EE611 Problem Set 4

1. Suppose that the transmitter signal pulse $g(t)$ has duration T and unit energy and the received signal pulse is $h(t) = g(t) + ag(t - T)$. Determine the equivalent discrete-time filter model for the sampled matched filter output.

2. The equivalent discrete-time model for an ISI channel is given by

$$v_k = I_k + 0.5I_{k-1} + \eta_k,$$

   where $\{I_k\}$ is the transmitted information sequence and $\{\eta_k\}$ is AWGN. In the absence of noise, sketch a constellation for the received signal if the transmitted signal constellation is:

   (a) a binary constellation: $+1$ and -1.

   (b) a 4-point constellation: $(1, 1)$, $(1, -1)$, $(-1, 1)$, and $-1, -1)$.

   Repeat the above problem for the discrete-time channel given by

$$v_k = I_k + I_{k-1} + \eta_k.$$

3. The equivalent discrete-time model for an ISI channel is given by

$$v_k = I_k + 0.5I_{k-1} + 0.25I_{k-2} + \eta_k,$$

   where $\{I_k\}$ is the transmitted information sequence and $\{\eta_k\}$ is AWGN. Binary signalling (± 1) is used.

   (a) Sketch the trellis diagram representing the channel. Label each state transition (branch) with the corresponding input and output.

   (b) Consider two paths in the trellis corresponding to the state sequences $\{A_k\}_{k=1}^{D}$ and $\{B_k\}_{k=1}^{D}$. If $A_l = B_l$ and $A_{l+1} \neq B_{l+1}$ for some l, then show that $A_{l+2} \neq B_{l+2}$.

   (c) Based on the answer to part (b), determine the minimum possible length for an error event for the above trellis.

   (d) Consider the error event described by the following 2 paths (sequence of states):

$$\textit{Path A}: \quad \{(1,1),\ (1,1),\ (1,1),\ (1,1)\},$$

   and

$$\textit{Path B}: \quad \{(1,1),\ (-1,1),\ (1,-1),\ (1,1)\}.$$

   Given that the all 1's sequence is transmitted, determine the probability that the likelihood metric for path B is larger than the likelihood metric for path A.

   (e) What is the minimum possible length for an error event for the following channel:

$$v_k = \sum_{n=0}^{L} f_n I_{k-n} + \eta_k,$$

4. Binary PAM is used to transmit information over a linear filter channel. The discrete-time model for the matched filter output is

$$y_k = 0.5I_{k-1} + 1.25I_k + 0.5I_{k+1} + \nu_k.$$

(a) Design a three-tap linear equalizer so that the output of the equalizer in the absence of noise is

$$\hat{I}_k = \sum_{n=-2}^{2} q_n I_{k-n},$$

where

$$q_m = \begin{cases} 1 & (m = 0) \\ 0 & (m = \pm 1) \end{cases}$$

(b) Determine q_m for $m = \pm 2$.

(c) Design a 5-tap equalizer so that

$$\hat{I}_k = \sum_{n=-3}^{3} q_n I_{k-n},$$

where

$$q_m = \begin{cases} 1 & (m = 0) \\ 0 & (m = \pm 1, \ \pm 2) \end{cases}$$

5. (a) Assuming that the variance of noise at the output of the matched filter is 0.125, design 3-tap and 5-tap linear MMSE equalizers for the channel in the previous problem.

   (b) Determine q_m, by convolving the impulse response of the equalizer with the channel response.