

(PS2 3.8, 3.10, 3.12, 3.13, <sup>3.14</sup>3.16, 3.27, 3.31)

Lecture 26: (8 Oct 2008)

## Cellular system design

Option 2 (Based on Code-Division Multiple Access)  
(CDMA)

### Generic cellular assumptions

- Area to be covered divided into cells
- Each cell has a base-station
- Mobile users communicate only to base-station & vice-versa.

⇒ Two types of links: Mobile to BS (Uplink)  
BS to Mobile (Downlink)

Assume

- uplink & downlink are separated using FDD or TDD.

Consider the design of the uplink:

→ All cells use the total available bandwidth



Spread-spectrum signaling employed by each mobile

BW  $\leftarrow W$   
Data rate  $\leftarrow R$   
 $\frac{W}{R} = \text{spreading factor}$

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⇒ No frequency  
planning

⇒ Interference between users (both intracell & inter-cell)

→ Spreading codes are designed for

- ideal autocorrelation properties
- ideal cross-correlation properties.

→ RAKE receiver (as a sub-optimal receiver)

(Multi-user detection can improve performance) → (See Verdu's book)

(by spreading code design)

→ Interference is limited, only if all users are received at the same power level

"Near-far" effect leads to varying power levels

⇒ Power control necessary to limit interference.

⊖ ⇒ Interference-limited system, capacity depends on how interference is mitigated.

Other important aspects of CDMA:

⊕ → Frequency diversity (multipath diversity) due to spread-spectrum signaling

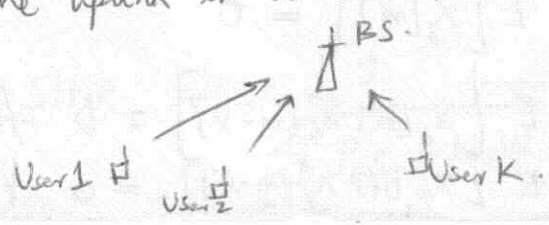
⊕ → Possibility of soft handoff: Simultaneous connection with multiple base-stations ⇒ Diversity

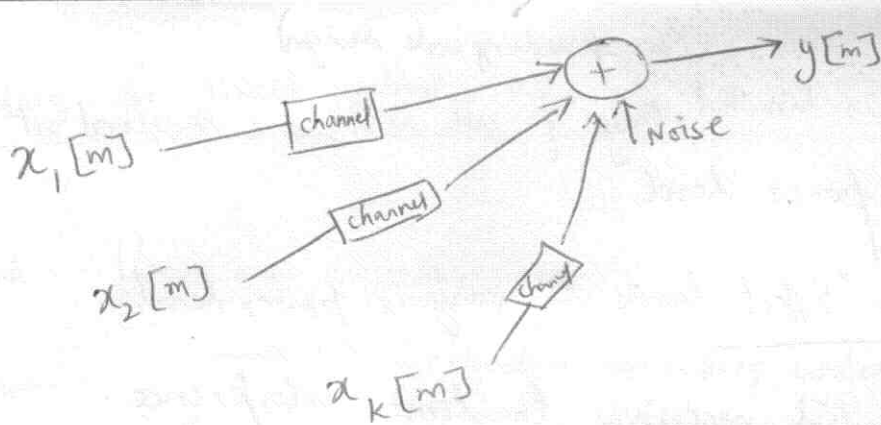
⊕ → Statistical Multiplexing advantage: For bursty traffic, power control can reduce average interference when multiple bursty sources are active

Eg: Voice-Activity-Detection gain

Lecture 27: (15 Oct 2008) CDMA uplink

Consider the uplink in a CDMA cellular system.

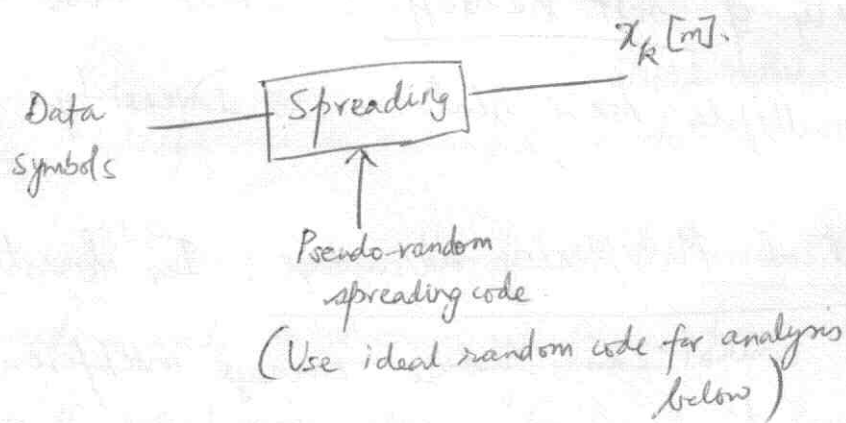




$$y[m] = \sum_{k=1}^K \left( \sum_l h_l^{(k)}[m] x_k[m-l] \right) + w[m]$$

$y[m]$  is the Received signal at BS.  
 $\sum_l h_l^{(k)}[m] x_k[m-l]$  is the Received signal from  $k^{\text{th}}$  user. (Sum of signals from all paths)  
 $w[m]$  is AWGN.

Transmit signal for each user generated as



Assumptions for analysis.

- Users are chip-synchronous (can be generalized to asynchronous scenarios)
- Random binary ( $\pm 1$ ) spreading codes are used.

$$\Rightarrow E[x_k[m]] = 0$$

$$E[x_k[m] x_k^*[m-l]] = 0 \text{ for } l \neq 0.$$

$$E[x_k[m] x_j^*[m-l]] = 0 \quad \forall m, l, k \neq j$$

SINR (Signal to interference + noise ratio) per chip:

Consider user 1

$$y[m] = S[m] + I[m] + w[m] \quad \rightarrow \text{Noise}$$

$$\begin{aligned} &\downarrow \qquad \qquad \qquad \downarrow \\ &\sum_l h_l^{(1)}[m] x_1[m-l] \qquad \sum_{k=2}^K \left( \sum_l h_l^{(k)}[m] x_k[m-l] \right) \\ &\text{Signal} \qquad \qquad \qquad \text{Interference} \end{aligned}$$

Received signal energy per chip from user  $k \triangleq E_k^c$

$$= E[|x_k[m]|^2] \sum_l E[|h_l^{(k)}[m]|^2]$$

For user 1  $I[m]$  is zero-mean

$$\& E[|I[m]|^2] = \sum_{k=2}^K E_k^c$$

$$E[I[m]I[m+l]] = 0 \text{ for } l \neq 0. \quad (\text{Random spreading})$$

Signal, Interference, noise are uncorrelated.

$$\Rightarrow \text{SINR}_1 = \frac{E_1^c}{\sum_{k=2}^K E_k^c + N_0}$$

$$E_k^c = \underset{\substack{\uparrow \\ \text{Transmit power} \\ \text{of user } k}}{P_k} \left( \frac{1}{W} \right) \underset{\substack{\uparrow \\ \text{One chip} \\ \text{duration}}}{g_k} \quad \rightarrow \text{channel attenuation}$$

$$\Rightarrow \text{SINR}_1 = \frac{P_1 g_1}{\sum_{k=2}^K P_k g_k + N_0 W}$$



$$\text{SINR per symbol (bit)} = \frac{\text{Energy per symbol (bit)}}{(\text{Interference + noise}) \text{ per chip}} = \frac{E_b}{I_0}$$

(after despreading)

↳ { Desired signal amplitude increases by  $G \Rightarrow$  Energy up by  $G$  }  
 { Interference + noise variance increases by factor  $G$  }

$$\text{SINR}_1 = \frac{G P_1 g_1}{\sum_{k=2}^K P_k g_k + N_0 W}$$

where  $G = \frac{W}{R} = \begin{matrix} \text{spreading factor} \\ \text{(or)} \\ \text{processing gain} \end{matrix}$

Suppose  $\beta$  is the reqd.  $\frac{E_b}{I_0}$  for each user.

Can we find  $P_k$ 's such that

$$\text{SINR} \geq \beta \text{ for each user?}$$

$$\frac{G P_k g_k}{\sum_{\substack{l \\ l \neq k}} P_l g_l + N_0 W} \geq \beta.$$

(Are there feasible power vectors?)

If no, how can this soln. be arrived at?)

Let  $Q_k \triangleq P_k g_k$ .

$$\frac{G Q_k}{\sum_{\substack{l \\ l \neq k}} Q_l + N_0 W} \geq \beta \text{ for } k = 1, 2, \dots, K.$$

$$GQ_k \geq \beta \left( \sum_{l \neq k} Q_l + N_0 W \right) \text{ for } k=1, 2, \dots, L \quad (46)$$

Sum of the above  $K$  equations gives

$$\sum_k GQ_k \geq \beta \sum_k \sum_{l \neq k} Q_l + \beta K N_0 W$$

$$(i.e.) \sum_k GQ_k \geq \beta(K-1) \sum_k Q_k + \beta K N_0 W$$

$$\Rightarrow (G - \beta(K-1)) \sum_k Q_k \geq \beta K N_0 W$$

If  $G - \beta(K-1) \leq 0$ , there is no soln. (LHS becomes -ve)  
RHS is +ve

Therefore,  $G - \beta(K-1) > 0$  is a necessary condition

(or)  $K < 1 + \frac{G}{\beta}$  is a necessary condition.

Lecture 28: (17 Oct 2008)

$\beta$  A

Now, Suppose  $K < 1 + \frac{G}{\beta}$ .

We can choose powers ( $P_k$ 's) such that

$$Q_k = \frac{\beta N_0 W}{G - \beta(K-1)} \text{ for each user}$$

$$i.e. P_k g_k = \frac{\beta N_0 W}{G - \beta(K-1)}$$

Therefore, A is also a sufficient condition.

Therefore  $K < 1 + \frac{G}{\beta}$  is a necessary & sufficient condition for existence of feasible powers to support a given  $\frac{E_b}{N_0}$  requirement.

Rewrite as

$$\frac{K}{G} < \frac{1}{G} + \frac{1}{\beta}$$

Since  $G = \frac{W}{R}$ ,

$$\frac{KR}{W} < \frac{1}{G} + \frac{1}{\beta}$$

$\frac{KR}{W}$  = overall spectral efficiency in bits/s/Hz

$G$  is typically much larger than  $\beta$

$\Rightarrow$  <sup>Max.</sup> Spectral efficiency  $\approx \frac{1}{\beta}$

Suppose  $\frac{E_b}{N_0}$  reqd. is 6dB, Max. spec. efficiency  $\approx 0.25$  b/s/Hz

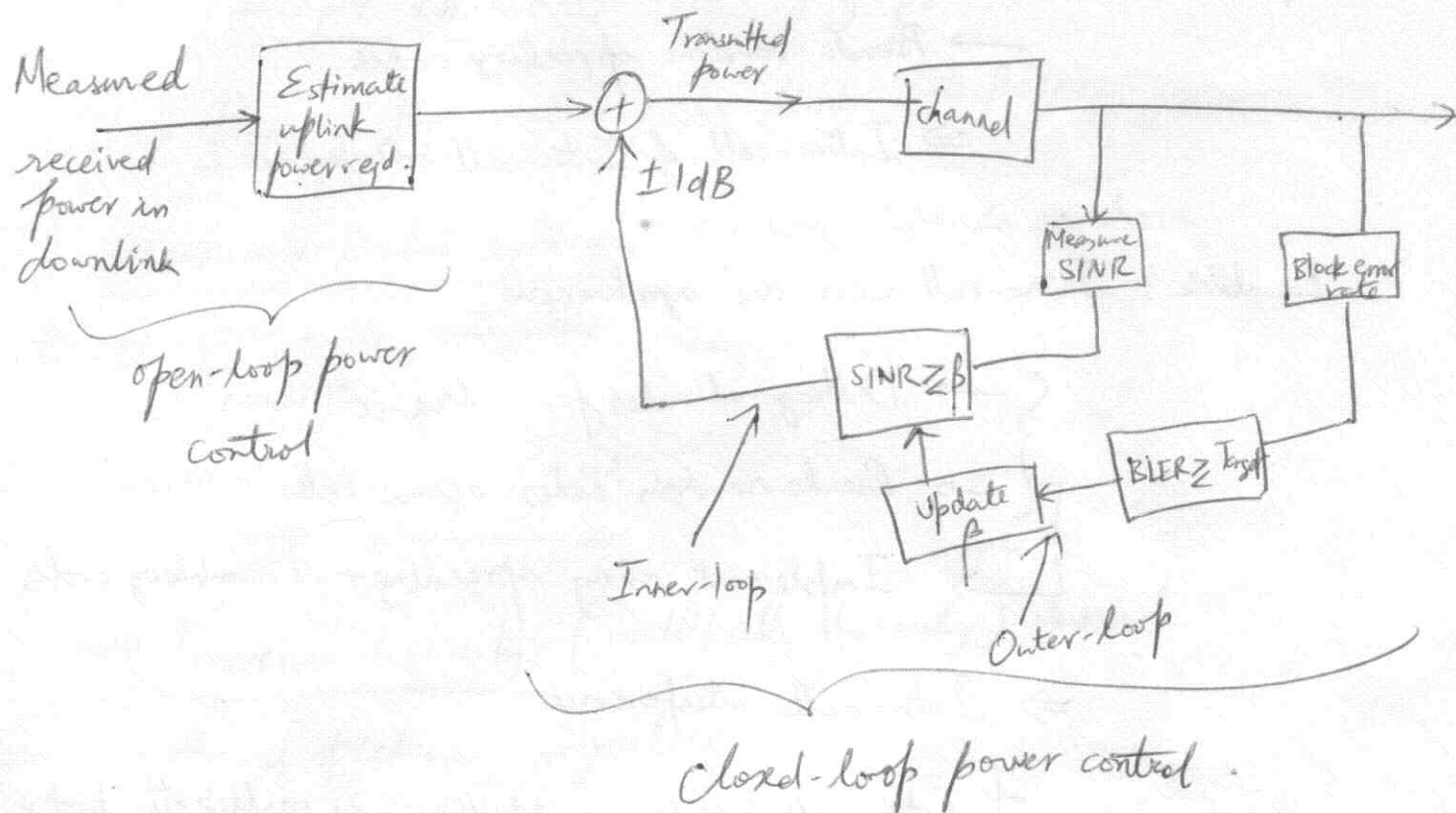
Statistical multiplexing gain from interference averaging:

Suppose traffic is bursty (R or 0 rate with prob.  $p$  &  $(1-p)$ ).

$\rightarrow$  When large no. of users are in the system, average interference is  $p$  times <sup>worst case</sup> interference from  $K$  users (like  $pK$  interferers).

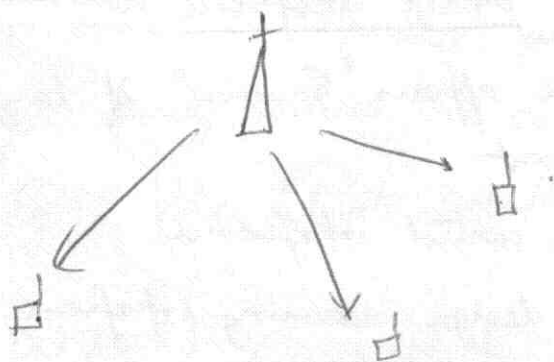
See analysis in pages 143-144.

# Power control in a real system:



Feedback rates in real systems : 800 Hz, 1500 Hz.  
(Power control bit)

## CDMA downlink power control



In uplink, "near-far" effect necessitates power control. Not so in the downlink.

Power control is used to

- minimize transmit power &
  - reduce interference to other cells.
- (Similar block diagram as above)

No "near-far" effect.



## Spreading in CDMA uplink vs Spreading in CDMA downlink.

Uplink: Users are asynchronous

→ Pseudo-random spreading codes

⇒ Intra-cell & Inter-cell interference.

Downlink: Intra-cell users are synchronous

{ → Orthogonal codes for intra-cell users

{ → Pseudo-random codes across cells

{ → Implement using spreading + scrambling codes

⇒ Inter-cell interference

+ Intra-cell interference if there is multipath propagation.

In summary, interference is managed <sup>in CDMA cellular systems</sup> as follows:

- All users share same bandwidth (Universal frequency reuse)  
Spreading used to average interference from several users
- Overall interference appears "Gaussian" if large number of interferers are present
- Power control to control interference
- Point-to-point link design assuming interference is AWGN.