EE613: Estimation Theory
Problem Set 5

1. The IID observations $x[n]$ for $n = 0, 1, \cdots, N-1$ have the Rayleigh PDF

$$p(x[n];\sigma^2) = \begin{cases} \dfrac{x[n]}{\sigma^2}\exp\left(-\dfrac{1}{2}\dfrac{x^2[n]}{\sigma^2}\right) & x[n] > 0 \\ 0 & x[n] < 0 \end{cases}.$$

   Find a sufficient statistic for σ^2.

2. The IID observations $x[n]$ for $n = 0, 1, \cdots, N-1$ are distributed according to $\mathcal{N}(\theta, \theta)$ where $\theta > 0$. Find a sufficient statistic for θ.

3. If $x[n] = A + w[n]$ for $n = 0, 1, \cdots, N-1$ are observed, where $w[n]$ is WGN with variance σ^2, find the MVUE for σ^2 assuming that A is known. You may assume that the sufficient statistic is complete.

4. Assume thet $x[n]$ is the result of a Bernoulli trial (a coin toss) with

$$\Pr\{x[n] = 1\} = \theta$$

$$\Pr\{x[n] = 0\} = 1 - \theta$$

   and that N IID observations have been made. Assuming that Neyman-Fisher factorization theorem holds for discrete random variables, find a sufficient statistic for θ. Then, assuming completeness, find the MVUE of θ.

5. If N IID observations are made according to the PDF

$$p(x[n];\theta) = \begin{cases} \exp\left[-(x[n] - \theta)\right] & x[n] > \theta \\ 0 & x[n] < \theta \end{cases},$$

   find the MVUE for θ. Note that θ represents the minimum value that $x[n]$ may attain. Assume that the sufficient statistic is complete.