EE5160: Error Control Coding
Problem Set 1

1. Construct the group under modulo-11 addition.

2. Construct the group under modulo-11 multiplication.

3. Let m be a positive integer. If m is not a prime, prove that the set $\{1, 2, \ldots, m-1\}$ does not form a group under modulo-m multiplication.

4. Find all the generators of the multiplicative group constructed in Problem 2.

5. Find a cyclic subgroup of the multiplicative group constructed using the prime integer 13 and give its cosets.

6. Let G be a group under the binary operation $*$. let H be a non empty subset of G. Prove that H is a subgroup of G if the following conditions hold.
   i. H is closed under the binary operation $*$.
   ii. For any element a in H, the inverse of a is also in H.

7. Let H be a subgroup of a group G with binary binary operation $*$. Prove that
   i. No two elements in a coset of H are identical.
   ii. No two elements in two different cosets of a subgroup H of a group G are identical.

8. Prove the following properties of a field F.
   i. For every element a in F, $a.0 = 0.a = 0$.
   ii. For any two nonzero elements a and b in F, $a.b \neq 0$.
   iii. For two elements a and b in F, $a.b = 0$ implies that either $a = 0$ or $b = 0$.
   iv. For $a \neq 0$, $a.b = a.c$ implies that $b = c$.
   v. For any two elements a and b in F, $-(a.b) = (-a).b = a.(-b)$.

9. Consider the integer group $G = \{0, 1, 2 \ldots, 31\}$ under modulo-32 addition. Show that H $=\{0,\ 4,\ 8,\ 12,\ 16,\ 20,\ 24,\ 28\}$ forms a subgroup of G. Decompose G into cosets with respect to H (or modulo H).

10. Prove the following properties of a vector space V over a field F .
    i. Let 0 be the zero element of F. For any vector $\mathbf{v}$ in V , $0.\mathbf{v} = \mathbf{0}$.
    ii. For any element a in F, $a.\mathbf{0} = \mathbf{0}$.
    iii. For any element a in F and any vector $\mathbf{v}$ in V , $(-a).\mathbf{v} = \mathbf{a}.(-\mathbf{v}) = -(\mathbf{a}.\mathbf{v})$.

11. Let S be a nonempty subset of a vector space V over a field F. Prove that S is a subspace of V if the following conditions are satisfied:
    i. For any two vectors $\mathbf{u}$ and $\mathbf{v}$ in S, $\mathbf{u} + \mathbf{v}$ is also a vector in S.
    ii. For any element a in F and any vector $\mathbf{u}$ in S, $a.\mathbf{u}$ is also in S.

12. Let m be a positive integer. Prove that the set $\{0, 1, \ldots, m-1\}$ forms a ring under modulo-m addition and multiplication.

13. Let $a(X) = 3X^2 + 1$ and $b(X) = X^6 + 3X + 2$ be two polynomials over the prime field GF(5). Divide $b(X)$ by $a(X)$ and find the quotient and remainders of the division.