EE 511 Solutions to Problem Set 8

1. $E[Y_t] = E[X_t]\cos 2\pi f_c t$. $E[Y_t]$ is periodic with period $1/f_c$.

$$R_Y(t,t+\tau) = E[X_{t+\tau}\cos 2\pi f_c(t+\tau) X_t \cos 2\pi f_c t] = \frac{R_X(\tau)}{2}\left[\cos 2\pi f_c\tau + \cos 2\pi f_c(2t+\tau)\right]$$

$R_Y(t,t+\tau)$ is periodic with period $1/(2f_c)$.

Therefore, Y_t is wide-sense cyclostationary with period $1/f_c$.

$E[Z_t] = E[X_t]E[\cos(2\pi f_c t + \Theta)] = 0$.

$$\begin{aligned} R_Z(t,t+\tau) &= E[X_{t+\tau}\cos(2\pi f_c(t+\tau)+\Theta)X_t\cos(2\pi f_c t+\Theta)] \\ &= \tfrac{R_X(\tau)}{2}\left[\cos 2\pi f_c\tau + E[\cos(2\pi f_c(2t+\tau)+2\Theta)]\right] \\ &= \tfrac{R_X(\tau)}{2}\cos 2\pi f_c\tau \end{aligned}$$

Therefore, Z_t is wide-sense stationary.

2. (a) As derived in class, we have

$$R_X(\tau) = \frac{1}{T}\sum_{k=-\infty}^{\infty} R_A(k)R_p(\tau - kT),$$

where

$$R_p(\tau) = \int_{-\infty}^{\infty} p(t)p(t+\tau)dt.$$

(b) $R_A(k) = 0$ for all $k \neq 0$. Therefore

$$R_X(\tau) = \frac{1}{T}R_A(0)R_p(\tau),$$

where $R_A(0) = 1$ and

$$R_p(\tau) = \begin{cases} A^2(2T - |\tau|) & |\tau| < 2T \\ 0 & \text{else} \end{cases}$$

Therefore, we have (see Figure 1)

$$R_X(\tau) = \begin{cases} A^2(2 - \frac{|\tau|}{T}) & |\tau| < 2T \\ 0 & \text{else} \end{cases}$$

(c)

$$R_p(\tau) = \begin{cases} A^2(T - |\tau|) & |\tau| < T \\ 0 & \text{else} \end{cases}$$

$R_A(0) = E[A_n^2] = E[(B_n + B_{n-1})(B_n + B_{n-1})] = E[B_n^2] + E[B_{n-1}^2] = 2$.

$R_A(-1) = R_A(1) = E[A_nA_{n-1}] = E[(B_n + B_{n-1})(B_{n-1} + B_{n-2})] = E[B_{n-1}^2] = 1$.

$R_A(k) = 0$ for all $|k| \geq 2$.

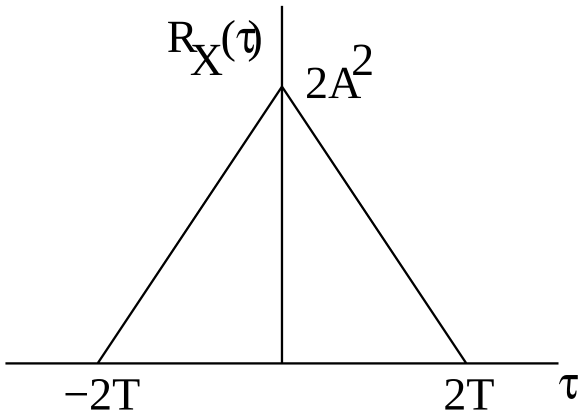

Figure 1:

We get the same result for $R_A(\tau)$ as in part (b) (See Figure 1).

$$R_X(\tau) = \begin{cases} A^2(2 - \frac{|\tau|}{T}) & |\tau| < 2T \\ 0 & \text{else} \end{cases}$$

(d) $R_A(k) = 0$ for $k \geq 2$. $R_A(0) = E[(2B_n + B_{n-1})(2B_n + B_{n-1})] = 4 + 1 = 5$. $R_A(1) = R_A(-1) = E[(2B_n + B_{n-1})(2B_{n+1} + B_n)] = 2$. Therefore, we have

$$R_X(\tau) = \frac{1}{T}\left[5R_p(\tau) + 2R_p(\tau - T) + 2R_p(\tau + T)\right],$$

which is as shown in Figure 2.

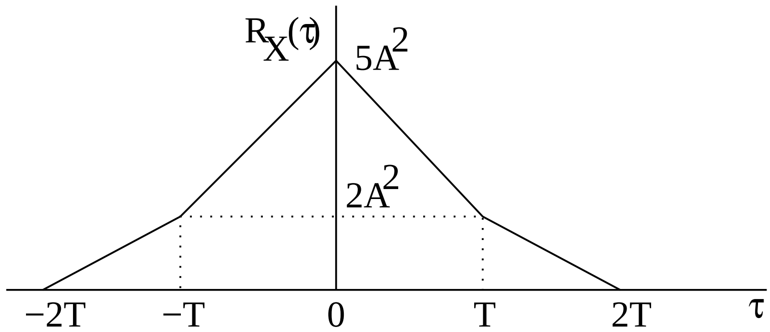

Figure 2: