EE613: Estimation Theory
Problem Set 9

1. If N IID observations $\{x[0], x[1], \ldots, x[N-1]\}$ are made from the Laplacian PDF

$$p(x;\sigma) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{\sqrt{2}|x|}{\sigma}\right)$$

   find a method of moments estimator for σ.

2. Assume that N IID samples from a bivariate Gaussian PDF are observed or $\{\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{N-1}\}$, where each $\mathbf{x}$ is a 2×1 random vector with PDF $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$. If

$$\mathbf{C} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

   find a method of moments estimator for ρ. Also, determine a cubic equation to be solved for the MLE of ρ. Comment on the ease of implementation of the different estimators.

3. If N IID observations $\{x[0], x[1], \ldots, x[N-1]\}$ are made from the $\mathcal{N}(\mu, \sigma^2)$ PDF, find a method of moments estimator for $\boldsymbol{\theta} = [\mu \ \ \sigma^2]^T$.

4. For a DC level in WGN or $x[n] = A + w[n]$ for $n = 0, 1, \ldots, N-1$, where $w[n]$ is WGN with variance σ^2, the parameter A^2 is to be estimated. It is proposed to use $\hat{A^2} = (\bar{x})^2$. For this estimator, find the approximate mean and variance using a first-order Taylor expansion approach.