EE5040: Adaptive Signal Processing
Problem Set 3: Linear least-mean-squares estimation

1. (Sayed II.13, Correlated component) Assume that a zero-mean random variable X consists of two components, $X = X_c + Z$, and that only X_c is correlated with the observation vector $\mathbf{Y}$. Show that the linear least-mean-squares estimator of X given $\mathbf{Y}$ is simply the linear least-mean-squares estimator of X_c given Y.

2. (Sayed II.8, Weighted error cost) Show that the linear least-mean-squares estimator of $\mathbf{X}$ given $\mathbf{Y}$, given by $\hat{\mathbf{X}} = K_0\mathbf{Y}$ where K_0 is any solution to the linear system of equations $K_0 R_Y = R_{XY}$, also minimizes $E[\tilde{\mathbf{X}}^H W \tilde{\mathbf{X}}]$ for any $W \geq 0$.

3. (Sayed II.5, Minimum of a quadratic form) Consider the quadratic cost function $J(\mathbf{x}) = (\mathbf{x} - \mathbf{c})^H A(\mathbf{x} - \mathbf{c})$ where A is a Hermitian nonnegative-definite matrix and $\mathbf{x}$ and $\mathbf{c}$ are column vectors. Argue that the minimum value of $J(x)$ is zero and it is achieved at $\mathbf{x} = \mathbf{c} + \mathbf{d}$ for any $\mathbf{d}$ satisfying $A\mathbf{d} = \mathbf{0}$.