EE613: Estimation Theory
Problem Set 2

1. Consider the data $\{x[0], x[1], \cdots, x[N-1]\}$, where each sample is distributed as $\mathcal{U}[0, \theta]$ and the samples are IID. Can you find an unbiased estimator for θ? The range of θ is $0 < \theta < \infty$.

2. Two samples $\{x[0], x[1]\}$ are independently observed from a $\mathcal{N}(0, \sigma^2)$ distribution. The estimator
$$\hat{\sigma^2} = \frac{1}{2}(x^2[0] + x^2[1])$$
is unbiased. Find the PDF of $\hat{\sigma^2}$ to determine if it is symmetric about σ^2.

3. For the estimation of a DC level in white Gaussian noise ($x[n] = A + w[n]$, as considered in class), consider the general estimator
$$\hat{A} = \sum_{n=0}^{N-1} a_n x[n]$$
is proposed. Find the a_n's so that the estimator is unbiased and the variance is minimized. Hint: Use Lagrangian multipliers with unbiasedness as the constraint equation.

4. Two unbiased estimators are proposed whose variances satisfy $\operatorname{var}(\hat{\theta}) < \operatorname{var}(\check{\theta})$. If both estimators are Gaussian, prove that
$$Pr\left\{|\hat{\theta} - \theta| > \epsilon\right\} < Pr\left\{|\check{\theta} - \theta| > \epsilon\right\}$$
for any $\epsilon > 0$. This says that the estimator with less variance is to be preferred since its PDF is more concentrated about the true value.

5. Consider $x[n] = A + w[n]$ as discussed in class. Assume that both A and σ^2 are unknown. We wish to estimate the vector parameter
$$\theta = \begin{bmatrix} A \\ \sigma^2 \end{bmatrix}.$$

Is the estimator
$$\hat{\theta} = \begin{bmatrix} \hat{A} \\ \hat{\sigma}^2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{N}\sum_{n=0}^{N-1} x[n] \\ \dfrac{1}{N-1}\sum_{n=0}^{N-1}\left(x[n] - \hat{A}\right)^2 \end{bmatrix}$$
unbiased?