EE613: Estimation Theory
Problem Set 1

1. Let $x = \theta + w$, where w is a random variable with PDF $p_w(w)$. If θ is a deterministic parameter, find the PDF of x in terms of p_w and denote it by $p(x;\theta)$. Next, assume that θ is a random variable independent of w and find the conditional PDF $p(x|\theta)$. Finally, do not assume that θ and w are independent and determine $p(x|\theta)$. What can you say about $p(x;\theta)$ versus $p(x|\theta)$?

2. The data $\{x[0],\ x[1],\ \cdots,\ x[N-1]\}$ are observed where $x[n]$'s are independent and identically distributed (IID) as $\mathcal{N}(0,\sigma^2)$. We wish to estimate the variance σ^2 as

$$\hat{\sigma^2} = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n].$$

Is this an unbiased estimator? Find the variance of $\hat{\sigma^2}$ and examine what happens as $N \to \infty$.

3. This problem illustrates what happens to an unbiased estimator when it undergoes a nonlinear transformation. Consider the example discussed in class: $x[n] = A + w[n]$, where $w[n]$ is zero mean white Gaussian noise. If we choose to estimate the unknown parameter $\theta = A^2$ by

$$\hat{\theta} = \left(\frac{1}{N}\sum_{n=0}^{N-1} x[n]\right)^2,$$

can we say that the estimator is unbiased? What happens as $N \to \infty$?

4. Given a single observation $x[0]$ from the distribution $\mathcal{U}[0, 1/\theta]$, it is desired to estimate θ. It is assumed that $\theta > 0$. Show that for an estimator $\hat{\theta} = g(x[0])$ to be unbiased we must have

$$\int_0^{\frac{1}{\theta}} g(u)du = 1.$$

Next, prove that a function g cannot be found to satisfy this condition for all $\theta > 0$.