EE 511 Solutions to Problem Set 2

1.

$$P(AB|A) = \frac{P(AB)}{P(A)} \quad \text{and} \quad P(AB|(A+B)) = \frac{P(AB(A+B))}{P(A+B)} = \frac{P(AB)}{P(A+B)}.$$

Since $P(A+B) \geq P(A)$, $P(AB|(A+B)) \leq P(AB|A)$.

2. Since A and B are mutually exclusive, $AB = \phi$ and $P(AB) = 0$. For A and B to be independent, we need $P(AB) = P(A)P(B)$. This is possible only if $P(A) = 0$ or $P(B) = 0$ or both.

3. (a) True. (b) False. (c) False, in general. True only if $X(s) = a \forall s \in S$. (d) True.

4. (i) For $f_X(x)$ to be a valid pdf, we need

$$\int_{-1}^{1} f_X(x)dx = 1 \quad \text{i.e.,} \quad \int_{-1}^{1} c(1-x^2)dx = 1$$

Therefore, we have

$$c\left(x - \frac{x^3}{3}\right)\bigg|_{-1}^{1} = 1$$

$$2c - \frac{2c}{3} = 1$$

Therefore, $c = 3/4$.

(ii)

$$P[X > 0] = \int_0^1 \frac{3}{4}(1-x^2)dx = \frac{1}{2}.$$

$$P[X < \frac{1}{2}] = \int_{-1}^{0.5} \frac{3}{4}(1-x^2)dx = \frac{27}{32}.$$

$$P[|X| > 0.75] = 2\int_{0.75}^{1} \frac{3}{4}(1-x^2)dx = \frac{11}{128}.$$

5. (i) This function cannot be a pdf since $f(x) < 0$ for $-1 \leq x < 0$.

(ii) This function can be a pdf since $f(x) \geq 0$ for all x and

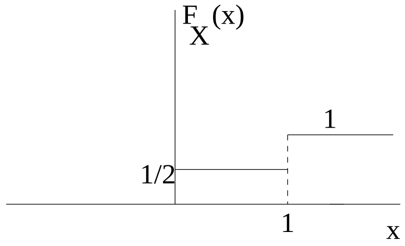
$$\int_{-\infty}^{\infty} f(x)dx = 1.$$

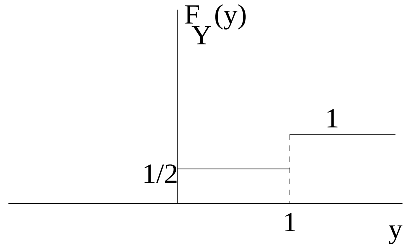
6.

$$F_X(x) = F_{X,Y}(x,\infty) = \begin{cases} 1 & x > 1 \\ \frac{1}{2} & 0 \leq x \leq 1 \\ 0 & x < 0 \end{cases}$$

Similarly, we have

$$F_Y(y) = F_{X,Y}(\infty,y) = \begin{cases} 1 & y > 1 \\ \frac{1}{2} & 0 \leq y \leq 1 \\ 0 & y < 0 \end{cases}$$





7. $F_X(\alpha) = P[X \leq \alpha]$ and $F_Y(\alpha) = P[Y \leq \alpha]$. We know that $X(s) \leq Y(s)$ for all $s \in S$. Therefore, we can say that $Y(s) \leq \alpha$ implies $X(s) \leq \alpha$, i.e., the set of all elements $s \in S$ such that $Y(s) \leq \alpha$ is a subset of the set of all elements $s \in S$ such that $X(s) \leq \alpha$.

$$\{s \in S : Y(s) \leq \alpha\} \subset \{s \in S : X(s) \leq \alpha\}$$

Therefore, we have

$$P\{s \in S : Y(s) \leq \alpha\} \leq P\{s \in S : X(s) \leq \alpha\}$$

which implies $F_Y(\alpha) \leq F_X(\alpha)$.