfering users all of whose signature waveforms are unknown. Our results show

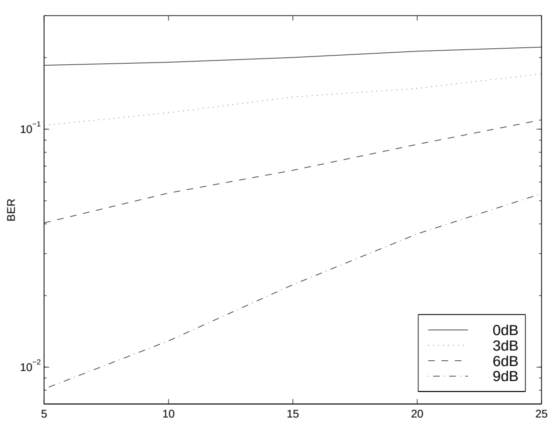

Fig. 3. Sensitivity of blind MMSE algorithm to background interference, all users have equal power.

that at 9dB signal to noise ratio (SNR) there is a significant gap in the bit error rate between a system with 10 interfering users and a system with 25 interfering users. In other words, if we can reduce the level of background interference, we can hope to significantly improve the performance of the blind MMSE algorithm.

C. *Design of partially blind multiuser detector*

In this section, we will describe ways of using the knowledge of the in-cell interferers to reduce interference and improve the performance of the blind detectors. We will describe two different partially blind detectors that can be employed, depending on the availability of amplitude information of the in-cell interferers.

C.1 Multistage Interference Cancellation

When the amplitudes of in-cell users are known along with their spreading codes, several interference cancellation techniques [7], [4] that have been exploited in the design of conventional multiuser receivers can be applied. The basic idea behind these interference cancellation algorithms is to estimate the information bits of interfering users and then, using their known signature waveforms and amplitudes, remove their contribution from the received signal to obtain a cleaner signal for the user of concern. In our system, if we estimate the information bits of the in-cell interferers, we can use the knowledge of their signature waveforms and amplitudes to obtain a cleaner signal for the user of concern c and then use some matched filtering technique to estimate b_c. Thus for a simple multistage algorithm would estimate

$$\hat{b}_c = S_c^\top (r - S_K \hat{b}_K).$$

However, even if we completely eliminate the contribution of the in-cell interferers K, we still have to deal with the out-of cell interference since it will limit the performance of the receiver.

If we can successfully eliminate the interference of the in-cell receiver, the new signal $\tilde{r} = (r - S_K \hat{b}_K)$ will have less background interference than the original signal r. From our earlier study, we have seen that the performance of the blind MMSE receiver improves if we have less background interference. This is the motivation behind the partially blind (*pblind*) receiver.

We should however note that in order to reduce the interference level we must feed back *correct* estimates of b_K. If the estimates are incorrect, instead of reducing the interference, we might actually increase the interference. This will adversely affect the performance of the blind MMSE receiver. So, when we feed back the estimated bits $\hat{b}_K$, instead of feeding back the hard ± 1 value for the information bits we use a soft decision statistic which is proportional to the soft estimate of the bits b_K. Thus, if the soft estimate of a bit b is x, we feed the conditional expected value

$$E[b|x] = (1)pr(b = 1|x) + (-1)pr(b = -1)|x)$$

back. If we assume that x is Gaussian with variance σ, then $pr(x|b=1) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{(x-1)^2}{2\sigma^2}\right)$. Using Bayes rule, $pr(b=1|x) = \frac{pr(x|b=1)pr(b=1)}{pr(x)}$ where $p(x) = pr(x|b=1)pr(b=1) + pr(x|b=-1)pr(b=-1)$ Using a similar expression for $pr(b=-1|x)$ and assuming that the transmitted bits are equally likely, we get

$$\begin{aligned} \mathrm{E}[b|x] &= \frac{\exp\left(\frac{(x-1)^2}{2\sigma^2}\right) - \exp\left(\frac{(x+1)^2}{2\sigma^2}\right)}{\exp\left(\frac{(x-1)^2}{2\sigma^2}\right) + \exp\left(\frac{(x+1)^2}{2\sigma^2}\right)} \\ &= \frac{\exp(\frac{-x}{\sigma^2}) - \exp(\frac{x}{\sigma^2})}{\exp(\frac{-x}{\sigma^2}) + \exp(\frac{x}{\sigma^2})} \\ &= \tanh\left(\frac{x}{\sigma^2}\right) \end{aligned}$$

C.2 Null-space projection

Another detector that does not require the knowledge of the amplitudes of the in-cell interferers is a null-space based detector. In this case, we project the received signal onto the null-space corresponding to the in-cell interferers to cancel their interference contribution.

$$\tilde{r} = (I - S_K(S_K^\top S_K)^{-1}S_K^\top)^\top r$$

Then, we apply the blind MMSE algorithm to recover the bits from the reduced interference. The reduction in interference due the projection onto the null-space improves the performance of the blind algorithm.

D. Simulation Results

In this section, we will present the simulation results obtained for the receivers described in sections III and IV and discuss their implications. The simulation results are obtained for a synchronous DS-CDMA system with 12 known interferers and 12 unknown interferers. Simulation results have been obtained for various interference conditions within and outside the cell. However, in each of the simulations, the out-of-cell interference is about 36% of the overall interference. The channel model is the additive white Gaussian noise model as described in section 2 and random spreading codes of length 31 were assigned to the users.

Initially, we will show the results for a simple system where all the in-cell users have the same power and all the out-of-cell users have the same power (which is 36% of the power of an in-cell user). In figure 4, the performance of the proposed partially blind multiuser receiver with soft decision feedback (Multistage pblind) is compared with that of the matched filter detector and the blind, crippled and conventional MMSE detectors. It can be seen that there

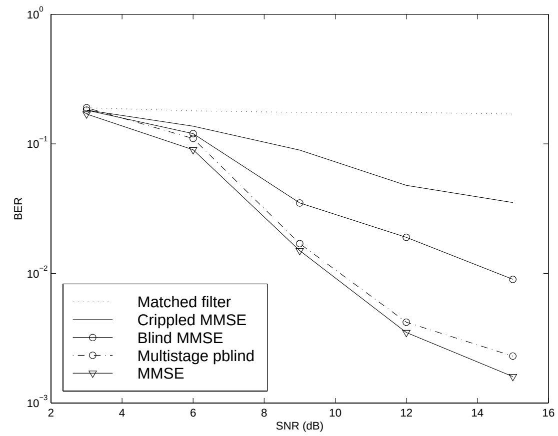

Fig. 4. Performance of partially blind multiuser detector based on soft-decision feedback – number of in-cell users = 12, number of out-of-cell users = 12, users within cell have equal power, power of out-of-cell users = 36% of power of in-cell users.

is a significant difference in performance between the conventional MMSE multiuser detector and the blind MMSE detector. Although the blind MMSE multiuser detector is asymptotically equivalent to the ideal MMSE detector, it does not achieve that performance under practical situations. The crippled MMSE detector also performs very poorly since it does not know the presence of the 12 out-of-cell interferers. However, this crippled detector uses more information than the blind MMSE detector. The partially blind detector uses the same additional information about the in-cell interferers and performs almost as well as the conventional MMSE detector.

Similar performance gains compared to the blind MMSE detector can be achieved in near-far situations also. In figure 5, we show the performance of the partially blind multiuser detector (Multistage pblind), the matched filter detector, the conventional MMSE detector and the blind MMSE detector in a near-far situation, where the 12 users in the cell have powers 0-6 dB higher than the user of concern. The 12 out-of-cell interferers also have powers 0-6 dB higher than the weakest out-of-cell interferer (which has 36% power as the user of concern). The crippled multiuser detector has been left out of this comparison since it was worse than the blind MMSE detector.

The null-space based partially blind detector developed in section IV-C.2 also provides gains compared to the blind MMSE detector and performs close to the conventional MMSE detector. In this case, the gain is achieved without requiring the knowledge of the amplitudes of the in-cell interferers. Amplitude information was used in the soft-decision feedback based detector. The simulated result in a near-far situation is shown in figure 6, where the null-space

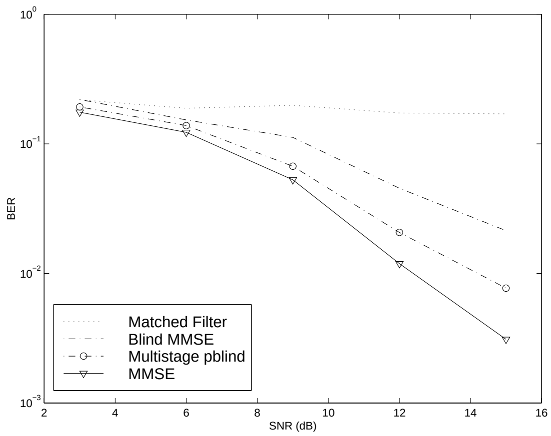

Fig. 5. Performance of partially blind multiuser detector based on soft-decision feedback – number of in-cell users = 12, number of out-of-cell users = 12, users within cell have power between 0 and 6 dB higher than the user-of-concern, power of out-of-cell users = 36% of power of in-cell users.

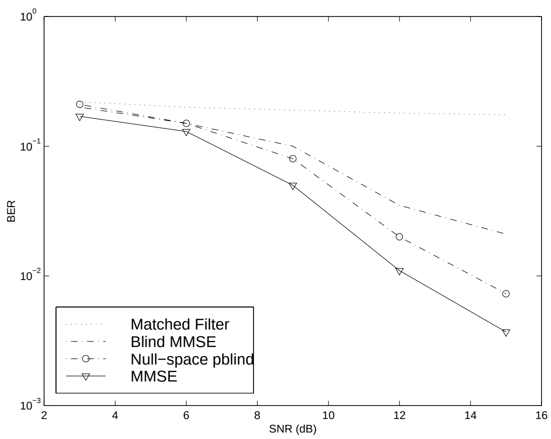

Fig. 6. Performance of null-space based partially blind multiuser detector – number of in-cell users = 12, number of out-of-cell users = 12, users within cell have power between 0 and 6 dB higher than the user-of-concern, power of out-of-cell users = 36% of power of in-cell users.

based partially blind detector (Null-space pblind) is compared with the conventional and blind MMSE detectors and the matched filter detector.

V. Conclusions

Partially blind multiuser detection schemes developed in this paper can perform significantly better than blind MMSE detectors. This gain is achieved using additional knowledge of a subset of interfering users. This gain is possible because blind detectors, although equivalent to conventional multiuser detectors asymptotically, suffer significant loss in performance in practice in the presence of a large number of interferers. While the blind MMSE detectors use the knowledge of the spreading code of the user of interest only and other existing multiuser detectors assume knowledge of all interfering users, partially blind detectors assume partial knowledge of the interference in the form of knowledge about a subset of interferers. This class of detectors would be applicable at the base-station of a cellular communication system which usually has knowledge of all the in-cell users and no information about the out-of-cell users.

The partially blind detectors perform almost as well as the conventional MMSE detector with information on all the interfering users. Two different detectors were obtained. One detector was based on parallel interference cancellation methods and assumed the knowledge of the spreading codes and amplitudes of a subset of interferers. Soft decision feedback was used for interference cancellation. The other detector, derived based on null space projection, assumed the knowledge of the spreading codes of the in-cell interferers only. Both these detectors allow us to bridge the performance gap between existing blind multiuser detectors and conventional multiuser detectors (that assume the knowledge of all the users in the system) by assuming partial knowledge of the interferers.

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