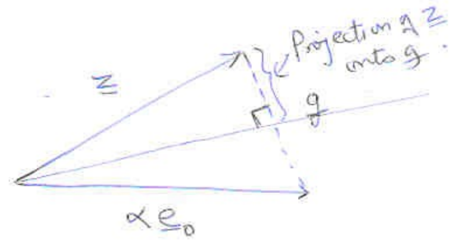
Problem Set 8

(1) $\underline{z} = [z(0) \quad z(1) \quad \cdots \quad z(n-1)]$ $\qquad \underline{e}_0 = [1 \quad 0 \quad \cdots \quad 0]$



$\underline{z}\,\Theta = \alpha\,\underline{e}_0$

$\Rightarrow |\alpha| = \|\underline{z}\|$ $\quad$ (Θ unitary)

& $\alpha z^*(0)$ is real (since $\underline{z}\,\Theta\,\underline{z}^H$ is real)

If $z(0) = |a| e^{j\phi_a}$, $\quad \alpha = \pm e^{j\phi_a}\|\underline{z}\|$.

Let $g = \underline{z} - \alpha\,\underline{e}_0$

g is also equal to 2(projection of $\underline{z}$ onto g)

$$\Rightarrow g = 2\left(\frac{\underline{z}\,g^H g}{g g^H}\right)$$

$$\Rightarrow \underline{z} - \alpha\,\underline{e}_0 = 2\,\underline{z}\,\frac{g^H g}{g g^H}$$

$$\Rightarrow \alpha\,\underline{e}_0 = \underline{z}\underbrace{\left(I - 2\frac{g^H g}{g g^H}\right)}_{\text{Required } \Theta}$$

where $g = \underline{z} \pm \|\underline{z}\| e^{j\phi_a}\,\underline{e}_0$

(2) $$A = \begin{bmatrix} 1 & 0.75 & 0.75 \\ 0.4 & 0.2 & 0.2 \end{bmatrix}$$

$\underline{x}_1 = [1 \quad 0.75 \quad 0.75]$ $\qquad \Theta_1 = I_3 - 2\dfrac{g_1^T g_1}{g_1 g_1^T}$

where $g_1 = \underline{x}_1 \pm \|\underline{x}_1\| [1 \quad 0 \quad 0]$

Let $g_1 = [1 \quad 0.75 \quad 0.75] + \|\underline{x}_1\|[1 \quad 0 \quad 0] = [2.2748 \quad 0.1500 \quad 1.0000]$

$\underline{x}_1 \Theta_1 = [-1.2748 \quad 0 \quad 0]$

$\underline{x}_2 \Theta_1 = [-0.4707 \quad -0.0871 \quad 0.1043]$

$$\begin{bmatrix} 1 & 0.75 & 0.75 \\ 0.4 & 0.2 & 0.2 \end{bmatrix}\Theta_1 = \begin{bmatrix} -1.2748 & 0 & 0 \\ -0.4707 & -0.0871 & 0.1043 \end{bmatrix}$$

If we choose $g_1 = x_1 - \|x_1\| [1 \; 0 \; 0]$, we get

$$\begin{bmatrix} 1 & 0.75 & 0.75 \\ 0.4 & 0.2 & 0.2 \end{bmatrix} \theta_1 = \begin{bmatrix} 1.2748 & 0 & 0 \\ 0.4707 & 0.0871 & -0.1043 \end{bmatrix}$$

To set this 0, we choose θ_2 such that $[0.0875 \;\; -0.1043]\,\theta_2 = [\alpha \;\; 0]$

$$\alpha = \| [0.0875 \;\; -0.1043] \| = 0.1359$$

$$\underbrace{A\,\theta_1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & \theta_2 \end{bmatrix}}_{\theta} = \underline{\underline{\begin{bmatrix} 1.2748 & 0 & 0 \\ 0.4707 & 0.1359 & 0 \end{bmatrix}}}$$

(3) $$H^H H = \begin{bmatrix} 1 & 0 & 1 \\ 1 & \epsilon & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & \epsilon \\ 1 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 & 2 \\ 2 & 2+\epsilon^2 \end{bmatrix}}}$$

rank = 2.

which in finite precision is $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

rank = 1

(4) $$\left[\begin{array}{c|c} \lambda^{1/2} \phi_{i-1}^{1/2} & h_i \\ \hline \lambda^{1/2} q_{i-1}^H & \bar{X}_i^* \end{array}\right] \theta = \left[\begin{array}{c|c} \phi_i^{1/2} & 0 \\ \hline q_i^H & e_a^*(i)\, \gamma^{1/2}(i) \end{array}\right]$$

+ve entries.

$$\left[\begin{array}{ccc|c} \times & 0 & 0 & \times \\ \times & \times & 0 & \times \\ \times & \times & \times & \times \\ \hline \times & \times & \times & \times \end{array}\right] \theta = \left[\begin{array}{ccc|c} \times & 0 & 0 & 0 \\ \times & \times & 0 & 0 \\ \times & \times & \times & 0 \\ \hline \times & \times & \times & \times \end{array}\right]$$

For $[a \;\; b]$ Given's rotation matrix $\dfrac{1}{\sqrt{1+|\rho|^2}} \begin{bmatrix} 1 & -\rho \\ \rho^* & 1 \end{bmatrix}$ where $\rho = \dfrac{b}{a}$, $a \neq 0$.

+ve diagonal entries.

$$\Theta = \Theta_1 \Theta_2 \Theta_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & -\rho_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \rho_1^* & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\rho_2 \\ 0 & 0 & 1 & 0 \\ 0 & \rho_2^* & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\rho_3 \\ 0 & 0 & \rho_3^* & 1 \end{bmatrix} \frac{1}{\sqrt{1+|\rho_1|^2}\sqrt{1+|\rho_2|^2}\sqrt{1+|\rho_3|^2}}$$

$x(-\rho_3) + x(1)$

$$= \begin{bmatrix} 1 & 0 & 0 & -\rho_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \rho_1^* & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & & 0 \\ & & & -\rho_2 \\ & & & -\rho_3 \\ & & & 1 \end{bmatrix} \frac{1}{\sqrt{1+|\rho_1|^2}\sqrt{1+|\rho_2|^2}\sqrt{1+|\rho_3|^2}}$$

$$= \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & X \end{bmatrix}$$

$$X = [\rho_1^* \ 0 \ 0 \ 1] \begin{bmatrix} 0 \\ -\rho_2 \\ -\rho_3 \\ 1 \end{bmatrix} = 1 > 0.$$