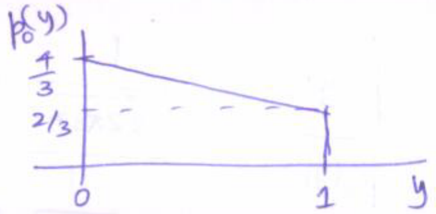
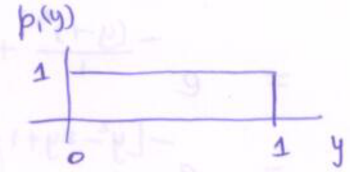
1) $p_0(y) = \begin{cases} \frac{4}{3}(1-0.5y) & 0 \le y < 1 \\ 0 & \text{else} \end{cases}$



$p_1(y) = \begin{cases} 1 & 0 \le y < 1 \\ 0 & \text{else} \end{cases}$



$\pi_0 = \pi_1 = \frac{1}{2}$

$C_{00} = C_{11} = 0\,; \quad C_{10} = C_{01} = 1$

$$L(y) = \frac{p_1(y)}{p_0(y)} = \begin{cases} \dfrac{3}{4(1-0.5y)} & 0 \le y < 1 \\ 0 & \text{else.} \end{cases}$$

$\tau = 1$

$$\delta_B(y) = \begin{cases} 1 & L(y) \ge 1 \\ 0 & L(y) < 1 \end{cases}$$

$\left[\begin{array}{l} L(y) \ge 1 \\ \Rightarrow 3 \ge 4(1-0.5y) = 4-2y \\ \Rightarrow 2y \ge 1 \Rightarrow y \ge \frac{1}{2}. \end{array} \right]$

$$\boxed{\delta_B(y) = \begin{cases} 1 & y \ge \frac{1}{2} \\ 0 & y < \frac{1}{2}. \end{cases}}$$

Minimum Bayes risk $= \dfrac{1}{2}\displaystyle\int_{1/2}^{1} p_0(y)\,dy + \frac{1}{2}\int_{0}^{1/2} p_1(y)\,dy$

$= \frac{1}{2}\left[\frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\right] + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}\left(\frac{5}{12}\right) + \frac{1}{4} = \underline{\frac{11}{24}}.$

(2) (a) $L(y) = \dfrac{p_1(y)}{p_0(y)} = \dfrac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-1)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2}}$ $\qquad \underline{\sigma^2 = 0.5}$

$= e^{-\frac{(y-1)^2}{1} + \frac{y^2}{1}}$

$= e^{-[y^2 - 2y + 1 - y^2]} = e^{2y-1}$

(b) $L(y) \gtreqless \tau$

$(\equiv) \quad y \gtreqless \tau'.$

$$\boxed{\delta(y) = \begin{cases} 1 & y > \tau' \\ \gamma & y = \tau' \\ 0 & y < \tau'. \end{cases}}$$

$P_F = \displaystyle\int_{\tau'}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} dy = Q\left(\frac{\tau'}{\sigma}\right) = Q(\sqrt{2}\tau') = \alpha = 0.01$

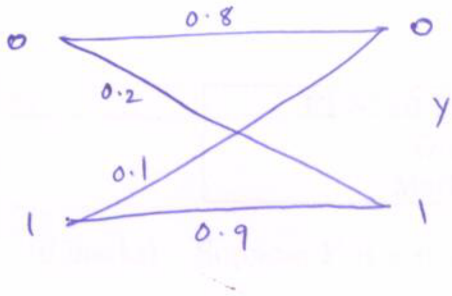
$\Rightarrow \tau' = \frac{1}{\sqrt{2}} Q^{-1}(\alpha) = \underline{\frac{1}{\sqrt{2}} Q^{-1}(0.01)}.$

$\underline{\gamma = 1.}$

(c) $P_D = \displaystyle\int_{\tau'}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-1)^2/2\sigma^2} dy = Q\left(\frac{\tau'-1}{\sigma}\right) = Q\left(\sqrt{2}(\tau'-1)\right)$

where $\tau' = \frac{1}{\sqrt{2}} Q^{-1}(0.01)$

$\underline{P_D = Q\left(Q^{-1}(0.01) - \sqrt{2}\right)}$



(a) 4 possible decision rules (deterministic)

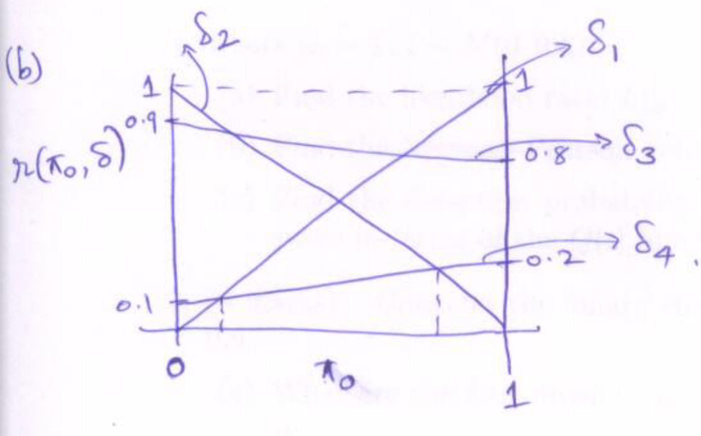
$$\delta_1(y) = 1$$

$$\delta_2(y) = 0$$

$$\delta_3(y) = \begin{cases} 1 & y=0 \\ 0 & y=1 \end{cases}$$

$$\delta_4(y) = \begin{cases} 0 & y=0 \\ 1 & y=1 \end{cases}$$

(b)



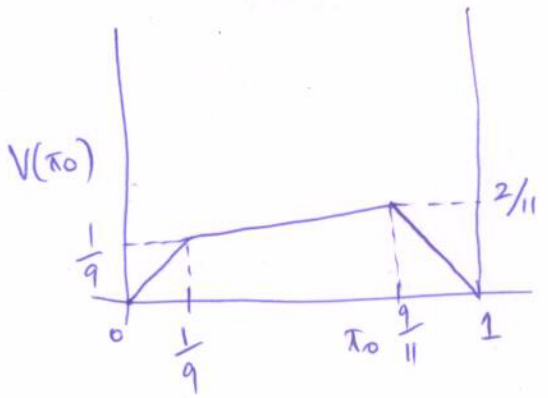
$$r(\pi_0, \delta_1) = \pi_0$$

$$r(\pi_0, \delta_2) = 1-\pi_0$$

$$r(\pi_0, \delta_3) = \pi_0(0.8) + (1-\pi_0)(0.9) = 0.9 - 0.1\pi_0$$

$$r(\pi_0, \delta_4) = \pi_0(0.2) + (1-\pi_0)\,0.1 = 0.1 + 0.1\pi_0.$$

(c)



$$0.1 + 0.1\pi_0 = \pi_0$$

$$\Rightarrow \pi_0 = \frac{1}{9}$$

$$0.1 + 0.1\pi_0 = 1-\pi_0$$

$$\Rightarrow \pi_0 = \frac{9}{11}.$$