EE5130: Detection and Estimation Theory
Problem Set 1

1. (Poor II.F.1) Find the minimum bayes risk $V(\pi_0)$ for the binary channel of example II.B.1 (discussed in class), where

$$p_j(y) = \begin{cases} \lambda_j, & y \neq j \\ (1-\lambda_j), & y = j \end{cases}.$$

   Sketch $V(\pi_0)$ versus π_0. If $\lambda_0 = \lambda_1$, sketch $V(\pi_0)$ versus π_0.

2. (Poor II.F.2) Suppose Y is a random variable that, under hypothesis H_0, has pdf

$$p_0(y) = \begin{cases} \frac{2}{3}(y+1), & 0 \leq y < 1 \\ 0, & \text{otherwise} \end{cases}$$

   and under hypothesis H_1, has pdf

$$p_1(y) = \begin{cases} 1, & 0 \leq y < 1 \\ 0, & \text{otherwise} \end{cases}$$

   Find the Bayes rule and minimum Bayes risk for testing H_0 versus H_1 with uniform costs and equal priors.

3. (Poor II.F.3) Repeat problem 2 for the situation in which p_j is given instead by

$$p_j(y) = \frac{j+1}{2} e^{-(j+1)|y|}, \text{ y} \in \mathbb{R}, \text{ j} = 0{,}1$$

$$C_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i = 1 \text{ and } j = 0 \\ \frac{3}{4}, & \text{if } i = 0 \text{ and } j = 1 \end{cases}$$

$$\pi_0 = \frac{1}{4} \text{ and } \pi_1 = \frac{3}{4}$$

4. (Poor II.F.4) Repeat problem 2 for the situation in which p_0 and p_1 are given instead by

$$p_0(y) = \begin{cases} e^{-y}, & y \geq 0 \\ 0, & y < 0 \end{cases}$$

$$p_1(y) = \begin{cases} \sqrt{\frac{2}{\pi}} e^{-\frac{y^2}{2}}, & y \geq 0 \\ 0, & y < 0 \end{cases}$$

   Consider prior probabilities π_0 and π_1.