EE5160: Error Control Coding
Problem Set 5

1. Find the codewords of the code $\mathcal{C}$ corresponding to the parity check matrix H=[1 1 1 1].

2. Consider a systematic (8,4) code whose parity-check equations are

$$\begin{aligned} v_0 &= u_1 + u_2 + u_3, \\ v_1 &= u_0 + u_1 + u_2, \\ v_2 &= u_0 + u_1 + u_3, \\ v_3 &= u_0 + u_2 + u_3. \end{aligned}$$

   where u_0, u_1, u_2 and u_3 are message digits, and v_0, v_1, v_2 and v_3 are parity-check digits. Find the generator and parity-check matrices for this code. Show analytically that the minimum distance of this code is 4.

3. Let $\mathbf{H}$ be the parity-check matrix of an (n, k) linear code $\mathcal{C}$ that has both odd and even-weight codewords. Construct a new linear code $\mathcal{C}_1$ with the following parity-check matrix:

$$\mathbf{H}_1 = \begin{bmatrix} 0 & & & \\ 0 & & & \\ \vdots & & \mathbf{H} & \\ 0 & & & \\ 1 & 1 & 1 \ldots & 1 \end{bmatrix}$$

   (Note that the last row of $\mathbf{H}_1$ consists of all 1's.)
   a) Show that $\mathcal{C}_1$ is an $(n+1, k)$ linear code.$\mathcal{C}_\infty$ is called an extension of $\mathcal{C}$.
   b) Show that every codeword of $\mathcal{C}_1$ has even weight.
   c) Show that $\mathcal{C}_1$ can be obtained from $\mathcal{C}$ by adding an extra parity-check digit,denoted by v_∞, to the left of each codeword $\mathbf{v}$ as follows:(1) if $\mathbf{v}$ has odd weight, then $v_\infty = 1$, and (2) if $\mathbf{v}$ has even weight, then $v_\infty = 0$. The parity-check digit v_∞ is called an overall parity-check digit.

4. Prove that the Hamming distance satisfies the triangle inequality; that is, let x, y and z be three n-tuples over $GF(2)$, and show that

$$d(x, y) + d(y, z) \geq d(x, z).$$

5. Prove that a linear code is capable of correcting λ or fewer errors and simultaneously detecting $l(l > \lambda)$ or fewer errors if its minimum distance $d_{min} \geq \lambda + l + 1$.

6. Show that the probability of an undetected error for Hamming codes of length $2^m - 1$ on a BSC with transition probability p satisfies the upper bound 2^{-m} for $p \leq 1/2$. (*Hint*: Use the inequality $(1 - 2p) \leq (1 - p)^2$.)

7. Compute the probability of an undetected error for the $(15, 11)$ generalized Hamming code on a BSC with transition probability $p = 10^{-2}$.